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PROGRESS REPORT, 1971 AIDJEX PILOT STUDY  

AIDJEX MAILING LIST  

Cover: Photograph of Camp 200, site of the 1971 AIDJEX pilot study, taken during a remote-sensing flight at 3,500 ft. by the NASA 990 research aircraft Galileo. The camera used is a Wild-Heerbrugg RC-8 metric mapping camera installed in the NASA aircraft.
Arctic Ice Dynamics Joint Experiment
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Division of Marine Resources
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MEASUREMENT OF TILT OF A FROZEN SEA

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Due to the effects of winds, ocean currents, tides, and atmospheric pressure gradients and density changes in the water, the surface of the ocean is tilted relative to the equipotential surface of the earth. It has been estimated that local tilts of up to $10^{-4}$ radians may occur in coastal waters or near the eye of a hurricane [1]. Extended tilts in the open ocean are probably less than $10^{-5}$ radians. No technique has yet been developed for measuring the tilt of an ice-free open ocean. However, it is possible to measure the tilt of a water surface relative to its local equipotential surface at the point of observation if the water is ice-covered.

In 1952, Browne and Crary [2] attempted to measure the tilt of the Arctic Ocean from Fletcher's Ice Island (T-3) by freezing a sensitive bubble level into the ice and observing the bubble over a five-month period. Fig. 1 shows the change of ocean tilt compared with the atmospheric pressure gradient obtained from synoptic maps over the same time period. The graph shows a remarkable correlation between the tilt, which changes over a range of two seconds of arc ($9.6 \times 10^{-6}$ radians), and the pressure gradient in three orientations with respect to the directions of the spirit level. The fact that the long period change of tilt occurs over weeks rather than over days or hours indicates that the tilt is not likely to be a local phenomenon but may extend over a large region. In view of the scale of the Arctic Ocean and the extent of the meteorological pressure, ocean tilts could conceivably persist for distances as large as 1000 km and considerable sea-level changes would take place.

The Earth Physics Branch of the Department of Energy, Mines and Resources is engaged in a continuing program of regional gravity mapping.
over the Arctic Ocean. Such sea level changes, if they occur, would significantly affect our gravity measurements. For instance, if an ocean tilt of two seconds of arc existed over a distance of 1000 km, an elevation change of the sea surface of 10 m would result. This would change the gravity anomaly by 2.7 mgal. Since gravity observations are made with an accuracy of ±0.1 mgal, we were motivated to measure the tilt of the fluid surface of the ocean in the vicinity of the North Pole by optical leveling using a DKM-3 theodolite [3]. We drilled three holes through the ice and leveled from water surface to water surface—first in one, then in the other, direction—

Fig. 1. Above: Change of tilt of the ice island T-3 over a five-month period in 1952 as measured with a bubble level frozen in the ice. Below: Change of atmospheric pressure gradient over the same time period as determined from synoptic weather maps. Drawn from original publication [2].
in order to provide a means of computing atmospheric refraction. Two sets of observations were taken at a distance of about 56 km from the North Pole at about 0400 gmt on May 11 and at 1800 on May 13, 1967. Both measurements showed a downward tilt of 8±2 seconds of arc in the direction of 100° and 150° east of Greenwich, respectively.

The fact that the direct and reversed measurements were not made concurrently created a problem related to the question of the consistency of atmospheric refraction, even though the basic technique corrects for this factor [3]. Nevertheless, this experiment showed that, if our measurements are correct, large ocean tilts in the polar region may occur. Because of atmospheric refraction, this technique is not suitable for the degree of precision which we desired. The search for a better method led to the development of a hydrostatic leveling system that was tested in the vicinity of the North Pole in April and May 1969 and in the Gulf of St. Lawrence in March 1970 [4].

The principle of hydrostatic leveling is shown schematically in Fig. 2. Two open reservoirs, 1 and 2, are connected by a tube and the system is filled with a low viscosity fluid. If the system is not moving, and under conditions of equal atmospheric pressures at 1 and 2 and constant fluid temperature throughout the system, the fluid surface in the two reservoirs form part of the same equipotential surface independent of the tube path.

Fig. 2. Principle of hydrostatic leveling for measuring ocean tilt.
If $\Delta h_1$ and $\Delta h_2$ are the level differences between the fluid levels of the reservoirs and the ocean surface, and $s$ is the separation between the two reservoirs, then the tilt is $\frac{\Delta h_1 - \Delta h_2}{s}$.

The actual leveling system consists of two insulated 3 m plastic pipes which form two wells. The pipes are frozen into the ice, and the water inside the wells is prevented from freezing by electric heating (Fig. 3). Rigidly bolted to the pipes are two reservoirs connected by a horizontal Tygon tube and filled with a silicone fluid of low viscosity.

Fig. 3. Schematic diagram of hydrostatic leveling system.

In developing the system we attempted to keep the measurement of the level difference $\Delta h_1 - \Delta h_2$ (Fig. 3) within a standard deviation of $\pm 2 \times 10^{-5}$ m, resulting in a sensitivity of $\pm 1.6 \times 10^{-7}$ radians, with a level length of $s = 120$ m.
On a drifting ice floe, a variety of disturbances act on the hydrostatic level, and their effects require consideration. The hydrostatic level is affected specifically by temperature changes in the fluid, by density changes of the water in the wells, by atmospheric pressure gradients, and by differential flow of the water below the wells (Bernoulli effect). In addition, inherent in any method of tilt measurements from a moving platform, the system is affected by horizontal accelerations and by the Coriolis force. The effect of some of these forces and disturbances can be minimized by proper instrument design; some can be corrected for, or neglected, depending on ice drift and atmospheric conditions.

The instrument is illustrated schematically in Figs. 3 and 4. The level difference \( \Delta h_1 - \Delta h_2 \) is determined by measuring the level differences \( f_1, w_1, f_2, w_2 \) relative to a level plate using pointed measuring rods connected to micrometers. The sensitivity of the system is given by the ratio of the smallest fluid level difference \( \Delta h_1 - \Delta h_2 \) that can be measured and the level length \( s \). The accuracy with which the quantity \( \Delta h_1 - \Delta h_2 \) can be determined is limited by the movement of the water level in the well. The results of many observations have shown that under calm conditions the level heights \( w_1 \) and \( w_2 \) (Fig. 4) can be measured with a standard deviation of about \( \pm0.8 \times 10^{-3} \) cm.

Level measurements with a rod assembly can be repeated to an accuracy of a few microns, and therefore the sensitivity of the instrument is not limited by the accuracy with which level differences of the fluid can be sensed, but rather by the effect of the movement of the water surface in the wells and by the effect of external disturbances such as temperature changes, atmospheric pressure changes, and ice movements. These external effects can be neglected if the resulting fluid level displacement does not exceed \( \pm1.0 \times 10^{-3} \) cm; they must be corrected if they are greater. The threshold value of those disturbances below which the effects can be neglected are summarized in Table I.

Commonly the pack ice drifts at speeds and accelerations which are a magnitude larger than the threshold values; of all the external effects, the system is most sensitive to drift speed and acceleration. Precise navigation is therefore a prerequisite to precise tilt measurements. In
Fig. 4. Diagram of instrument. Level differences $f$ and $w$ of Fig. 3 are measured relative to a level plate with pointed steel rods attached to micrometers. The temperature of the fluid in the reservoir is measured with thermistors. The reflection of the pilot light in the well serves to indicate when the point touches the water.
the Arctic Ocean, we determined our absolute position from astronomical fixes and our relative motion by ranging from acoustic transponders on the ocean floor. In the Gulf of St. Lawrence, we used Decca navigation.

Table I

<table>
<thead>
<tr>
<th>Error Source</th>
<th>Threshold Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Temperature of the hydrostatic fluid</td>
<td>±0.06°C</td>
</tr>
<tr>
<td>2. Density change of the well water</td>
<td>±5 x 10^{-6} g cm^{-3}</td>
</tr>
<tr>
<td>3. Atmospheric pressure gradient</td>
<td>30 µbar km^{-1}</td>
</tr>
<tr>
<td>4. Differential flow (Bernoulli effect)</td>
<td>$V_1^2 - V_2^2 &lt; 2 \text{ cm}^2 \text{s}^{-2}$</td>
</tr>
<tr>
<td>5. Horizontal accelerations</td>
<td>$1.6 \times 10^{-4} \text{ cm s}^{-2}$</td>
</tr>
<tr>
<td>6. Drift velocity (Coriolis force)</td>
<td>1.1 cm s^{-1}</td>
</tr>
</tbody>
</table>

(1) As long as the connecting tube is perfectly horizontal only the temperature differences of the fluid between the two reservoirs matters. It is measured to an accuracy of ±0.01°C using thermistors. (2) The possibility of density changes of the seawater in the wells is avoided by pumping the old water out of the wells before each measurement. (3) Atmospheric pressure gradients in excess of 30 µbar km^{-1} are rare. (4) $V_1$ and $V_2$ are the current speeds of the water flowing past the bottom openings of the wells which cause a pressure decrease in the wells. The Bernoulli effect is insignificant for current speeds of less than 8 cm s^{-1}. During periods of rapid drift the flow rate must be measured with current meters. (5) The component of the acceleration in the direction of the level axis and (6) the velocity component at right angles to the level axis must be considered. The system is made insensitive to vertical accelerations (wave motions) by choosing its dimensions such that the dynamic response of the level fluid is critically damped.
The results of the tilt measurements near the North Pole, determined with two hydrostatic levels, are listed in Table II. Directions are west of Greenwich. The observed tilt has been calculated from the mean of 40 measurements of the level differences $f_1, f_2, w_1, w_2$ (Fig. 3) after corrections for the temperature changes of the reservoir fluids had been applied.

<table>
<thead>
<tr>
<th>Date</th>
<th>Position</th>
<th>Drift speed (cm/sec, constant)</th>
<th>Direction of drift</th>
<th>Direction of level axis</th>
<th>Observed tilt*</th>
<th>True tilt (Coriolis corrected)*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level I</td>
<td>April 30/69</td>
<td>5.6</td>
<td>300°</td>
<td>248°</td>
<td>4.71±0.07</td>
<td>5.22±0.07</td>
</tr>
<tr>
<td>Level II</td>
<td>May 2/69</td>
<td>1.8</td>
<td>56°</td>
<td>129°</td>
<td>3.15±0.08</td>
<td>3.23±0.08</td>
</tr>
</tbody>
</table>

*Down-slope in direction of level axis, in units of micro-radians.

Tilt observations in the Gulf of St. Lawrence were carried out in an area between Amherst Island and the eastern tip of Prince Edward Island. The sea surface sloped down in a northerly direction. Preliminary results show that the observed tilt ranges from $-0.4$ to $3.5 \times 10^{-6}$ radians [4].

Tests have shown that it is quite feasible, by using floats and displacement transducers, to modify the system so that it measures tilt automatically and continuously.

In conclusion, we have shown a method that has been experimentally tested and which shows promise of measuring the tilt of a frozen ocean with an accuracy which is almost a hundred times greater than previous methods. We believe that it is important, from both geophysical and oceanographic points of view, to use sensors of this type over baselines which are of the order of $10^2$ km in length to determine the extent and persistence of ocean
tilts. A desirable experiment of the frozen Arctic Ocean might involve four pairs of sensors at the corners of a 100 km square with an additional pair in the center. These would permit the statistical calculation of planar, quadratic, and cubic tilt surfaces with varying degrees of over-determination, since there will be ten distinct measurements of ocean tilt at these five stations.

REFERENCES


ICE BALANCE IN THE ARCTIC OCEAN

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INTRODUCTION

The problem of the ice balance of the Arctic Ocean has hitherto been considered indirectly. For example, Vowinckel (1964) determined ice export from a study of wind and current ice drift through the Greenland-Spitsbergen "gate." Ahlmann (1953), by comparing the ice thickness recorded by the Fram expedition with that taken by the Sedov personnel, concluded a negative balance for the period 1894-1938. With the advent of increasingly sophisticated remote sensing systems, it should prove feasible to determine the ice balance more directly. Put in its simplest form, the ice balance of the Arctic Ocean can be expressed by

\[ B = C - A - G \]  

(1)

where \( C \) and \( A \) are the accumulation and ablation of ice, \( G \) is the ice exported by drift out of the Arctic Ocean, and \( B \) is the net balance.

If a continuous record of ice thickness can be taken by remote sensing, then the solution to (1) can be based on data from a series of representative flights over the Arctic Ocean made at the same time each year. This is the "fixed date" method ("Mass Balance Terms," Journal of Glaciology, 1969) often used in glacier mass balance calculations. Failing this, it will be essential to determine the relative coverage of each ice type and then obtain a best estimate of the mean thickness. From recent work (e.g., Ketchum, 1970) the LASER profiler appears the most promising remote sensor in this respect.

There is to date scant information on the distribution of the various ice types in the Arctic Ocean, so that the data recorded by the British Trans-Arctic expedition in 1968 and 1969 are of value if only to bring a better perspective to the problem of ice balance.
METHODS

Between February 21, 1968, and May 27, 1969, the British Trans-Arctic Expedition crossed the Arctic Ocean between Barrow, Alaska, and a small island off the coast of Spitsbergen (Fig. 1). During the crossing, a log of the topography was kept and more than 250 holes were drilled through old ice and more than 100 through one-year ice to measure thickness. In July and August, 1968, the ablation of a multi-year floe and the change of topography due to melting were measured. In winter 1968-1969, ice production in the local camp area was estimated by closely monitoring ice growth in fractures and under old ice.

ACCURACY

As the prime motive of the expedition was not a scientific one, the majority of the observations made during the travel periods were recorded while on the move. Briefly, the more serious errors result from the difficulty of determining the age of the ice, and the possible nonrepresentativeness of the traverse. While the extent of the latter error is impossible to determine at present, the dating error is believed to have led to an underestimating of the coverage of first-year ice by 2 to 3 per cent.

RELATIVE COVERAGE OF ICE TYPES AND ICE-FREE AREAS

The relative coverage of deformed, old, and young ice, and ice-free areas is shown in Table I and Fig. 2.

The extent of ridges and hummocks below the waterline (i.e., keels and bummocks) has been calculated from a pressure ridge model similar to Wittmann and Schule's (1966). From a study of under-ice profiles recorded by U.S. nuclear submarines, Wittmann and Schule (1966) found that 18 per cent of the ice was ridged in the winter of 1960 and 13 per cent in the summer of 1962. There is a difference of 3 per cent between the BTAE and the submarine data. Old bummocks and ice keels have presumably been included in the calculation from the submarine profiles, whereas in the present data, ridges and hummocks more than two years old and smoothed by summer ablation were recorded as unridged old ice. The difference in definition and a genuine
Fig. 1. The Arctic Ocean, route of the British Trans-Arctic Expedition. The broad arrows show the two main directions of ice drift. The Pacific Gyral lies between Pt. Barrow and the Pole, and the Trans-Polar Drift Stream is on the Russian side of the Pole. The BTAE drifted from A to B between July 1968 and February 1969.
Fig. 2. Distribution of ice types between 89°N and 81°N along approximately 30°E expressed as percent of total area.  
1. mean value in the Pacific Gyral, 2. mean value in the Transition Zone.  
A. hummocks and ridges, B. hummocks and keels.
annual variation may account for the 3 per cent discrepancy between the BTAE and the submarine data.

The term ice-free is defined in the WMO ice glossary (Dunbar, 1969) as "no sea ice present." In the present context, however, the term is loosely used to include areas in which the ice cover consists of frazil or grease ice, both of which form in a few hours in midwinter. Between the North Pole and Spitsbergen, only 0.6 per cent of the area crossed by the expedition was ice-free (Table I and Fig. 2). In March and early April, when the expedition was within a few degrees of the North Pole, only 0.2 per cent of the area was ice-free. Badgley (1966), on theoretical grounds, considered that less than 1 per cent of the Central Arctic Ocean should be ice-free in winter; Untersteiner (1964) considered the coverage to be less than 0.5 per cent. Profiles taken by the USS Sargo in the winter of 1960 found an ice-free coverage of 1.8 per cent (Wittmann and Schule, 1966). "Birds-eye" flight data give a mean winter ice-free coverage which is 11.0 per cent of the total area (Wittmann and Schule, 1966); this percentage seems unreasonably high.

Nilas, young, and first-year ice will be referred to collectively as first-year ice. The Gyral shows the smallest first-year ice coverage. This is in agreement with theoretical considerations of ice drift in the Arctic Ocean (e.g., Campbell, 1965) which predict a higher occurrence of convergent ice flow in the Pacific Gyral than elsewhere. The most extensive coverage of first-year ice crossed by the BTAE was between 85°N and 88°N along longitude 140°W (Fig. 1).

**ICE THICKNESS**

Despite the fact that 75 per cent of the thickness measurements were taken through old ice, there is a wide range of thicknesses (Fig. 3). However, a mean multiyear floe thickness is more important for the purposes of the present study. This may be termed a mean steady state thickness $\bar{h}_{88}$ which is dependent on the relative coverage and thickness of hummocked/ponded and level/unponded ice. The relative coverages have been measured from a set of aerial photographs taken by the Polar Continental Shelf Project in the Lincoln Sea in July, 1968, and from photographs taken by the BTAE. The
mean thickness of undeformed old ice is taken from the ice thickness data. A mean hummock thickness of 10 m has been estimated from a leveling profile across a multiyear floe and an above/below relationship of 1:7.

\[ \bar{h}_{gs} \] is calculated to be 347 cm in the Pacific Gyral, 300 cm in the Trans-Polar Drift Stream, and 323 cm in the transition zone between the two. The high value for \( \bar{h}_{gs} \) in the Pacific Gyral is due to the large number of hummocks on the relatively old multiyear floes there.

The mean thickness of the entire ice cover crossed by the expedition has been calculated from the extent and thickness of each ice type, including both undeformed and deformed ice. The results are shown in Table II. \( \langle h_t \rangle \) is greatest in the Pacific Gyral, which again agrees with both the theoretical and the observed patterns of ice drift in the Arctic Ocean.

**ACCUMULATION**

The accumulation of snow, superimposed ice, and fresh-water ice (i.e., ice forming at the salt-water/fresh-water interface in summer) was measured on the expedition; the results are shown in Fig. 4 and Table II.

First-year ice may be in either unridged or ridged forms. In the process of ridging of new ice, some old ice is often incorporated into the ridge. Between the North Pole and Spitzbergen, 15.8 per cent of the new ridges contained some old ice. However, the total amount of old ice in first-year ridges is probably no more than 5 per cent, and the calculations of the amount of new ice in ridged form have been corrected by this amount. The amount of new and first-year ice crossed by the expedition (i.e., the winter balance of ice less than one year old) is given in Table III.

Between October 2, 1968, and February 23, 1969, ice production in fractures forming in an area approximately 5 km square was measured. The area was traversed daily and the width of new fractures estimated. The

*Throughout the text, a symbol in angle brackets, such as \( \langle \bar{b} \rangle \), denotes a value meaned over the total ice and ice-free area. A symbol in square brackets, such as \([b_i]\), refers to a value meaned over the area covered by the type of ice denoted by the suffix. Areas (for example, \( A_i \)) are always expressed as percentages of the total ice and ice-free areas \( A_e \).*
Table I

Ice type, expressed as percent of distance covered between 90°N and 81°N along longitude 30°E

<table>
<thead>
<tr>
<th>Ice Type</th>
<th>% of Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Old ice</td>
<td>73.0</td>
</tr>
<tr>
<td>New ice, nilas, young ice, first-year ice</td>
<td>17.0</td>
</tr>
<tr>
<td>Ice-free</td>
<td>0.6</td>
</tr>
<tr>
<td>Hummocked or ridged ice (a)</td>
<td>9.0</td>
</tr>
<tr>
<td>Hummocked or ridged ice (b)</td>
<td>(14.5)</td>
</tr>
</tbody>
</table>

(a) surface  (b) bummocks and keels - for calculation, see text.

Table II

Ice export (g), accumulation (σ), ablation (a), balance, and thickness in the Arctic Ocean (cm of ice)

<table>
<thead>
<tr>
<th>Region</th>
<th>&lt;g_t&gt;</th>
<th>&lt;a_t&gt;</th>
<th>&lt;σ_t&gt;</th>
<th>&lt;b_1&gt;</th>
<th>&lt;b_x&gt;</th>
<th>&lt;b_y&gt;</th>
<th>&lt;h_p&gt;</th>
<th>&lt;h_e&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trans-Polar Drift Stream</td>
<td>62.8</td>
<td>58.1</td>
<td>61.2</td>
<td>46.3</td>
<td>16.2</td>
<td>0</td>
<td>0.3</td>
<td>387.0</td>
</tr>
<tr>
<td>Pacific Gyral</td>
<td>48.8</td>
<td>58.7</td>
<td>43.9</td>
<td>35.0</td>
<td>13.6</td>
<td>0</td>
<td>0.2</td>
<td>456.0</td>
</tr>
<tr>
<td>Transition Zone</td>
<td>30.5</td>
<td>70.9</td>
<td>37.8</td>
<td>25.2</td>
<td>4.9</td>
<td>0</td>
<td>0.4</td>
<td>441.0</td>
</tr>
</tbody>
</table>

All values are the equivalent of a continuous ice layer covering the region referred to in the first column. t is a total value, \( l \) - first-year ice, \( ss \) - floes of steady-state thickness, \( x \) - floes older than first-year but younger than steady-state floes, \( y \) - ice formed at the salt-water/fresh-water interface in summer, \( p \) - perimeter thickness (see text), \( e \) - effective thickness (see text).
Fig. 3. Ice thickness (m) of floes more than 1.5 m and less than 5.5 m thick.
thickness of ice forming in new fractures was measured by drilling. The total ice production in fractures in the study area is shown in Table III. Between October 2, 1968, and January 2, 1969, the figures refer essentially to three 400 m by 600 m floes and the fractures at their edges. After January 2, 1969, the study area included two large polynyas which opened at the edges of the floes. The polynyas increased the new-ice coverage in the study area to 70 per cent, which is clearly unrepresentative of the Pacific Gyral as a whole. If the growth rate of ice in fractures between October 2, 1968, and January 2, 1969, is extrapolated to cover a September 1 to mid-May growth period, the annual new-ice (or first-year ice) growth in the study area becomes 26-56 cm/cm². A calculation from the ice-log data of first-year ice growth in the Pacific Gyral gives a comparable figure of 49.1 cm/cm².

ABLATION (a), BALANCE (b), AND EXPORT (g)

Ablation measurements on the BTAE were made only on a multiyear floe where the mean ablation (including hummocks and ponded ice) was 48-6 cm/cm². The ablation of younger ice has been related to this figure (see Table IV). In the case of undeformed first-year ice, according to Yanes (1962),

\[ \bar{\alpha}_1 = 1.4 \bar{\alpha}_{ss} \]  

(2)

where the suffixes 1 and ss refer to first-year and multiyear ice, respectively. From (2), \( \bar{\alpha}_1 \) in 1968 was 77.0 cm/cm².

BTAE measurements showed that new ridges and hummocks ablate at 2.6 times the rate of undeformed first-year ice, but as they contain 15 per cent air voids,

\[ \bar{\alpha}_n = \bar{\alpha}_h = 0.85 \cdot 2.6 \bar{\alpha}_{ss} = 110 \text{ cm/cm}^2. \]  

(3)

Ice growth under multiyear floes of less than mean steady-state thickness is greater than ablation. As no separate study of floes of this age was made, the ice balance on them (i.e., \( \alpha - \alpha \)) must be calculated.

To attain a mean steady-state thickness \( \bar{h}_{ss} \), a first-year floe of thickness \( \bar{h}_1 \) must have a total net balance \( \bar{B}_x \) of \( [\bar{h}_{ss}] - [\bar{h}_1] \) over a
period $m_1 - 1$ years, where $m_1$ is the age, in years, of a floe which has just attained mean steady-state thickness. To estimate a maximum value for $[h_1]$, thickness measurements taken in May, 1968, and May, 1969, through first-year floes where the uppermost 30 cm was fresh-water ice (i.e., beginning its growth in late August) have been used. The mean of these measurements is 204 cm. Using the previous values of $[h_{ss}]$ in $[h_{ss}] - [h_1]$, the value of $[P_x]$, where $1 < x < m_1$ years, is 142 cm in the Pacific Gyral, 119 cm in the Transition Zone, and 96 cm in the Trans-Polar Drift Stream.

Ice drift in the Trans-Polar Drift Stream may be considered as stream flow where ice leaves the stream between Greenland and Spitsbergen. As a first approximation, therefore,

$$A_1 = A_2 = A_3 = \ldots = A_\sigma$$

where $A$ is the area of ice 1, 2, 3, and $\sigma$ years old. $\sigma$ is the age, in years, of the oldest ice commonly present and is given by

$$\sigma = A_t/A_1.$$  (5)

In this case, $A_1$ includes areas of ridged and hummocked new and first-year ice. In the Trans-Polar Drift Stream, $A_1$ is equal to 19.9 per cent of $A_t$, so that $\sigma$ is 5.0 years. However, hummocking annually reduces the area of old ice by an amount $R$ which can be calculated from

$$R = \frac{1}{\sigma} \left( A_1 \frac{[h_q]}{[h_d]} - \frac{A_q}{\sigma} \right)$$

(6)

where $A_q$ is the area of hummock fields composed of old ice (6 per cent in the Trans-Polar Drift Stream), $[h_q]$ is the mean hummock-field thickness (10 m), and $[h_d]$ is the mean thickness of the ice before hummocking (i.e., $\frac{[h_1] + [h_{ss}]}{2}$). $\frac{A_q [h_q]}{[h_d]}$ in (6) gives the original extent of undeformed ice that the hummock fields formed from; $\frac{1}{\sigma}$ reduces the term to an annual value.

In the Trans-Polar Drift Stream $R$ is equal to 3.5 per cent of the total ice and ice-free area.
Fig. 4. Snow accumulation between Pt. Barrow (71°N) and Spitsbergen (81°N).
Table III
Ice growth in a small area in the Pacific Gyral, October 5, 1968, to February 22, 1969. (in cm of ice a day)

<table>
<thead>
<tr>
<th></th>
<th>10/5-10/20</th>
<th>10/20-12/14</th>
<th>12/14-1/2</th>
<th>1/2-2/22</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total new ice growth</td>
<td>.312</td>
<td>.193</td>
<td>.425</td>
<td>1.644</td>
</tr>
<tr>
<td>New ice growth in recurring fracture</td>
<td>.155</td>
<td>.161</td>
<td>.283</td>
<td>.246</td>
</tr>
<tr>
<td>New ice growth in polynyas</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>1.255</td>
</tr>
<tr>
<td>New ice growth elsewhere</td>
<td>.157</td>
<td>.032</td>
<td>.142</td>
<td>.143</td>
</tr>
</tbody>
</table>

Ice growth under old ice is not included in these figures, which are expressed as a continuous layer covering the entire area examined.

Table IV
Ice ablation, lowering of the surface on a multiyear floe.

<table>
<thead>
<tr>
<th></th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>New ridges</td>
<td>260</td>
</tr>
<tr>
<td>Old hummocks</td>
<td>61</td>
</tr>
<tr>
<td>Level areas</td>
<td>100</td>
</tr>
<tr>
<td>Ponds, ice surface</td>
<td>168</td>
</tr>
<tr>
<td>Ponds, water surface</td>
<td>73</td>
</tr>
</tbody>
</table>

Measurements are expressed as percentages of lowering of level areas where surface ablation of ice July and August 1969 was 40-45 g/cm.
The area of ice $n$ years old ($A_n$) for $n \geq 1$ year is then given by

$$A_n = A_1 (1 - R)^{n-1}. \quad (7)$$

A more accurate value for $\sigma$, in years, can then be found from

$$A_t - A_1 = \int_{1}^{\sigma} A_1 (1 - R)^{n-1} \, dn. \quad (8)$$

Putting $A_t = 100$, $R = 3.5$, and $A_1 = 19.9$, $\sigma$ is 5.4 years. This $\sigma$ value is in reasonably close agreement with the drift period of Russian floe stations and the Maud and Fram (Dunbar and Wittmann, 1962).

As from thermodynamic considerations, there is no ice growth where $\bar{h} \leq \bar{h}_{SS}$; the area of hummock fields more than a year old must be subtracted from the area of old ice, so that

$$A_x = A_t - A_1 - A_q \quad (9)$$

where $A_x$ is the area of old ice of less than steady-state thickness. Untersteiner (1961) has calculated that $\bar{h}_{SS}$ is attained in 7-8 years from ice-free conditions. However, there is no significant difference between the set of old-ice thickness measurements from the Pacific Gyral and the set from the Trans-Polar Drift Stream, despite the fact that ice may circulate for considerably longer periods in the Pacific Gyral. Similar old-ice thicknesses in each area, therefore, indicate that $\bar{h}_{SS}$ is attained in both--i.e., in less than 5.4 years. Equation (9) assumes that $\bar{h}_{SS}$ is not attained in less than 5.4 years; this assumption introduces a maximum error of 10 per cent (i.e., if $\bar{h}_{SS}$ is attained in 2 years).

The annual balance of ice $x$ years old ($\langle \bar{B}_x \rangle$) where $1 < x < 5.4$ is given by

$$\langle \bar{B}_x \rangle = A_x \cdot \left[ \bar{B}_x \right]/(\sigma-1) \quad (10)$$

with $A_x = 74.1$ per cent and $\bar{B}_x = 96$ cm. $\langle \bar{B}_x \rangle$ is 16.2 cm of ice.
The same arguments do not apply to the Pacific Gyral where, due to the circulatory pattern of the drift (Fig. 1), ice of all ages may drift out of the Gyral in any one year. If there is a constant decrease in the area of any one year's ice with time $R'$ (due to both ice export and ice deformation), then

$$A_t = A_1 + (A_1 - R') + (A_1 - 2R') + \cdots + [A_1 - (n-1)R'] \quad (11)$$

from which

$$\sigma = 1 + \left(2A_t/A_1\right). \quad (12)$$

In 1968, 11.6 per cent of the ice crossed in the Gyral was in its first year. Substituting this value in (12) gives $\sigma$ equal to 18.3 years. As there is an exponential rather than a linear decrease in area of any one year's ice with time, 18.3 years must be regarded as a lower limit of $\sigma$.

As ice attains $h_{88}$ in the Trans-Polar Drift Stream where $\sigma$ is 5.4 years, $x$ can be approximated by $1 < x < 5.4$ years. The area of ice $x$ years old, where there is a positive ice balance, is given by

$$A_x = \frac{5^h}{1} A_1 - \frac{1}{\sigma} [A_1(x-1) - A_q] \, dx \quad (13)$$

where $\frac{A_q}{\sigma}$ is the area of ice hummocked annually and where the ice balance is zero. Putting $A_1 = 11.6$ per cent, $A_q = 10.8$ per cent, and $\sigma = 18.3$ years in (10), $A_x$ is 42.3 per cent of the total ice and ice-free area in the Pacific Gyral, and [from (10)] $\overline{h_x}$ there is 13.6 cm of ice.

Using similar arguments, in the Transition zone $\overline{h_x}$ is 4.85 cm of ice.

If the ice balance in the Arctic Ocean ($\overline{B}$) is assumed to be in a steady-state condition, then

$$B = C - A - G = 0 \quad (14)$$

and

$$G = C - A.$$

Ice export ($G$) can now be estimated using the values of accumulation ($C$) and ablation ($A$) already calculated. The results are shown in Table III.
The area of the Pacific Gyral, the Trans-Polar Drift Stream, and the Transition Zone between has been estimated from the pattern of ice drift shown in Dunbar and Wittmann (1962, p. 92). The three regions cover 41 per cent, 55 per cent, and 4 per cent of the Arctic Ocean, respectively. If the ice export values in Table III are representative, the total annual ice export from the Arctic Ocean $\langle G_t \rangle$ is equal to a layer 55.8 cm thick.

CONCLUSIONS

The BTAE data, if they are representative, shown that the equivalent of a continuous layer of ice 114 cm thick may form in the Arctic Ocean in a year. Of this ice, 47 per cent forms in ice-free areas and under first-year ice, giving a mean thickness for first-year ice $[\bar{h}_1]$ of 353 cm. As no more than about 200 cm of ice forms from the ice-free water in a single winter, the remainder—153 cm—is formed when new ice is piled into ridges and hummocks; new ice-free areas then open for ice to form at a very high growth rate. Thus, 20.5 per cent of the ice production is indirectly due to ridging and hummocking. It seems likely, therefore, that variations in the amount of ice production each year in the Arctic Ocean are more dependent on variations in the amount of ridging and hummocking than on variations in the winter temperature. Consequently, comparisons between the ice thickness measurements taken in different years are not meaningful in terms of varying ice production and climatic change unless the measurements are representative of the entire range of ice types.

ACKNOWLEDGMENTS

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ON THE RELATION BETWEEN TURBULENT 
AND AVERAGED ICE-FLOE MOTION 

by 
L. A. Timokhov 

The equations of motion for a single ice floe [2, 6] can be used to describe the motion of an ice floe when the compactness of the ice cover is nonvanishing but not very large. In this case, one must introduce the additional force $F_{\text{int}}$ of interaction between the ice floe under consideration (denoted by the subscript $i$) and the surrounding ice floes:

\[
\begin{align*}
\frac{dV_{ix}}{dt} & = - \beta_{ix} |V_i - u_i| (V_{ix} - u_{ix}) + T_{ix} + F_{\text{int}}^{ix}, \\
\frac{dV_{iy}}{dt} & = - \beta_{iy} |V_i - u_i| (V_{iy} - u_{iy}) + T_{iy} + F_{\text{int}}^{iy}, \\
|V_i - u_i| & = \sqrt{(V_{ix} - u_{ix})^2 + (V_{iy} - u_{iy})^2},
\end{align*}
\]

(1)

where $\beta_{ix} |V_i - u_i| (V_{ix} - u_{ix})$, $\beta_{iy} |V_i - u_i| (V_{iy} - u_{iy})$ denote the components of the drag or reactive force which results from the motion of the ice floe relative to the water; $u_{ix}$ and $u_{iy}$ are the components of the flow velocity at the external boundary of the layer in which the ice floe experiences a disturbance; $\beta_{ix}$ is the drag coefficient for ice floe $i$; $T_{ix}$ and $T_{iy}$ denote the coefficients of tangential wind stress; and $\alpha$ is the Coriolis parameter.

In practical applications, it is of greater importance to know the average drift velocity of the ice cover than the velocity of individual ice floes. The average drift velocity is obtained by averaging the drift velocities of individual floes comprising the ice cover of a particular ocean area and then averaging these results over a time interval. We will consider the effect of turbulence upon the average drift velocity, assuming that the motion of individual ice floes is given by equations of the form (1).
Averaging over the drift velocities of the floes can be replaced with a simple averaging of the drift velocities over the area considered. But this operation cannot be directly applied to the equations of (1). In equations (1), the velocity components $V_{ix}$ and $V_{iy}$ as well as the coefficient $\beta_i$ are defined only for the point at which floe $i$ is located and cannot be considered field functions.

In general, the tangential wind stress, the drag forces, and the interactions are turbulent. Fluctuations of the acting forces modify the position of the floe in accordance with the actual force conditions. We will consider equations (1) as one of the possible forms of ice-floe motion. We will sum over all possible trajectories passing through the particular element in space. We will then divide by the total number of possible ice-floe motions. We introduce the probability $\rho$ of finding the floe in a particular element of space. Then equations (1) transform into the following equations:

$$
\rho \left( \frac{dV_{ix}}{dt} - \alpha V_{iy} \right) = \rho \left( -\beta_i |V_i - u_i| (V_{ix} - u_{ix}) + T_{ix} + F_{ix}^{int} \right),
$$

$$
\rho \left( \frac{dV_{iy}}{dt} + \alpha V_{ix} \right) = \rho \left( -\beta_i |V_i - u_i| (V_{iy} - u_{iy}) + T_{iy} + F_{iy}^{int} \right). \tag{1'}
$$

The function $\rho$ expands the area in space in which the characteristics of $V_{ix}$, $V_{iy}$, and $\beta_i$ are determined at a particular time. The areas overlap where the function $\rho$ and, consequently, $V_{ix}$, $V_{iy}$, and $\beta_i$ are defined for neighboring floes, provided that the ice-floe drift is sufficiently turbulent.

Since the areas in which $\rho$ is defined overlap, equations (1) can be treated as equations for the instantaneous drift velocities of floes of low compactness at that particular point. Therefore, the equations can be rewritten in the form

$$
\frac{\partial V_x}{\partial t} + V_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial y} - \alpha V_x = -c(V_x - u_x) + T_x + F_{x}^{int},
$$

$$
\frac{\partial V_y}{\partial t} + V_x \frac{\partial V_y}{\partial x} + V_y \frac{\partial V_y}{\partial y} + \alpha V_y = -c(V_y - u_y) + T_y + F_{y}^{int}, \tag{2}
$$

where $c = \beta |V - u|$. 

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We represent the functions $V_x, V_y, u_x, u_y, \sigma, T_x, T_y, F_x, F_y$ as a sum of average values and fluctuations, both taken over a certain time interval $\Delta t$ and over some area $\Delta x \cdot \Delta y$:

$$\phi = \overline{\phi} + \phi', \quad (3)$$

where $\phi = V_x, V_y, \sigma, u_x, u_y, T_x, T_y, F_x$, and $F_y$.

The fluctuations of the interaction forces between ice floes can be ignored, because, in the case of dispersed ice floes, these fluctuations are small compared to the fluctuations of the other forces. We assume that the variations of the parameter $\sigma$ result from the space-dependent variability of the drag coefficient $\beta$.

We insert the expansion of equation (3) into the expressions of (2), average over both time and space, and subtract from equations (2) the result of the averaging. This results in an equation system for the average drift velocity of the floes and the velocity fluctuations:

\[
\begin{align*}
\frac{\partial \overline{V}_x}{\partial t} + \overline{V}_x \frac{\partial \overline{V}_x}{\partial x} + \overline{V}_y \frac{\partial \overline{V}_x}{\partial y} &+ \overline{V}_x \frac{\partial \overline{V}_x}{\partial x} + \overline{V}_y \frac{\partial \overline{V}_y}{\partial y} + \\
&+ \overline{c} \overline{V}_x - \alpha \overline{V}_y = \overline{u}_x + \overline{T}_x + \overline{F}_x, \\
\frac{\partial \overline{V}_y}{\partial t} + \overline{V}_x \frac{\partial \overline{V}_y}{\partial x} + \overline{V}_y \frac{\partial \overline{V}_y}{\partial y} &+ \overline{V}_x \frac{\partial \overline{V}_y}{\partial x} + \overline{V}_y \frac{\partial \overline{V}_y}{\partial y} + \\
&+ \overline{c} \overline{V}_y - \alpha \overline{V}_x = \overline{u}_y + \overline{T}_y + \overline{F}_y, \\
\end{align*}
\]

In the expressions (5) for the velocity fluctuations, we have neglected the Coriolis parameter as the smallest quantity.
For practical purposes, it is important to know how the average drift velocity changes; equations (4) are the basis of our considerations. Equations (4) do not form a closed system. The various terms of equations (4) depend upon the turbulent motion of the floes. To obtain closure of the problem of determining the average ice-floe drift velocity, we must establish expressions for the turbulent terms

\[
\frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial y}, \quad V_x \frac{\partial V_y}{\partial x} + V_y \frac{\partial V_y}{\partial y}.
\]

To do this, we use equations (5), which we solve for the functions \(V'_x\) and \(V'_y\), neglecting the nonlinear terms \(f_x\) and \(f_y\).

We represent the stochastic functions which appear in equations (5) in the form of Fourier series:

\[
\Phi' = \sum_{n=\infty} \phi_n e^{i(w_i t + u_i x + v_i y)}.
\]

where \(\Phi' = V'_x, V'_y, \mu'_x, \mu'_y, \sigma'_x, T'_x, T'_y\); \(i = \sqrt{-1}\).

After some transformations, we obtain the following formulas for the fluctuations of the ice-floe drift velocities \(V'_x\) and \(V'_y\):

\[
V'_x = \sum_{n=\infty} \left\{ \frac{1}{B} \left[ (\bar{c} - i\omega) (\bar{c}^2 + \omega^2) - (\bar{c}^2 + \omega^2) \left( \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} \right) + \right. \right.
\]

\[
\left. + (\bar{c} - i\omega) \frac{\partial V_y}{\partial y} \left[ \mu'_x + \mu'_y (\bar{V}_x - \bar{V}_y) \right] + T'_x \right\} e^{i(w_i t + u_i x + v_i y)}
\]

\[
V'_y = \sum_{n=\infty} \left\{ \frac{1}{B} \left[ (\bar{c} - i\omega) (\bar{c}^2 + \omega^2) + \bar{c}^2 + \omega^2 \left( \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} \right) + (\bar{c} - i\omega) \frac{\partial V_x}{\partial x} \left[ \mu'_y + \right. \right. \right.
\]

\[
\left. + \mu'_y (\bar{V}_y - \bar{V}_y) + T'_y \right\} e^{i(w_i t + u_i x + v_i y)}
\]

\[
+ \mu'_y (\bar{V}_y - \bar{V}_y) + T'_y \right\} e^{i(w_i t + u_i x + v_i y)}
\]
where
\[ B = (\bar{c}^2 + \omega^{(v)})^2 + 2\bar{c}(\bar{c}^2 + \omega^{(v)}) \left( \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} \right), \]
\[ \omega^{(v)} = \omega_x^{(v)} + \bar{V}_x \omega_x^{(v)} + \bar{V}_y \omega_y^{(v)}. \]

We differentiate equations (7) and (8) with respect to \( x \) and \( y \) and calculate the correlational moments of the quantities \( V'^x \) and \( V'^y \) [complex conjugates to the functions stated in equations (7) and (8)] and the derivatives of \( V_x' \) and \( V_y' \) with respect to \( x \) and \( y \):
\[
K \left( V'^x \frac{\partial V'^x}{\partial x} \right), \quad K \left( V'^x \frac{\partial V'^y}{\partial x} \right), \quad K \left( V'^y \frac{\partial V'^x}{\partial y} \right), \quad K \left( V'^y \frac{\partial V'^y}{\partial y} \right).
\]

When we use the real parts of the correlation functions, we obtain the following equations for the turbulent terms:

\[
\overline{V_x' \frac{\partial V'^x}{\partial x} + V_y' \frac{\partial V'^y}{\partial y}} = \text{Real} \left( K \left( V'^x \frac{\partial V'^x}{\partial x} \right) + K \left( V'^y \frac{\partial V'^y}{\partial x} \right) \right) = \nonumber
\]
\[
= -k_{xx} \frac{\partial \bar{V}_x}{\partial x^2} - k_{yy} \frac{\partial \bar{V}_y}{\partial y^2} - 2k_{xy} \frac{\partial \bar{V}_x}{\partial x} \frac{\partial \bar{V}_y}{\partial y} + \frac{\partial p_1}{\partial x} + F_1, \quad (9)
\]

\[
\overline{V_x' \frac{\partial V'^y}{\partial x} + V_y' \frac{\partial V'^y}{\partial y}} = \text{Real} \left( K \left( V'^x \frac{\partial V'^x}{\partial y} \right) + K \left( V'^y \frac{\partial V'^y}{\partial y} \right) \right) = \nonumber
\]
\[
= -k_{xx} \frac{\partial \bar{V}_x}{\partial x^2} - k_{yy} \frac{\partial \bar{V}_y}{\partial y^2} - 2k_{xy} \frac{\partial \bar{V}_x}{\partial x} \frac{\partial \bar{V}_y}{\partial y} + \frac{\partial p_2}{\partial y} + F_2, \quad (10)
\]

where
\[
k_{xx} = \sum_{-\infty}^{\infty} \frac{c}{(\bar{c}^2 + \omega^{(v)})^2} \left[ \bar{c}^2 D_{u(v)} + (\bar{u}_x - \bar{V}_x) D_{\varepsilon(v)} + D_{T(v)} \right],
\]
\[
k_{yy} = \sum_{-\infty}^{\infty} \frac{c}{(\bar{c}^2 + \omega^{(v)})^2} \left[ \bar{c}^2 D_{u(v)} + (\bar{u}_y - \bar{V}_y) D_{\varepsilon(v)} + D_{T(v)} \right],
\]
\[
k_{xy} = \sum_{-\infty}^{\infty} \frac{c}{(\bar{c}^2 + \omega^{(v)})^2} \left[ \bar{c}^2 K_{u(v)}^2 \omega_{v(v)} + \right.
\]
\[+ (\bar{u}_x - \bar{V}_x)(\bar{u}_y - \bar{V}_y) D_{\varepsilon(v)} + K_{T(v)}^2 \tau_{v(v)} \left. \right],
\]
\[
p_1 = \sum_{-\infty}^{\infty} \frac{c}{(\bar{c}^2 + \omega^{(v)})^2} \left[ \bar{c}^2 D_{u(v)} + (\bar{u}_x - \bar{V}_x)^2 D_{\varepsilon(v)} + D_{T(v)} \right],
\]
\[
p_2 = \sum_{-\infty}^{\infty} \frac{c}{(\bar{c}^2 + \omega^{(v)})^2} \left[ \bar{c}^2 D_{u(v)} + (\bar{u}_y - \bar{V}_y)^2 D_{\varepsilon(v)} + D_{T(v)} \right],
\]

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\[
F_1 = \sum_{\nu=1}^{\infty} \frac{1}{\varepsilon^2 + \omega(\nu)^2} \left( c^2 K_{\nu} \frac{\partial u_{\nu}(\nu)}{\partial y} + \frac{1}{2} \frac{\partial}{\partial x} \left[ (u_y - \overline{V}_y)(u_x - \overline{V}_x) \frac{\partial D_{\nu}(\nu)}{\partial y} \right] + K_{\nu} \frac{\partial \gamma_{\nu}(\nu)}{\partial x} \right),
\]

\[
F_2 = \sum_{\nu=1}^{\infty} \frac{1}{\varepsilon^2 + \omega(\nu)^2} \left( c^2 K_{\nu} \frac{\partial u_{\nu}(\nu)}{\partial x} + \frac{1}{2} \frac{\partial}{\partial y} \left[ (u_y - \overline{V}_y)(u_x - \overline{V}_x) \frac{\partial D_{\nu}(\nu)}{\partial x} \right] + K_{\nu} \frac{\partial \gamma_{\nu}(\nu)}{\partial y} \right),
\]

\[
D_{\Phi(\nu)} \text{ denotes the dispersion of the components } \Phi^{(\nu)} \text{ with the subscript } \nu, \text{ and } K_{\Phi F} \text{ is the correlational moment of the quantities } \Phi \text{ and } F.
\]

Let us assume that the dispersion of the elementary components is independent of the number \(\nu\) and numerically equal to the dispersions of the corresponding stochastic functions. After summing the series, we obtain for the isotropic turbulence of the wind and the flow:

\[
k_{xx} = \frac{\Delta t}{2\pi c_2 \varepsilon^2} \left[ c^2 E_u + c^2 E_T + \overline{(u_x - \overline{V}_x)^2 c^2} \right],
\]

\[
k_{yy} = \frac{\Delta t}{2\pi c_2 \varepsilon^2} \left[ c^2 E_u + c^2 E_T + \overline{(u_y - \overline{V}_y)^2 c^2} \right],
\]

\[
k_{xy} \approx 0,
\]

\[
p_1 = \frac{\Delta t}{2\pi c_2 \varepsilon^2} \left[ c^2 E_u + c^2 E_T + \overline{(u_x - \overline{V}_x)^2 c^2} \right],
\]

\[
p_2 = \frac{\Delta t}{2\pi c_2 \varepsilon^2} \left[ c^2 E_u + c^2 E_T + \overline{(u_y - \overline{V}_y)^2 c^2} \right],
\]

\[
F_1 \approx 0,
\]

\[
F_2 \approx 0,
\]

\[
E_u = \frac{1}{2} \overline{u_x^2} + \frac{1}{2} \overline{u_y^2} = \overline{u_x} = \overline{u_y},
\]

\[
E_T = \frac{1}{2c^2} \overline{T_x^2} + \frac{1}{2c^2} \overline{T_y^2} = \frac{1}{c^2} \overline{T_x} = \frac{1}{c^2} \overline{T_y}.
\]
It follows from equations (12) that the turbulent coefficients \( k_{xx} \) and \( k_{yy} \) and the functions \( p_1 \) and \( p_2 \) depend upon the energy of the turbulent flow \( E_u \) and the energy \( E_T \) of the external forces, as well as on the dispersion of the parameter \( \sigma \). The functions \( p_1 \) and \( p_2 \) can be considered pressures, whereas the components \( \partial p_1 / \partial x \) and \( \partial p_2 / \partial y \) play the role of forces resulting from pressure inhomogeneities in space.

We wish to mention that D. L. Laikhtman [3] introduced the following expressions for the turbulent terms of ice-floe motion in analogy with the description of viscous liquids (in our notations):

\[
\begin{align*}
V_x' \frac{\partial V_x'}{\partial x} + V_y' \frac{\partial V_y'}{\partial y} &= -k_{xx} \frac{\partial^2 V_x'}{\partial x^2} - k_{yy} \frac{\partial^2 V_y'}{\partial y^2}, \\
V_x' \frac{\partial V_y'}{\partial x} + V_y' \frac{\partial V_x'}{\partial y} &= -k_{xx} \frac{\partial^2 V_y'}{\partial x^2} - k_{yy} \frac{\partial^2 V_x'}{\partial y^2}.
\end{align*}
\]

(13)

It follows from a comparison of equations (13) and our analytical expressions (9) and (10) that equations (13) are only a first approximation of the expressions for the turbulent terms.

Thus, the terms

\[
\begin{align*}
V_x' \frac{\partial V_x'}{\partial x} + V_y' \frac{\partial V_y'}{\partial y}, \quad V_x' \frac{\partial V_y'}{\partial x} + V_y' \frac{\partial V_x'}{\partial y}
\end{align*}
\]

in the expressions for the average drift velocity of the ice floes result from the nonlinearity of the equations of motion (1) for the floes. This constitutes an analog to the hydrodynamic equations [4]. However, contrary to a liquid, one can obtain analytical expressions for the turbulent terms in the case of an ice cover, provided that certain restrictions are accepted [see equations (9) and (10)].

The average motion of ice floes is related to the turbulence by the turbulence coefficients \( k_{xx} \), \( k_{yy} \), and \( k_{xy} \), the functions \( p_1 \), \( p_2 \), \( E_1 \), and \( E_2 \), which depend upon both the type and the extent of the turbulence of the external media (water and air), and the dispersion of the parameters \( \sigma \) and \( \beta \) for the ice cover.

At the present time, it is not possible to check equations (9) and (10), because direct measurements of the turbulent terms have not been performed. The dispersion of the drift velocity of the ice floes \( V_x' V_x', V_y' V_y' \) can be treated analytically or can be evaluated from ice-drift observations.
The agreement between the results can be considered an indirect confirmation of the validity of equations (9) and (10).

We use equations (7) and (8) for the fluctuations of the ice-drift velocity to obtain formulas for the dispersions:

\[
\overline{V_x'V_x'} = \sum_{\nu=\infty}^{\infty} \frac{1}{\sigma^2+\nu^2} \left[ \hat{\rho}^2 D_u^{(\nu)} + D_{\tau_x^{(\nu)}} + (\hat{u}_x - \hat{V}_x) D_{\delta^{(\nu)}} \right],
\]

\[
\overline{V_y'V_y'} = \sum_{\nu=\infty}^{\infty} \frac{1}{\sigma^2+\nu^2} \left[ \hat{\rho}^2 D_u^{(\nu)} + D_{\tau_y^{(\nu)}} + (\hat{u}_y - \hat{V}_y) D_{\delta^{(\nu)}} \right].
\]

(14)

Let us consider the results of observations on dispersed ice floes; these observations were made in 1964 by Zh. A. Pavlikov and the author [5]. The actual values \( \overline{V_x^2}, \overline{V_y^2} \) were calculated as follows. For a square of 400 m x 400 m situated at a distance of 300 m southeast of the ship's position, the drift velocities of ice floes whose trajectories ran through the particular square were sampled for four hours. The average values, the deviations, and the dispersions were calculated from a series of velocity components. The results are:

\[
\overline{V_x^2}_{\text{act}} = 1.15 \cdot 10^2 \text{cm}^2/\text{sec}^2
\]

\[
\overline{V_y^2}_{\text{act}} = 0.65 \cdot 10^2 \text{cm}^2/\text{sec}^2
\]

As in the derivation of equations (12), we assume that

\[
D_{u^{(\nu)}} = \overline{u'^2_x}, \quad D_{\tau_x^{(\nu)}} = \overline{\tau_x^{(\nu)}} = \overline{\sigma^2 (|W| W_x)^2},
\]

\[
D_{u^{(\nu)}} = \overline{u'^2_y}, \quad D_{\tau_y^{(\nu)}} = \overline{\tau_y^{(\nu)}} = \overline{\sigma^2 (|W| W_y)^2},
\]

where \( |W| = \sqrt{\overline{W_x^2} + \overline{W_y^2}} \), and \( W_x \) and \( W_y \) are the components of the wind velocity. We ignore the changes of the parameter \( \sigma (D_\sigma^{(\nu)} = 0) \).

We obtain for the averaging period \( \Delta t = 4 \) hours, the linear dimensions of the square \( \Delta x = \Delta y = 400 \) m, and the average drift velocity of the ice floes \( \overline{V} = 0.24 \) m/sec:

\[
\sum_{\nu=\infty}^{\infty} \frac{1}{\sigma^2+\nu^2} \overline{\sigma^2} = \frac{3.85 \cdot 10^2}{\overline{\sigma}}.
\]
Instead of equations (14), we obtain the simplified formulas

\[
\bar{V}_x^2 = 3.85 \cdot 10^2 \frac{c}{k_w} u_x^2 + \frac{3.85 \cdot 10^4}{c} \alpha^2 \left(<W_x^2>/<W_x^4>\right),
\]
\[
\bar{V}_y^2 = 3.85 \cdot 10^2 \frac{c}{k_w} u_y^2 + \frac{3.85 \cdot 10^4}{c} \alpha^2 \left(<W_y^2>/<W_y^4>\right),
\]

where

\[
\bar{c} = \beta \sqrt{\frac{(\bar{V}_x - \bar{u}_x)^2 + (\bar{V}_y - \bar{u}_y)^2}{\bar{W}_x^2 + \bar{W}_y^2}},
\]

\[
\beta = k_w \frac{\rho_w}{2h_i}, \quad \alpha = k_a \frac{\rho_a}{2h_i},
\]

\(k_w\) and \(k_a\) denote the coefficients of friction between the ice and water and the ice and air, respectively; \(\rho_w\), \(\rho_a\), and \(\rho_i\) denote the densities of water, air, and ice, respectively; and \(h_i\) denotes the thickness of the ice.

In our case, the ice was assumed to have an average thickness of 0.8 m. According to [1], the coefficient of friction between ice and water was assumed to be \(4.5 \cdot 10^{-3}\). The coefficient of friction between ice and air is \(7.8 \cdot 10^{-3}\) according to our measurements performed on the Ladoga Lake (at an altitude of 6 m for the wind measurements).

The dispersions \(\bar{u}_x^2\), \(\bar{u}_y^2\), \(\bar{W}_x^2\), and \(\bar{W}_y^2\) were calculated from ship observations of the flow at a depth of 2 m and of the wind and an altitude of 6 m. The final result obtained with equation (14') is:

\[
\bar{V}_x^2 = 1.14 \cdot 10^2 \text{cm}^2/\text{sec}^2
\]
\[
\bar{V}_y^2 = 0.74 \cdot 10^2 \text{cm}^2/\text{sec}^2
\]

Obviously, there exists a rather strong agreement between the calculated and the actual dispersions of the velocity components of the ice-floe drift.
REFERENCES


We consider the drift of ice fields whose horizontal dimensions are much greater than the ice thickness. We use D.L. Laikhtman's hypothesis [6] in order to describe the interaction of the ice floes and introduce the equations of ice-floe motion. In the case of a gradient flow which is independent of time and the horizontal coordinates, the equations assume the form [6, 4]:

\[
\begin{align*}
\frac{m}{\partial t} \frac{\partial u}{\partial t} &= m \lambda \dot{u}_0 + \alpha_{ax} + \nabla \times \sigma_{ax} + k_0 \frac{\partial^2 u}{\partial x^2} + k_0 \frac{\partial^2 u}{\partial y^2}, \\
\frac{m}{\partial t} \frac{\partial \dot{v}}{\partial t} &= - m \lambda \dot{v}_0 + \alpha_{ay} + \nabla \times \sigma_{ay} + k_0 \frac{\partial^2 \dot{v}}{\partial x^2} + k_0 \frac{\partial^2 \dot{v}}{\partial y^2}.
\end{align*}
\]

The notations are interpreted as follows: \( u_0 = u^* - \hat{u}; \dot{v}_0 = v^* - \hat{v}; \)
\( u^* \) and \( v^* \) denote the components of the velocity vector of the entire ice drift; \( \hat{u} \) and \( \hat{v} \) are the vector components of the flow velocity; \( m \) denotes the ice mass per unit surface; \( \lambda \) is the Coriolis parameter; \( k_0 = \overline{m k_0} \) denotes the coefficient of the ice-floe interaction; and \( \sigma_{ax} \) and \( \sigma_{ay} \) denote the tangential stresses at the ice/air boundary and the ice/water boundary, respectively.

Nonstationary ice drift without interaction of ice floes has been previously considered in [10, 5] and, according to the results of those calculations, the time required for the motion to become stationary amounts to about 2 hours. Thus, a quasi-stationary model can be adopted for calculating a purely wind-induced drift. Additional terms, which result in equations (1) and (2) from interactions between ice floes, do not a priori justify the use of a quasi-stationary model. One must consider the effect of the ice-floe interaction upon the time required

for the motion to become stationary. A linear drift theory suffices for quantitative estimates of the effect. According to the linear drift theory, the components of the tangential stresses on the ice surface have the form [6,3]:

\[
\begin{align*}
\tau_{ax} &= k \rho a (v_0 - u_0 + U - V), \\
\tau_{ay} &= k \rho a (U + V - u_0 - v_0), \\
\tau_{wx} &= \bar{k} \tilde{\rho} \bar{a} (v_0 - u_0), \\
\tau_{wy} &= -\bar{k} \tilde{\rho} \bar{a} (u_0 \mp v_0),
\end{align*}
\]

(3)

where \( k \) and \( \bar{k} \) denote the turbulence coefficients for the atmosphere and the ocean, respectively; \( \rho \) and \( \tilde{\rho} \) are the density of air and water, respectively; \( a = \sqrt{\frac{\lambda}{2k}} \); \( \bar{a} = \sqrt{\frac{\lambda}{2k}} \); and \( U \) and \( V \) denote the components of the velocity vector of the geostrophic wind in the atmosphere.

Taking into account equations (3), we obtain from equations (1) and (2) one relation for the complex quantity \( \tilde{W}_0 = u_0 + i v_0 \):

\[
\frac{\partial \tilde{W}_0}{\partial t} + k \tilde{W}_0 \left( \frac{\partial \tilde{W}_0}{\partial x^2} + \frac{\partial \tilde{W}_0}{\partial y^2} \right) = B \cdot G(x, y, t),
\]

(4)

where

\[
A = \frac{1}{m} \left[ k \rho a + \bar{k} \tilde{\rho} \bar{a} + i (m \lambda + k \rho a + \bar{k} \tilde{\rho} \bar{a}) \right];
\]

\[
B = \frac{1+i}{m} k \rho a; \quad G = U + iV.
\]

Equation (4) will be solved under the following conditions:

\[
W_0 \big|_{t=0} = 0, \quad \text{and} \quad \tilde{W}_0 \big|_{x, y \rightarrow \pm \infty} \text{ bounded}
\]

(5)

(6)

We apply the Laplace transform to equation (4) and use condition (5).

Thus, we obtain

\[
\frac{\partial^2 \tilde{W}_0}{\partial x^2} + \frac{\partial^2 \tilde{W}_0}{\partial y^2} - \frac{A + \frac{1}{m} k}{k_0} \tilde{W}_0 = -\frac{B}{k_0} \tilde{G}(x, y, p),
\]

(7)

which has the solution [9]

\[
\tilde{W}_0 = \frac{B}{A + \frac{1}{m}} \sum_{n=0}^{\infty} \frac{-k_0^n}{(A + \frac{1}{m})^{n+1}} \cdot \Delta^n \tilde{G}(x, y, p),
\]

(8)

where \( \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \).
When \( G = G(x,y) \) is independent of time, we have
\[
\overline{G}(x, y) = \frac{1}{p} G(x, y),
\]
and consequently,
\[
\overline{W}_0 = \frac{B}{p(A + p)} \sum_{n=0}^{\infty} \frac{k_0^n}{(A + p)^n} \Delta^n G(x, y).
\] (9)

When we switch from the image of the function to its original, we obtain the following expression:
\[
W_0 = \frac{B}{A} \sum_{n=0}^{\infty} \frac{k_0^n}{A^n} \left[ 1 - e^{-At} \sum_{j=0}^{n} \frac{(A \cdot t)^j}{j!} \right] \Delta^n G.
\] (10)

In order to simplify the calculations, we assume \( V = 0 \) and \( \Delta G = \Delta U = \text{const.} \) We separate into real and imaginary parts:
\[
\begin{align*}
\overline{u}_0 &= \overline{u}_{0a} \left[ 1 - x_1(t) e^{-ReA \cdot t} \right] + \overline{u}_{0i} \left[ 1 - \beta_1(t) e^{-ReA \cdot t} \right], \\
\overline{v}_0 &= \overline{v}_{0a} \left[ 1 - x_2(t) e^{-ReA \cdot t} \right] + \overline{v}_{0i} \left[ 1 - \beta_2(t) e^{-ReA \cdot t} \right],
\end{align*}
\] (11)
\[
\begin{align*}
\overline{u}_0 &= \overline{u}_{0a} \left[ 1 - x_1(t) e^{-ReA \cdot t} \right] + \overline{u}_{0i} \left[ 1 - \beta_1(t) e^{-ReA \cdot t} \right], \\
\overline{v}_0 &= \overline{v}_{0a} \left[ 1 - x_2(t) e^{-ReA \cdot t} \right] + \overline{v}_{0i} \left[ 1 - \beta_2(t) e^{-ReA \cdot t} \right],
\end{align*}
\] (12)

where \( \overline{u}_{0a} \) and \( \overline{v}_{0a} \) denote the components of the velocity vector of a purely wind-induced stationary drift \([6]\); and \( \overline{u}_{0i} \) and \( \overline{v}_{0i} \) are the components of the velocity vector of a stationary drift caused by inhomogeneities in the wind field;
\[
\begin{align*}
\overline{u}_{0a} &= \frac{P_1}{|A|^2} U; \overline{v}_{0a} = \frac{Q_1}{|A|^2} U; \\
\overline{u}_{0i} &= \frac{k_0 m}{|A|^2} \cdot P_2 \cdot \Delta U; \overline{v}_{0i} = \frac{k_0 m}{|A|^2} \cdot Q_2 \cdot \Delta U; \\
\alpha_1(t) &= \cos \psi + \frac{Q_1}{P_1} \sin \psi; \quad \gamma_1(t) = \frac{C_1}{P_2} (1 + ReA \cdot t) - \frac{C_2}{P_2} \psi, \\
\alpha_2(t) &= \cos \psi - \frac{P_1}{Q_1} \sin \psi; \quad \gamma_2(t) = \frac{C_2}{Q_2} (1 + ReA \cdot t) + \frac{C_1}{Q_2} \psi; \\
\psi &= ImA \cdot t; \quad |A|^2 = (ReA)^2 + (ImA)^2; \\
P_1 &= \frac{k_0 m}{|A|^2} (ReA + ImA); \quad Q_1 = \frac{k_0 m}{|A|^2} (ReA - ImA); \\
P_2 &= P_1; \quad ReA + Q_1 \cdot ImA; \quad Q_2 = Q_1 \cdot ReA - P_1 \cdot ImA; \\
C_1 &= P_2 \cdot \cos \psi + Q_2 \cdot \sin \psi; \quad C_2 = Q_2 \cdot \cos \psi - P_2 \cdot \sin \psi.
\end{align*}
\]
It follows from equations (11) and (12) that the time required for the motion to become stationary is determined by the maximum of the four quantities $\alpha_1$, $\alpha_2$, $\beta_1$, and $\beta_2$, i.e., by the maximum of the stationary times of each of the drift components $u_{\theta_B}$, $v_{\theta_B}$, $u'_{\theta}$, and $v'_{\theta}$.

A calculation which was performed with the following numerical parameter values

\[ U = 10 \text{ m/sec}; \quad k = 15 \text{ m}^2/\text{sec}; \quad \kappa = 2 \cdot 10^{-2} \text{ m}^2/\text{sec}; \quad \lambda = 1.4 \cdot 10^{-4} \quad (1/\text{sec}); \]
\[ m = 2.10^3 \text{ kg/m}^2 \]

revealed that two hours after the start of a drift motion, the purely wind-induced drift had reached 98% of its stationary value, whereas the additional drift had reached 85% of its stationary value, and amounted to 95% of the latter after three hours. These estimates lead to the conclusion that though a stationary drift motion is established less rapidly due to the effect of the surrounding ice fields, the actual calculations of the ice drift can be made with the stationary approximation.

Keeping these results in mind, we consider again the stationary equations for the ice drift within the framework of a nonlinear theory of the turbulence of boundary layers at horizontal ice surfaces [7].

In a fixed coordinate system, the tangential stress components at the boundary ice/water can be written in the form:

\[
\begin{align*}
\tau_{wx} &= \tilde{\rho} \tilde{v}^2 \cos \gamma = \tilde{\rho} \tilde{v}^2 \cos(\varphi - \beta) = \tilde{\rho} \tilde{v}^2 \left( c_1 \frac{u'}{V_0} + c_2 \frac{v'}{V_0} \right) \\
\tau_{wy} &= \tilde{\rho} \tilde{v}^2 \sin \gamma = \tilde{\rho} \tilde{v}^2 \left( c_1 \frac{v_0}{V_0} - c_2 \frac{u_0}{V_0} \right)
\end{align*}
\]

(13)

where $\gamma$ denotes the deviation of the tangential stress from the x axis, $\varphi$ denotes the deviation of the drift from the x axis; $\beta$ denotes the deviation of the drift from the tangential stress at the ice/water boundary; for a particular stratification in the boundary layer, the deviation can be assumed to be independent of the drift velocity so that $\cos \beta = c_1$ and $\sin \beta = c_2$ (for the stratification parameter $\mu = 0 \rightarrow \beta \approx 163^\circ$); $V_0$ denotes the absolute value of the drift velocity; and $v_4$ is the drag velocity.

The drag velocity can be expressed by the drift velocity and the universal functions $\eta'_0$ and $\phi'_0$ from [1] when one uses the equations of motion in the boundary layer under the ice (equations of motion written for the roughness level $z = z_0$). According to the results of [7] and [4], the
equations of motion assume the following form in any coordinate system:

\[
\begin{align*}
\tau_x &= \frac{v_x}{z}(\eta_x \cos \gamma - \eta_z \sin \gamma), \\
\tau_y &= \frac{v_x}{z}(\eta_y \cos \gamma + \eta_z \sin \gamma),
\end{align*}
\]  

so that

\[
\frac{v^2}{x} = \frac{v_0^2}{c_0 + v_0^2}.
\]  

By inserting the equation (15) into equation (13) we obtain

\[
\begin{align*}
\tau_{wx} &= \frac{\rho x^2}{c_0 + v_0^2} \cdot V_0 (c_1 u_0 + c_2 v_0), \\
\tau_{wy} &= \frac{\rho y^2}{c_0 + v_0^2} \cdot V_0 (c_1 v_0 - c_2 u_0).
\end{align*}
\]

It follows from observations which were made on drifting stations that the drift velocity at distances far from the shore is primarily determined by the wind, and the effect of the surrounding ice fields can be considered a small addition to the wind-induced drift velocity. Therefore, we assume

\[
u_0 = u_0 + \nu_0', \quad v_0 = v_0 + \nu_0'.
\]

The quantity \(\eta_0'^2 + \sigma_0'^2\) changes slightly when the drift velocity changes (changes of the drift velocity by a factor of 2 involve changes of \(\eta_0'^2 + \sigma_0'^2\) by 20%). One can therefore assume that this quantity is only a function of the wind-induced drift. Thus, when we insert equation (17) into equation (16) and assume that \(u_0'\) and \(v_0'\) are small, we obtain

\[
\begin{align*}
\tau_{wx} &= \tau_{wx} + A_0 \left( c_1 + c_1 \frac{u_{0x}^2}{V_{0x}^2} + c_2 \frac{u_{0x} \cdot v_{0x}}{V_{0x}^2} \right) \cdot \nu_0' + \\
&\quad + A_0 \left( c_2 + c_1 \frac{u_{0x} \cdot v_{0x}}{V_{0x}^2} + c_2 \frac{v_{0x}^2}{V_{0x}^2} \right) \cdot \nu_0', \\
\tau_{wy} &= \tau_{wy} - A_0 \left( c_2 + c_2 \frac{u_{0y}^2}{V_{0y}^2} - c_1 \frac{u_{0x} \cdot v_{0x}}{V_{0x}^2} \right) \cdot \nu_0' + \\
&\quad + A_0 \left( c_1 - c_2 \frac{u_{0y} \cdot v_{0x}}{V_{0y}^2} + c_1 \frac{v_{0y}^2}{V_{0y}^2} \right) \cdot \nu_0'.
\end{align*}
\]
where

\[ A_0 = \frac{\rho z^3}{v_0^3 + c_0^3} \cdot V_0; \]

\[ \tau_{\text{we}} = A_0 (c_1 u_{0a} + c_2 v_{0a}); \]

\[ \tau_{\text{wy}} = A_0 (c_1 v_{0a} - c_2 u_{0a}). \]

Furthermore, when we assume that the tangential stress at the ice/air boundary is determined only by the wind velocity, and when we subtract the equations of the purely wind-induced drift from equations (1) and (2) for the stationary drift, we obtain with equations (17) and (18) relations for the drift components \( u'_0 \) and \( v'_0 \):

\[ A_1 v'_0 + A_2 u'_0 + k_0 \cdot \Delta u'_0 = -k_0 \cdot \Delta u_{0a}, \]
\[ A_3 v'_0 - A_4 u'_0 + k_0 \cdot \Delta v'_0 = -k_0 \cdot \Delta v_{0a}, \]

where

\[ A_1 = A_0 \left( \frac{m^3}{\lambda_0} + c_2 + c_1 \cdot \sin \varphi \cdot \cos \varphi + c_2 \cdot \sin^2 \varphi \right); \]

\[ A_2 = A_0 (c_1 + c_1 \cdot \cos^2 \varphi + c_2 \cdot \sin \varphi \cdot \cos \varphi); \]

\[ A_3 = A_0 (c_1 - c_2 \cdot \sin \varphi \cdot \cos \varphi + c_1 \cdot \sin^2 \varphi); \]

\[ A_4 = A_0 \left( \frac{m^3}{\lambda_0} + c_2 \cdot \cos^2 \varphi - c_1 \cdot \sin \varphi \cdot \cos \varphi \right). \]

It is easy to derive the following equation system from equations (19) and (20):

\[ A_5 \cdot v'_0 + k_0 (A_4 \cdot \Delta u'_0 + A_2 \cdot \Delta v'_0) = -k_0 (A_4 \cdot \Delta u_{0a} + A_2 \cdot \Delta v_{0a}), \]
\[ A_5 \cdot u'_0 + k_0 (A_3 \cdot \Delta u'_0 - A_1 \cdot \Delta v'_0) = -k_0 (A_3 \cdot \Delta u_{0a} - A_1 \cdot \Delta v_{0a}), \]

where \( A_5 = A_1 \cdot A_4 + A_2 \cdot A_3 \).

We switch to dimensionless variables in equations (21) and (22) by introducing characteristic velocity scales so that

\[ u'_0 = u'_0^* \cdot \overline{V}_0; \quad v'_0 = v'_0^* \cdot \overline{V}_0; \]
\[ u_{0a} = u_{0a}^* \cdot \overline{V}_{0a}; \quad v_{0a} = v_{0a}^* \cdot \overline{V}_{0a}, \]

where \( \overline{V}_0 \) and \( \overline{V}'_0 \) denote the absolute values of the purely wind-induced drift and the additional drift at the point under consideration, respectively.

We use the characteristic size of a baric formation, i.e., the radius of an isobar curvature, as the scale \( R \) for the length. According to [2], this radius amounts to about 500 km. In dimensionless notation, equations
(21) and (22) assume the form

\[ v_0^* + \varepsilon (b_1 \cdot \Delta v_{0a}^* + b_2 \cdot \Delta v_{0a}^*) = -\mu (b_4 \cdot \Delta u_{0a}^* + b_5 \cdot \Delta v_{0a}^*), \tag{24} \]

\[ u_0^* + \varepsilon (b_2 \cdot \Delta u_{0a}^* - b_1 \cdot \Delta v_{0a}^*) = -\mu (b_5 \cdot \Delta u_{0a}^* - b_1 \cdot \Delta v_{0a}^*), \tag{25} \]

where

\[ \varepsilon = \frac{k_0}{R^2 \cdot \bar{A}_0}; \quad \mu = \varepsilon \cdot \frac{\bar{V}_m}{\bar{V}_0}; \]

\[ b_1 = \frac{A_1}{A_0}, \quad b_2 = \frac{A_2}{A_0}, \quad b_3 = \frac{A_3}{A_0}, \quad b_4 = \frac{A_4}{A_0}, \quad b_5 = \frac{A_5}{A_0}; \]

and \( \bar{A}_0 \) denotes the \( A_0 \) value at the point of interest.

When we use Campbell's estimate \([11]\) \( (k_0 \sim 10^{12} \ g/\sec) \) for \( k_0 \), we obtain \( \varepsilon < 0.3 \) even for \( R = 300 \) km. When the drift far from the shore is considered, this estimate justifies a solution to equations (24) and (25) in the form of a power series of the small parameter \( \varepsilon \). The two equations are used in succession. When we neglect terms of the order of \( \varepsilon^2 \), we obtain

\[ v_0^* = -\mu (b_4 \cdot \Delta u_{0a}^* + b_5 \cdot \Delta v_{0a}^*) + \varepsilon \cdot b_1 \Delta (b_1 \cdot \Delta v_{0a}^*) + \varepsilon (b_5 \cdot \Delta u_{0a}^* + b_1 \cdot \Delta v_{0a}^*) + 0(\varepsilon^2); \]

\[ u_0^* = -\mu (b_2 \cdot \Delta u_{0a}^* - b_1 \cdot \Delta v_{0a}^*) + \varepsilon \cdot b_1 \Delta (b_1 \cdot \Delta v_{0a}^*) - \Delta (b_1 \cdot \Delta v_{0a}^*) + \varepsilon \cdot b_1 \Delta (b_5 \cdot \Delta u_{0a}^* - b_4 \cdot \Delta v_{0a}^*) + 0(\varepsilon^2), \]

or, in dimensionless form,

\[ v_0' = -k_0 \cdot M + \frac{k_0^2}{A_0} (A_4 \cdot \Delta N + A_2 \cdot \Delta M), \tag{26} \]

\[ u_0' = -k_0 \cdot N + \frac{k_0^2}{A_0} (A_3 \cdot \Delta N - A_1 \cdot \Delta M), \tag{27} \]

where

\[ M = \frac{1}{A_0} (A_4 \cdot \Delta u_{0a} + A_2 \cdot \Delta v_{0a}); \]

\[ N = \frac{1}{A_0} (A_3 \cdot \Delta u_{0a} - A_1 \cdot \Delta v_{0a}). \]

Equations (26) and (27) were used to calculate the additional drift of the station "Severnui Polyus-4" (North Pole 4) during the four months of July, September, October, and November 1954 (free drift of the ice floe was observed during August). The velocity of the wind-induced drift of the surrounding ice fields was determined from the geostrophic wind. The geostrophic wind was evaluated by calculations based on daily 24-hour pressure maps. The step length in the calculations of all derivatives was assumed equal to 250 km. Only the first terms of the right side of equations
(26) and (27) were used in the calculations so that 

\[ S'_y = -k_0 \sum_{i=1}^{n} M_i \cdot \delta t; \quad S'_x = -k_0 \sum_{i=1}^{n} N_i \cdot \delta t, \]

where \( \delta t \) is equal to 24 hours and \( n \) denotes the number of days.

The wind-induced drift was determined from the results of D.L. Laikhtman's work [7]. The gradient flow was assumed equal to the flow velocity which we had determined during August [4]. All three drift components are plotted in Figure 1. Optimum agreement between the calculations and the observations on the drift of the station "Severny Polyus-4" (North Pole 4) was obtained for the above-mentioned time intervals when the following values were employed for the coefficient \( k_0 \):

<table>
<thead>
<tr>
<th>Time Period</th>
<th>( k_0 ) (g/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>July 5-August 4</td>
<td>( 3.8 \cdot 10^{12} )</td>
</tr>
<tr>
<td>September 5-30</td>
<td>( 4.2 \cdot 10^{12} )</td>
</tr>
<tr>
<td>October 1-31</td>
<td>( 8.5 \cdot 10^{12} )</td>
</tr>
<tr>
<td>November 1-30</td>
<td>( 10.8 \cdot 10^{12} )</td>
</tr>
</tbody>
</table>

The increase which is observed in the coefficient \( k_0 \) from July to December may be caused by the increased compactness and thickness of the ice fields in the winter months, because the relation \( k_0 = \kappa_0 \cdot m \) holds and this means, generally speaking, that the coefficient depends upon both the compactness and the thickness of the ice cover.

Figure 1.
Coordinates and components of the drift of the station "Severny Polyus-4" (North Pole 4) in 1954.

\( S_P \) denotes the vector of the wind-induced drift; 
\( S_T \) denotes the drift with the constant current; and 
\( S' \) denotes the vector of the drift resulting from the effect of the surrounding ice fields.
REFERENCES


Thermal Cracks in Floating Ice Sheets
R. J. Evans and N. Untersteiner

When the air temperature drops below the water temperature under a floating ice sheet, thermal cracks often occur. To acquire quantitative information on these cracks from an analytic point of view, the ice has been represented as a homogeneous elastic floating plate. The effect of thermal contraction then becomes equivalent to a lateral surface load. After the problem has been formulated in general terms, three special conditions which lend themselves to analytic treatment are considered: the wide ice sheet under conditions of plane strain, the narrow ice sheet under plane stress conditions and the axisymmetric ice sheet. The first two lead to simple solutions that illustrate general effects, the third is of more practical significance. Typical stress distributions prior to cracking are shown and for particular numerical values, some of which are applicable to arctic sea ice; typical crack spacing is related to the temperature difference between the upper and lower surface. Finally, the assumptions on which the analysis rests are examined critically with regard to establishing the validity of the results and to indicate ways in which improvements in the analytic treatment can be made.

Some Results from a Time-dependent Thermodynamic Model of Sea Ice
Gary A. Maykut and Norbert Untersteiner

A one-dimensional thermodynamic model of sea ice is presented that includes the effects of snow cover, ice salinity, and internal heating due to penetration of solar radiation. Surface-energy balances determine rates of ablation and accretion; diffusion equations govern heat transport within the ice and snow. The incoming radiative and turbulent fluxes, oceanic heat flux, ice salinity, snow accumulation, and surface albedo are specified as functions of time. Starting from an arbitrary initial condition, the model is integrated numerically until annual equilibrium patterns of temperature and thickness are achieved. The model is applied to the central Arctic. Input values for the initial test of the model are based on observational data.
predicted by the model for the average ice thickness (288 cm), amount of surface ablation (40 cm), and the temperature field all agree closely with field observations. Other results from the model indicate that, under present conditions, the ocean must supply 1 to 2 kcal/cm² year to the ice; an additional 4 kcal/cm² year would cause the ice to vanish. Annual snow depths greater than 70 cm would result in much thicker ice. Comparison of observed and calculated temperature profiles suggest that about 2.0 to 2.5 kcal/cm² year of the incoming short-wave radiation penetrates the ice and contributes to internal heating. Average ice albedos under 0.50 would cause the ice to vanish in a few years.
As the AIDJEX Bulletin went to press, the 1971 pilot study was nearing completion at Camp 200 with almost all its scientific objectives accomplished. More complete reports on the pilot studies will appear in future Bulletins.

After minor delays caused by adverse weather conditions and aircraft unavailability, Camp 200 was established at 73°45'N, 130°15'W, on an old floe 16-20 feet thick and 10 miles across. An airstrip was laid out on new ice about six feet thick adjacent to the camp site. On March 1 and 3, three C-130 flights carried 16 prefabricated buildings and a five-man construction team to Tuktoyaktuk from Point Barrow. Bristol aircraft then began ferrying buildings and equipment to Camp 200. The Weeks-Campbell party flew to the camp on March 5 to prepare for ground truth studies in connection with the NASA 990 flights.

High winds and low visibility at Camp 200 prevented supply flights on March 7 and 8, and on the 8th the Bristol aircraft was grounded for an engine change. Its replacement and a Twin Otter aircraft resumed transportation of supplies and personnel. By March 8 all personnel except the Brown party and five of the Oceanography group had arrived at Camp 200.

The first flight of the NASA 990 research aircraft was successfully completed March 9, although only one tellurometer site had been established by that time because of bad weather on the previous three days. The full strain network was occupied for NASA flights on March 11, 12, and 15. A whiteout on the 16th prevented helicopter flying, and extreme ice fracturing kept the personnel from reaching the tellurometer stations for the final NASA flight. Despite these difficulties, the ground truth program exceeded expectations.

While the NASA flights were under way, Coachman's group established the remote hydroholes. Smith's under-ice diving program began March 14. The Coast Guard C-130 flew a SLAR and photographic mission on March 15, and on March 21 and 23 the NAVOCEANO "Birdseye" aircraft flew remote sensing missions over the AIDJEX area. The ice deformation mentioned earlier forced the relocation of one tellurometer site.
The Weeks-Campbell party was evacuated after the aircraft remote sensing program had been completed. By April 1, the Coachman, Hunkins, and Smith parties had accomplished their objectives and were reducing operations in preparation for leaving. Brown's first tower became operational on March 28, the second tower the next day. Plans call for the evacuation of all personnel by April 10.

The Camp 200 position as of April 1 was 73°50'N, 131°02'W.
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