What Is the Trajectory of Arctic Sea Ice?

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We consider the trajectory in phase space of the Arctic sea ice thickness distribution, in which each dimension or component is the time series of sea ice area for a given ice thickness bin. We analyze the trajectory as determined by output from an ice-ocean model, finding that the first two principal components account for 98% of the variance. Simplifying the ice thickness distribution into thin ice, thick ice, and open water, we construct a simple empirical linear model that converges to a stable annual cycle from any initial state. When we include a quadratic nonlinearity to simulate a crude ice-albedo feedback, the model exhibits two stable states, one with perennial ice and one with ice-free summers, resembling the projections of some climate models for the late 21st century. We discuss the interplay between external forcing, internal dynamics, and “tipping points” in the decline of Arctic sea ice.

1. INTRODUCTION

Significant changes in the Arctic sea ice, ocean, atmosphere, ice sheets, and freshwater cycle over the past few decades are well documented [Intergovernmental Panel on Climate Change, 2007]. Several recent articles refer to the “trajectories” of these components, in which the word trajectory is used in a figurative sense [e.g., Overpeck et al., 2005; Peterson et al., 2006]. The literal meaning of a trajectory is a path in physical space or phase space. In this work we consider the trajectory of sea ice in the ice thickness phase space. We then analyze that trajectory as determined by model sea ice thickness distributions. Our use of the word trajectory does not refer to the physical motion of ice floes.

The sea ice thickness distribution $g(h)$ over some region $R$ gives the fractional area of $R$ covered by ice of thickness $h$. It is the fundamental description of the Arctic sea ice cover [Thorndike et al., 1975]. The processes controlling $g(h)$ are ice growth, melt, divergence, ridging, and import/export (into or out of $R$). Models that simulate the evolution of $g(h)$ often use discrete bins of ice thickness. Let $g_k$ represent the fractional area covered by ice with thickness in the range $h_k < h < h_{k+1}$, for $k = 1$ to $n$, where $n$ is the number of bins. The area with $h < h_1$ is considered open water. The sum of the fractional areas $g_1 + \ldots + g_n$ is the total ice concentration. Now consider $(g_1, \ldots, g_n)$ as a point in $n$-dimensional space. As time progresses, the point moves in the $n$ space, tracing out a trajectory. That is what we mean by the trajectory of Arctic sea ice. A point on the trajectory gives the ice thickness distribution at a particular time. Each component $g_k(t)$ is a time series of the fractional ice area in thickness bin $k$. The region $R$ over which this description applies may be chosen to be as small as one model grid cell or as large as the entire Arctic Ocean.
We analyze the trajectory of Arctic sea ice from two models. One is a retrospective coupled ice-ocean model that assimilates sea ice concentration data and is run for the years 1958–2005. Using the output of this model, we construct a pair of equations for the evolution of thin ice and thick ice that exhibits two stable annual cycles: one with ice-free summers and one with sea ice year-round. The second model is a coupled global general circulation climate model that is forced with increasing greenhouse gas and aerosol concentrations and is run for the years 1870–2099. This model attains ice-free summers by the second half of the 21st century. We use these models to contrast the influence of external forcing versus internal dynamics. We end with a brief discussion of tipping points and the transition from one state to another.

2. MODELS

The first model used in this study is the Pan-Arctic Ice-Ocean Modeling and Assimilation System (PIOMAS), which has been used in a wide range of retrospective climate studies [Zhang and Rothrock, 2005]. It is based on the parallel ocean and ice model of Zhang and Rothrock [2003]. It consists of the Parallel Ocean Program (POP) ocean model developed at Los Alamos National Laboratory coupled to a multicategory ice thickness and enthalpy distribution sea ice model [Zhang and Rothrock, 2001; Hibler, 1980]. The POP model is a Bryan-Cox-Semtner type ocean model [Bryan, 1969; Cox, 1984; Semtner, 1976] with numerous improvements. The sea ice model consists of five main components: (1) a momentum equation that determines ice motion, (2) a viscous-plastic rheology that determines the internal ice stress, (3) a heat equation that determines ice temperature profiles and ice growth or decay, (4) an ice thickness distribution equation that conserves ice mass, and (5) an enthalpy distribution equation that conserves ice thermal energy. The model has seven ice thickness categories plus open water (Table 1), and a latitude/longitude grid with mean spacing 22 km, with the

<table>
<thead>
<tr>
<th>Table 1. PIOMAS Sea Ice Thickness Bins</th>
<th>Bin</th>
<th>Lower Limit (m)</th>
<th>Upper Limit (m)</th>
<th>Designation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.00</td>
<td>0.10</td>
<td>open water</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.10</td>
<td>0.66</td>
<td>thin ice</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.66</td>
<td>1.39</td>
<td>thin ice</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1.39</td>
<td>2.47</td>
<td>thick ice</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2.47</td>
<td>4.57</td>
<td>thick ice</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>4.57</td>
<td>infinity</td>
<td>thick ice</td>
<td></td>
</tr>
</tbody>
</table>

The second model used in this study is the Community Climate System Model version 3.0 (CCSM3) [Collins et al., 2006]. It is a state-of-the-art fully coupled climate model composed of four separate models simultaneously simulating the Earth’s atmosphere, ocean, land surface, and sea ice. The sea ice model has five ice thickness categories plus open water (Table 2), and a nominal 1° grid with the pole displaced over Greenland. The model assimilates open-ocean sea surface temperature (SST) data using a nudging method with a 15-day time constant, and sea ice concentration data using a nudging method that emphasizes the ice extent and minimizes the effect of observational errors in the interior of the pack ice [Lindsay and Zhang, 2006]. The assimilated data (HadISST [Rayner et al., 2003]) consists of monthly averaged values on a 1° grid. The effect of the assimilation of total ice concentration is that the sum of the model ice concentrations over all the bins (g1 + ... + g7) is constrained to match (approximately) the observations. The assimilation of ice concentration also improves the modeled ice thickness compared to submarine and moored upward looking sonar data [Lindsay and Zhang, 2006]. Thus the PIOMAS ice concentration and ice thickness are in reasonable agreement with observations. The model is run for 48 years (1958–2005), forced with daily fields of 10 m surface wind velocities, 2 m air temperature and humidity, precipitation, and downwelling longwave and shortwave radiative fluxes. These are obtained from the ERA-40 reanalysis for 1958–2001, and from the ECMWF operational analysis for 2002–2005. The daily sea ice thickness distributions computed by the model are then temporally averaged over each month, and spatially averaged over 6.6 × 106 km2 of the central Arctic Ocean shown in figure 1. Thus the end result is one sea ice thickness distribution (with seven bins) for each month, 1958 through 2005.

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Report on Emissions Scenarios A1B scenario [Meehl et al., 2006] which assumes a moderate level of conservation and technical advance to limit emissions (i.e., greenhouse gas levels rise more slowly after about 2050). The distribution and thickness of Arctic sea ice have improved in CCSM3 relative to earlier versions. The model simulates sea ice extent, thickness, and trends reasonably well in the late 20th and early 21st centuries [Holland et al., 2006a]. The model is run for 230 years, of which we analyze the last 150 years (1950–2099). The sea ice thickness distributions are averaged in a manner similar to those from PIOMAS: monthly in time and Arctic-wide in space. The spatial domain is essentially the same as that shown in Figure 1 (although slightly more area along the northern edge of the Canadian Archipelago is included).

3. ANALYSIS

3.1. Principal Component Analysis

Consider the trajectory of sea ice from the PIOMAS model, which is a curve in seven-dimensional space. It is natural to ask whether this curve occupies the full seven dimensions, or whether it is primarily confined to a lower-dimensional subspace. Thus we look for linear combinations of the components $g_i$ through $g_7$ that account for as much variance as possible, i.e., principal component analysis. After removing the mean from each time series $g_i(t)$, we form the $7 \times 7$ covariance matrix and compute its eigenvalues and eigenvectors. The sum of the eigenvalues is the total variance of all the components, which includes the seasonal cycle. It turns out that the first two eigenvalues account for 98% of the variance; the trajectory of Arctic sea ice is essentially two-dimensional. The first two principal components are given by

$$\begin{align*}
PC_1 &= -0.56g_1 + 0.76g_2 + 0.33g_3 + 0.03g_4 \\
PC_2 &= 0.35g_1 + 0.56g_2 - 0.69g_3 - 0.29g_4 - 0.07g_5 - 0.02g_6.
\end{align*}$$

The two-dimensional trajectory in principal component space is shown in Figure 2. The annual cycles go clockwise, drifting slowly over time toward increasing values of $PC_2$. Notice from equation (1) that $PC_2$ is roughly “thin ice minus thick ice.” Thus the trajectory indicates increasing thin ice and decreasing thick ice.

To investigate the effect of the number of ice thickness bins on the principal component analysis, we also analyzed a 56-year run (1948–2003) of a version of the PIOMAS model with 11 ice thickness bins. We found that the first two principal components account for 91% of the total variance.

Figure 1. The sea ice thickness distribution from PIOMAS is averaged over the 6.6 million km$^2$ of the central Arctic Ocean shown by the gray cells.

Figure 2. Trajectory of Arctic sea ice in principal component (PC) space. The annual cycles go clockwise, drifting slowly over time toward increasing values of the second principal component.
Thus the two-dimensionality of the trajectory is fairly robust. Nevertheless, it is not straightforward to give physical interpretations to the principal components in equation (1). Therefore we take a slightly different approach, as follows.

### 3.2. Thin Ice and Thick Ice

Motivated by the results in the previous section, we form linear combinations of the bins $g_1$ through $g_7$ that have simple physical interpretations, at the expense of accounting for less than the maximum possible variance. The combinations are simply thin ice ($g_1 + g_2$) and thick ice ($g_3 + g_4 + g_5 + g_6 + g_7$). Unlike the principal components, which are uncorrelated, thin ice and thick ice have nonzero covariance, and thus the total variance cannot be neatly partitioned into thin and thick contributions. The sum of $\text{var}(g_1) + \ldots + \text{var}(g_7)$ is $300 \times 10^{-4}$. The covariance matrix of the first two principal components is

$$
\begin{bmatrix}
69 & 0 \\
0 & 224
\end{bmatrix} \times 10^{-4},
$$

while the covariance matrix of thin ice (first row/column) and thick ice (second row/column) is

$$
\begin{bmatrix}
69 & -50 \\
-50 & 113
\end{bmatrix} \times 10^{-4}.
$$

Thus thin ice and thick ice are negatively correlated, but their variances ($69$ and $113 \times 10^{-4}$) capture a reasonably large fraction (61%) of the total variability.

Figure 3 shows the trajectory of Arctic sea ice in thin/thick space. The axes give the concentration of each ice type. All years (1958–2005) are shown in gray. The annual cycles go counterclockwise. Monthly values for January to December are shown for the years 1966 and 2005. The distance from a point on the trajectory to the line thin + thick = 1 is the fraction of open water. Consider the physical sources and sinks of thin ice and thick ice throughout the year. During the winter, thin ice grows thicker (sink of thin ice, source of thick ice). This can be seen in January through April as the total ice concentration remains close to 1 but the trajectory moves toward the upper left. May is a transition month, when thick ice reaches a maximum. From June through September, thick ice decreases. This melting of thick ice is a source of thin ice, which is offset to some extent by the melting of the thin ice itself. In 1966 the thin ice increased from June to September (being replenished by melting thick ice more quickly than its own melt rate), whereas in 2005 the thin ice decreased from June to September (being replenished by melting thick ice more slowly than its own melt rate). Notice that the total ice concentration in September 1966 was about 0.8 while in September 2005 it was 0.6. October and November are freeze-up months, with rapid growth of thin ice and hardly any increase in thick ice. December is another transition month, when full winter conditions are reached, with a total ice concentration close to 1 again.

The gradual shift over time toward less thick ice and lower total ice concentration in September is clearly evident in Figure 3 (more so than in the principal component trajectory of Figure 2). What are the factors driving this transition? They can be separated into two broad categories: external forcing and internal dynamics. External forcing refers to (for example) increasing air temperature that melts more ice; internal dynamics refers to the response of the system to perturbations in its state. For example, if the system has multiple stable equilibrium states, a small perturbation could knock the system from one basin of attraction into another, sending it on a course toward less ice even when normal forcing conditions are restored. Several investigators have found multiple stable states in sea ice models [e.g., Flato and Brown, 1996; Hibler et al., 2006; Merryfield et al., this volume], which we discuss in more detail later. These were physical models, in which processes such as ice growth and melt were explicitly
3.3. Empirical Models

While Merryfield et al. [this volume] constructed a simple physical model to mimic the essential behavior of the sea ice component of CCSM3, we adopt an empirical approach toward “modeling the model” in which we postulate a simple form for the evolution equations of thin ice and thick ice that includes undetermined coefficients, use the output of PIOMAS or CCSM3 to find the coefficients that best reproduce the thin/thick trajectory, and then study the stability properties of the resulting equations.

3.3.1. One-dimensional linear model. We start with a one-dimensional formulation. Let \( z \) equal the total sea ice concentration in the Arctic. We postulate \( dz/dt = p(t)(1 - z) \), where \( p(t) \) is an annually periodic function of the form \( p(t) = A \cos(\omega(t - \tau)) \). With \( t \) measured in months, the frequency \( \omega \) is \( 2\pi/12 \). The constant \( \tau = 10.3 \) months is a phase shift to align the minima of \( z(t) \) with the end of summer. The motivation for the form of this equation is that it is linear in \( z \), the solutions are periodic, and it gives the correct shape of the annual cycle (as shown presently). If \( A \) is a constant then the amplitude of each annual cycle is the same. If we allow a different value of \( A \) each year, we can reproduce interannual variability. Figure 4 shows the solution of this one-dimensional equation (solid line), together with the monthly Arctic sea ice concentration from PIOMAS (black dots). The annual values of \( A \) used in this 10-year interval are \( [0.98, 0.96, 0.97, 0.99, 0.98, 0.83, 0.81, 0.90, 0.84, 1.19] \). The fit is remarkably good: the bias between \( z \) and the PIOMAS ice concentration is less than 0.002, with a standard deviation of 0.008. Apparently the annual cycle has only one degree of freedom per year, an amplitude that determines the September minimum. The success of this one-dimensional fit motivates the following two-dimensional case.

3.3.2. Two-dimensional linear model. Let \( x \) equal thin ice concentration and \( y \) equal thick ice concentration. We wish to formulate simple equations for \( x \) and \( y \) that reproduce, in an average sense, the annual trajectories of Figure 3. Based on the one-dimensional case, we choose linear equations with annually periodic coefficients:

\[
\begin{align*}
\frac{dx}{dt} &= a(t)(1 - x - y) + b(t)x + c(t)y \\
\frac{dy}{dt} &= d(t)(1 - x - y) + e(t)x + f(t)y.
\end{align*}
\]

The term \( 1 - x - y \) is the open water concentration. The six functions \( a, b, c, d, e, \) and \( f \) are determined from the 48-year time series of PIOMAS thin ice and thick ice concentrations using a least squares method. Let time be measured in months, and replace the continuous \( dz/dt \) with \( \Delta x_k \), which represents the change in thin ice concentration from one month to the next. Consider the change from, say, January to February. We write

\[
\Delta x_k = a(1 - x_k - y_k) + bx_k + cy_k + \text{error}_k,
\]

where \( x_k \) and \( y_k \) are the concentrations of thin ice and thick ice in January of year \( k \) and \( \Delta x_k \) is the change in thin ice concentration from January to February of year \( k \). This is 48 equations (48 years, \( k \) = 1 to 48) with three unknowns: \( a, b, \) and \( c \). We find \( a, b, \) and \( c \) by using a standard least squares procedure that minimizes the variance of the error term. The process is repeated for the change from February to March, March to April, etc. Thus \( a(t) \) consists of 12 values, one for each month, and similarly for \( b(t) \) and \( c(t) \). Finally, the same procedure is applied to the thick ice equation (for \( \Delta y_k \)) to obtain the 12 values of \( d(t), e(t), \) and \( f(t) \). The equations (2) can then be integrated in time, starting from some initial state \( (x_0, y_0) \). Figure 5 shows the resulting equilibrium trajectory that is obtained, no matter what the initial state. The full 48-year thin ice/thick ice trajectory from PIOMAS is shown in gray. The equilibrium trajectory is clearly an average of the 48 years, by construction. The coefficients \( a(t), b(t), c(t), d(t), e(t), \) and \( f(t) \) parameterize the annual cycle of forcing fields such as air temperature (i.e., cold in winter and warm in summer). If the coefficients were constants instead of periodic functions, the equilibrium would be a single point.

Figure 4. Solid curve is the solution of \( dz/dt = p(t)(1 - z) \), where \( p(t) \) is a cosine function with a period of 1 year and a different amplitude for each year shown by solid curve. Black dots indicate monthly Arctic sea ice concentration from PIOMAS.
The periodicity of the coefficients allows for an annual cycle of thin ice and thick ice. There is no interannual variability in this empirical model. The point of this two-dimensional exercise is that a simple linear model can reproduce the annual cycle of thin ice and thick ice fairly well, and it has one stable state: the trajectory always converges to that of Figure 5 no matter what the initial condition. It will never evolve to an ice-free Arctic.

If we add together equations (2), we obtain a single equation for the total ice concentration, \( z = x + y \), of the form\( \frac{dz}{dt} = (a + d)(1 - z) + (b + e)x + (c + f)y \). On the basis of the one-dimensional case above, we would expect \( a + d \) to be approximately a cosine function with amplitude close to 1, and \( b + e \) and \( c + f \) should be zero or small. This is indeed the case, as shown in Figure 6. This confirms that the two-dimensional model is consistent with the one-dimensional model.

A trend in external forcing (such as increasing atmospheric CO\(_2\)) is capable of driving the trajectory of Arctic sea ice toward a state with ice-free summers, according to the recent predictions of several models [Zhang and Walsh, 2006]. Figure 7 shows the trajectory of the CCSM3 model for 150 years (1950–2099). The annual cycles through 2005 are similar to those from PIOMAS (Figure 3). From 1950 through about 2005, the amount of thick ice in September decreases but the amount of thin ice in September is fairly constant. After 2005 the September trajectory swings toward (0,0), with decreasing amounts of thin ice nearly every year. The annual cycle for 2044 is shifted downward relative to 2005, i.e., less thick ice, but there is also a noticeable change in the shape of the annual cycle: (1) In 2044 there is a loss of thin ice from June to July, whereas in 2005 and earlier years there is a gain of thin ice. (2) In 2044 the loss of thin ice from July to August is far greater than in the earlier years. This might be understood as follows:

1. When there is a relatively large amount of thick ice in June, and it begins to melt, it creates thin ice faster than the thin ice itself can melt. But when there is relatively little thick ice in June, its conversion to thin ice cannot keep up with the melting of the thin ice itself.
2. With the creation of more open water in July, the ice-albedo feedback may be exerting more influence. The fraction of open water in July goes from 0.19 (1966) to 0.27 (2005) to 0.51 (2044). That is an increase of 42% in the first 39-year period and 89% in the second 39-year period. With more open water in July, more solar radiation is absorbed by the ocean, leading to more melting of thin ice and the creation of more open water.

These processes can also be understood in terms of open water formation efficiency (OWFE) [Holland et al., 2006b; Merryfield et al., this volume], which is the change in open water fraction per meter of ice melt, over the course of the melt season (May to September). OWFE increases dramatically in CCSM3 simulations as the mean winter ice thickness decreases; that is, it becomes much easier to create open water for a given amount of melt if one starts with thinner ice. The strength of the OWFE depends on the ice thickness distribution, i.e., on the internal model dynamics. The ice-free summers in CCSM3 toward the end of the 21st century are undoubtedly due to both the trend in forcing and the internal model dynamics, but the relative contributions of these factors are unknown.

What role might internal dynamics play in the transition to an ice-free Arctic summer? Flato and Brown [1996] constructed a one-dimensional thermodynamic model to study landfast sea ice. The model successfully reproduced seasonal and interannual variability of ice thickness at two locations in the Canadian Arctic for which data were available. The model was also shown to have two stable states: thin seasonal ice and thick perennial ice. A relatively modest change in climate was sufficient to cause the model to jump from the thick perennial state to the thin seasonal state. An unstable equilibrium existed between the two stable states (as is always the case) which arose because of the contrast in albedo between thick ice and thin ice/open water. Merryfield et al. [this volume] studied the phenomenon of abrupt sea ice reductions in CCSM3 (as documented by Holland et al. [2006b]) by constructing a simplified physical model of the essential processes with just two dependent variables: winter sea ice thickness and summer sea ice extent. They found that pulses of ocean heat transport into the Arctic, along with increased sensitivity of summer sea ice to declining ice extent, were the likely causes of the abrupt drops in summer sea ice. Their nonlinear model also exhibited multiple stable states in a physically relevant parameter regime. These examples suggest that internal dynamics could play a role in guiding the trajectory of sea ice toward a new equilibrium state after the system crosses from one basin of attraction into another, and that the ice-albedo feedback and increasing summer open water are likely to be important factors.

3.3.3. Two-dimensional nonlinear model. We investigate the above ideas by making a small modification to the equations (2). We add a quadratic term to the equation for thin ice:

\[
\frac{dx}{dt} = a(t)(1 - x - y) + b(t)x + c(t)y + a(t)(1 - x - y)^2
\]

\[
\frac{dy}{dt} = d(t)(1 - x - y) + e(t)x + f(t)y ,
\]

where \(a(t)\) is negative in summer and zero otherwise. The quadratic term simulates enhanced melting of thin ice in summer as the fraction of open water increases. Equation (3) is therefore modified by adding a corresponding quadratic term:

\[
\Delta x_k = a(1 - x_k - y_k) + bx_k + cy_k + a(1 - x_k - y_k)^2 + \text{error}_k ,
\]

where we specify a negative value of \(a\) for the June-July and July-August equations, and \(a = 0\) otherwise. We then solve for \(a, b, c\) and \(d\) as before using the PIOMAS model output and a standard least squares procedure. We need only do this for the June-July and July-August thin ice equations; all the other coefficients remain the same. Having found the new coefficients, we then integrate the equations (4), with specified initial conditions for thin ice and thick ice in September. (If a negative value of \(x\) or \(y\) is obtained for a given month, it is reset to zero before continuing the integration.) The results are shown in Figure 8, using ad hoc values of \(a = -3\) for June-July and \(a = -6\) for July-August. There are now two equilibrium cycles, one with perennial ice and one with seasonal ice, depending on the initial conditions.

If we start the integration in September within the gray triangle (bottom left), the trajectory evolves to the seasonal ice cycle. If we start outside the gray triangle, the trajectory evolves to the perennial ice cycle. Comparison of the perennial ice cycle with that of figure 5 obtained from the linear equations (2) shows that the thin ice and thick ice fractions match within 0.02 in every month: the perennial ice cycle is nearly unchanged. The quadratic nonlinearity has allowed the emergence of a seasonal ice cycle, which can only be sustained if the summer open water is sufficiently large. The seasonal ice cycle resembles the annual cycle for the year 2044 in CCSM3 (Figure 7). Comparing the size of terms in equations (2) and (4), we find that when the open water fraction \(1 - x - y\) becomes greater than about 0.2, the nonlinearity becomes important, with increasing dominance as \(1 - x - y\) increases.
Figure 9 shows the approach to equilibrium. The black dots are different September initial conditions from throughout the domain. Black and gray paths trace subsequent Septembers obtained by integrating equations (4) with $\alpha = -3$ for June-July and $\alpha = -6$ for July-August. Black trajectories lead to the equilibrium September value for perennial ice; gray trajectories lead to the equilibrium September value for seasonal ice. Notice how the September trajectories all rapidly approach a curving “attractor” and then continue along the attractor to the equilibrium point (gray circle). The triangles are the 48 Septembers of the PIOMAS model, clustered loosely around the perennial ice equilibrium point and along the attractor. Although the black and gray paths are derived from equations (4) with coefficients fit to the PIOMAS model output, we also show the 150 Septembers of the CCSM3 model (squares and pluses). The CCSM3 Septembers for 1950–2005 (squares) do not match those of PIOMAS because of the different thin ice/thick ice cutoffs (Tables 1 and 2) and because of differences between the two models. However, after about 2025 the CCSM3 Septembers follow a path close to the curving attractor toward the seasonal ice equilibrium near (0,0).

4. DISCUSSION

We reiterate that the equations (2) and (4) do not explicitly model physical processes. The annually periodic coefficients, derived empirically, are meant to simulate the annual cycles of the mean external forcing fields, with no interannual variability. The linear equations (2) are perhaps the simplest possible nontrivial evolution equations for thin ice and thick ice, and the quadratic term in equation (4) is a simple nonlinearity. Nevertheless, these empirical models illustrate that (1) it is not difficult to formulate a system with multiple stable states and (2) the inclusion of a quadratic nonlinearity that mimics the ice-albedo feedback leads to the emergence of a stable annual cycle with ice-free summers (August–September). Serreze et al. [2007] raised the question of how a seasonally ice-free Arctic Ocean might be realized: through gradual decline or through a rapid transition once the ice thins to a more vulnerable state? Our empirical model would allow the second scenario to exist if there were a mechanism to nudge the system across the “tipping point” from one stable state to another.

The concept of a tipping point has recently gained popularity in the press [Walker, 2006]. A tipping point is
unstable equilibrium between two stable ones. (In our nonlinear empirical model, the tipping point for September is actually the entire hypotenuse of the gray triangle in Figure 8). The transition across a tipping point could be precipitated by a particular event, or it could be gradual. Lindsay and Zhang [2005] hypothesized that the thinning of Arctic sea ice since 1988 was triggered by the export of older, thicker ice out of the Arctic basin in the late 1980s and early 1990s, driven by atmospheric circulation patterns associated with high values of the Arctic Oscillation (AO) and the Pacific Decadal Oscillation (PDO). The thinning continued in the following years, even though the AO and PDO returned to near normal levels, because the ice-albedo feedback was then exerting a larger influence because of the increase in summer open water and thin ice. The abrupt reductions in future Arctic sea ice in CCSM3 simulations were triggered by “pulse-like” events of ocean heat transport into the Arctic Ocean [Holland et al., 2006b, this volume], followed by a similar increase in the influence of the ice-albedo feedback due to more open water. Winton [2006] examined two climate models that both became seasonally ice free gradually as the air temperature increased, but further warming caused an abrupt loss of sea ice year-round in one of the models (MPI) because of the ice-albedo feedback. The other model (CCSM3) lost all of its sea ice more gradually, driven primarily by the ocean heat flux. Thus we have examples of both abrupt and gradual changes between states.

A simple model that exhibits a tipping point is \( dz/dt = -k(z-a)(z-b) + F(t) \) where \( k > 0 \) is a constant, \( 0 < a < b \) are constants, and \( F(t) \) is the external forcing. (We are thinking of \( z \) as the September sea ice extent). First consider \( F = 0 \). The two stable equilibria are \( z = 0 \) and \( z = b \). They are separated by the unstable equilibrium point (tipping point) \( z = a \). Any trajectory that starts with \( z > a \) converges to \( z = b \); any trajectory that starts with \( z < a \) converges to \( z = 0 \). The constant \( k \) (with units 1/time) determines the rate of convergence. Large \( k \) means strong internal dynamics (short response time of the system); small \( k \) means weak internal dynamics (long response time). Now turn on the external forcing \( F \). Clearly \( F \) can be formulated to kick the system back and forth between its two regions of attraction, either randomly or cyclically, suddenly or gradually. If \( F \) is of the form \( A \sin(\omega t) \) and we nondimensionalize time by \( t' = kt \) then the nondimensional external forcing is \( F' = (A/k)\sin((\omega/k)t') \).

The ratio \( A/k \) is the strength of the external forcing relative to the strength of the internal dynamics. The ratio \( \omega/k \) is the response time of the system relative to the period of the external forcing. If \( A/k \) is small, the system will not cross the tipping point; if \( A/k \) is large, it will cross back and forth regularly. Suppose instead that \( F' \) is a random process with mean zero and standard deviation \( S \). Then \( S/k \) is analogous to \( A/k \), and the autocorrelation time scale of \( F' \) is analogous to the period \( 2\pi/\omega \). If \( S/k \) is small, the system will fluctuate about one of the stable equilibrium points. At larger values of \( S/k \), the system will make occasional, gradual transitions across the tipping point, sometimes flipping back and forth several times before approaching one of the stable equilibrium points. At still larger values of \( S/k \), the transitions are abrupt. In the limiting case of large \( A/k \) or \( S/k \), the system is dominated by the external forcing, and the internal dynamics become negligible.

We are led to the following thoughts about tipping points:

1. A tipping point may be approached or crossed suddenly because of an externally forced event, or the transition may be more gradual as the external forcing changes gradually.
2. When the system is near a tipping point, it is more sensitive or susceptible to being nudged into a new state by small perturbations.
3. Crossing a tipping point is not an irreversible event. The system can be driven back into the previous state if the external forcing changes course.
4. The trajectory of the system is determined by a balance between the amplitude and period of the external forcing and the response time of the system (internal dynamics).

5. SUMMARY AND CONCLUSIONS

We have defined the trajectory of sea ice to be the path in phase space of the ice thickness distribution, \((g_1(t),\ldots, g_6(t))\), where \( g_i(t) \) is the fractional area of sea ice in bin \( i \) at time \( t \). We analyzed the 48-year monthly output of an ice-ocean model with seven bins and found that the first two principal components of the trajectory account for 98% of the variance: the trajectory is essentially two-dimensional. Simplifying the ice thickness distribution into thin ice, thick ice, and open water, we constructed a linear model with empirically determined periodic coefficients that matches the mean annual cycle of the 48-year ice-ocean model output.

The linear model was found to be stable. We then modified the linear model by adding a quadratic term to simulate enhanced melting of thin ice in summer when the amount of open water is large, i.e., a crude ice-albedo feedback. The nonlinear model was found to have two stable annual cycles, one with ice-free summers and one with perennial summer ice, qualitatively similar to the results of Plato and Brown [1996]. The annual cycle with ice-free summers resembles the late 21st century annual cycles of the CCSM3 model projection.

The projection of an ice-free Arctic in summer is not new, nor is the idea that Arctic sea ice may have multiple stable states. Twenty-eight years ago, Parkinson and Kellogg [1979] ran a sea ice model forced by atmospheric warming commensurate with a doubling of \( CO_2 \). They found that a 5°C
increase in surface air temperature led to the disappearance of sea ice in August and September. Also in 1979, Kellogg [1979, p. 85] wrote, “There are good reasons to believe that the Arctic Ocean may have just two stable states, a largely frozen-over one (as at present) and an ice-free one.” What is now new compared to 1979 is the sustained downward trend in Arctic sea ice, capped by record-shattering losses in August and September 2007. Using data from the National Snow and Ice Data Center [Fetterer et al., 2002], we find that the best linear fit of September sea ice extent versus year (1979–2006) leaves residuals with standard deviation 4.2 × 10^4 km², in terms of which the September 2007 sea ice extent falls 4 standard deviations below the trend line. This is the type of abrupt loss of summer sea ice simulated by CCSM3 starting in the year 2024 [Holland et al., 2006b, this volume], and it continues the pattern of observed ice extent declining faster than model predictions [Stroeve et al., 2007]. One could argue that Arctic sea ice is now entering a new regime. In any case, it seems likely that increasing greenhouse gases, driving increasing air temperatures, will eventually lead to summers without sea ice in the Arctic Ocean. The interplay between the internal dynamics of the climate system and the external forcing will determine the extent to which this outcome is achieved sooner rather than later. Whether or not CCSM3 or the real climate system actually has a “tipping point” is still unknown.

Several extensions of this work are possible. One could divide the Arctic Ocean and peripheral seas into regions, and construct and analyze the trajectory of sea ice in each region. One could add a third category of ice thickness to the present thin ice and thick ice (e.g., medium thick ice), thereby capturing more of the variance. A more interesting extension would be to construct a simplified physical model of the evolution of thin ice and thick ice, rather than an empirical model. This was proposed by Stern et al. [1995], based on the framework of Thorndike et al. [1975] and Hibler [1980]. One could then analyze the stability of the physical model and attempt to attribute the observed changes in sea ice to changes in the external forcing and to the influence of internal dynamics.

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