AIDJEX BULLETIN

ARCTIC ICE DYNAMICS JOINT EXPERIMENT
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**Cover:** Photograph of Camp 200, site of the 1971 AIDJEX pilot study, taken during a remote-sensing flight at 3500 ft. by the NASA 990 research aircraft Galileo. The camera used is a Wild-Heerbrugg RC-8 metric mapping camera installed in the NASA aircraft.
INVESTIGATION OF THE ICE CONDITIONS IN THE ARCTIC SEAS
AND METHODS OF FORECASTING AND COMPUTATION

(translated for the National Science Foundation
under the Public Law 480 Program)

* * * *

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* * * *
The AIDJEX Bulletin aims to provide both a forum for discussing AIDJEX problems and a source of information pertinent to all AIDJEX participants. Issues—numbered, dated, and sometimes subtitled—contain technical material closely related to AIDJEX, informal reports on theoretical and field work, translations of relevant scientific reports, and discussions of interim AIDJEX results.

The collection of papers which comprise Volume 303 of the Proceedings (Trudy) of the Arctic and Antarctic Scientific Research Institute in Leningrad covers a wide range of topics related to the ice conditions in the Arctic seas. According to the editor's Preface, it is intended for oceanographers and graduate students of hydro-meteorological specialization.

Because of the length of the translation, we are printing the volume in two halves. The first half, AIDJEX Bulletin No. 16, contains one group of papers which deal with the variations of ice coverage and methods of computation and forecasting, and another group addressing problems related to ice drift.

The second half of Volume 303, which will appear as AIDJEX Bulletin No. 17, covers (1) formation and deterioration of the ice cover and (2) methodological topics connected with the study of the parameters and deformation of the ice cover. A list of papers in this second half is given on page 133.

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INVESTIGATION OF THE ICE CONDITIONS IN THE ARCTIC SEAS AND METHODS OF FORECASTING AND COMPUTATION

Edited by N. A. Volkova

Hydrometeorological Press, Leningrad, 1971

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The ice coverage of an Arctic sea characterizes the ice conditions of that sea in a highly general though not quite comprehensive way. Taking into account the long-term cumulative effects of ice accumulation and deterioration, this ice coverage is subjected to significant seasonal and annual fluctuations.

From freeze-up through the end of winter (from November through May or June), the sea surface is almost completely covered with ice. Ice accretion, ice transport, and hummocking increase the thickness of the ice cover in this period.

With the onset of summer breakup, the ice starts to melt and the nearly continuous ice cover begins to open, its area shrinking steadily though nonuniformly. At the end of summer (September), the ice coverage of Arctic seas is generally at its minimum; it then starts to increase rapidly through autumnal freezing.

Studies of the general features and the factors responsible for the variation in summer ice coverage in space and in time are necessary for developing techniques for long-range ice forecasting.

Over the last 30-35 years, systematic ice surveys have yielded extensive information on the ice coverage of various regions in the Arctic Ocean. Statistical treatment of these data provides definite conclusions about the multiyear variations. That part of the Arctic Ocean covered by ice surveys was divided into ten regions in accordance with natural geographical features.

Since the regions involved in this survey were of different sizes, the ice coverage figures were reduced to a common (unit) area

1
by multiplying by numerical coefficients calculated as the ratio of
the unit area to the area of the individual regions.

The specific ice coverage obtained by this method was reduced
graphically to the first day of each ten-day period so as to ensure
availability of initial data corresponding to identical starting
times. The averaged values of the specific ice coverage for each
of the summer months (July, August, September) and for the entire
summer were used in this analysis.

Analysis of the year-to-year changes of ice coverage in the
ten regions indicates that these changes are apparently caused by
a superposition and interaction of cycles with different amplitudes
and periods.

The multiyear ice coverage curves are decomposed into the
cyclic component by simple filtration, a climatic smoothing or
averaging followed by subtracting the averaged series from the
natural series.

Figure 1 shows an example of such a filtration procedure. The
ttrue ice coverage curve (solid curve in the figure) was smoothed
using the relation

\[ 0.25L_1 + 0.50L_2 + 0.25L_3 \]

The difference between the natural and the smoothed series (the dashed
curve, Figure 1a) gave the ordinates of some short-period "two-year"
cycles. To achieve better filtration of these cycles, the smoothing
procedure can be applied again to form second-order differences.

The resultant difference series was again subjected to climatic
smoothing, which is equivalent to the smoothing of the initial series
according to the relation

\[ 0.0625L_1 + 0.1875L_2 + 0.500L_3 + 0.1875L_4 + 0.0625L_5 \]

A new difference series was formed between the first and the second
smoothed series, which characterizes "five-year" cycles (Figure 1b). The seven-to-eight-year "nutation" cycles were isolated by smoothing only the adjoining ice coverage minima and maxima. The intermediate characteristics were then obtained by trigonometric interpolation (Figure 1c). The residual curve (Figure 1d) obtained by this filtration constitutes a many-year curve on which other component cycles are superimposed.
This method was applied to process the entire series of monthly and ten-day ice coverage data of all the regions. Comparison of the cyclic ice-coverage curves for these regions reveals certain identical features in the structure of the cyclic oscillations, i.e., certain common cause-and-effect relations in their formation.

Other statistical methods were also used: correlation analysis, autocorrelations, significance estimates of the correlation coefficients, significance estimates of differences between two samples, and separation between random and regular results.

**General Description of the Multiyear Fluctuations of Ice Coverage in the Arctic Regions**

Each region has its own characteristic physico-geographical features which are reflected in the prevailing ice conditions. Therefore, before proceeding with the analysis of the cyclic variations, we have to consider the general background variation. This is particularly important because the main features of the general fluctuations of ice coverage are closely related to the space distribution of the individual spectral components of the cyclic variation in ice coverage (Figure 2 and Table 1). We see from Figure 2 that two distinct groups of regions—IV, V and VII, VIII, IX—have increased ice coverage. Regions I, II, III and X have a specific ice coverage which is less than half that of the other regions.

In summer, the specific ice coverage decreases nonuniformly. The ice coverage in all regions except VII decreases faster in July than in August. The ice melting process is particularly fast in III and VI, where most of the ice cover disappears in this period. In VII, VIII, IX, where ice melting begins later and proceeds at a slower rate, the minimum coverage is observed in August.

Let us consider the space distribution of the multiyear ice-coverage amplitudes over the regions (Table 2).
Fig. 2. Geographic distribution of the (reduced) monthly norms of ice coverage by region (ice-coverage norms reckoned in the downward vertical direction).

TABLE 1
MULTIYEAR AVERAGES OF (REDUCED) ICE COVERAGE IN THE ARCTIC (by thous. km²)

<table>
<thead>
<tr>
<th>Region*</th>
<th>Months</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>July</td>
<td>August</td>
</tr>
<tr>
<td>I</td>
<td>34.4</td>
<td>15.8</td>
</tr>
<tr>
<td>II</td>
<td>51.0</td>
<td>27.3</td>
</tr>
<tr>
<td>III</td>
<td>95.3</td>
<td>28.2</td>
</tr>
<tr>
<td>IV</td>
<td>125.8</td>
<td>87.9</td>
</tr>
<tr>
<td>V</td>
<td>128.3</td>
<td>94.5</td>
</tr>
<tr>
<td>VI</td>
<td>119.3</td>
<td>64.5</td>
</tr>
<tr>
<td>VII</td>
<td>144.6</td>
<td>111.4</td>
</tr>
<tr>
<td>VIII</td>
<td>135.5</td>
<td>136.9</td>
</tr>
<tr>
<td>IX</td>
<td>151.6</td>
<td>137.3</td>
</tr>
<tr>
<td>X</td>
<td>101.6</td>
<td>62.5</td>
</tr>
<tr>
<td>Average</td>
<td>107.1</td>
<td>76.8</td>
</tr>
</tbody>
</table>

*In all figures and tables, the regions are numbered according to the figure and table on this page.
TABLE 2
REDUCED MULTIYEARAMPLITUDES OF ICE COVERAGE IN THE ARCTIC FOR 1940-1966 (in thous. km²)

<table>
<thead>
<tr>
<th>Regions</th>
<th>Months</th>
<th>July</th>
<th>August</th>
<th>Sept.</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>III</td>
<td>125.7</td>
<td>130.4</td>
<td>40.8</td>
<td>99.0</td>
<td></td>
</tr>
<tr>
<td>IV</td>
<td>64.4</td>
<td>120.8</td>
<td>130.3</td>
<td>104.0</td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>70.7</td>
<td>130.6</td>
<td>135.7</td>
<td>104.8</td>
<td></td>
</tr>
<tr>
<td>VI</td>
<td>69.5</td>
<td>116.0</td>
<td>125.8</td>
<td>106.5</td>
<td></td>
</tr>
<tr>
<td>VII</td>
<td>31.2</td>
<td>77.1</td>
<td>122.5</td>
<td>74.5</td>
<td></td>
</tr>
<tr>
<td>VIII-IX</td>
<td>13.8</td>
<td>39.4</td>
<td>83.1</td>
<td>45.2</td>
<td></td>
</tr>
<tr>
<td>X</td>
<td>74.7</td>
<td>97.8</td>
<td>86.9</td>
<td>79.1</td>
<td></td>
</tr>
</tbody>
</table>

The maximum change in ice coverage in July is noted in region III, and in August-September in IV, V, and VI. The smallest amplitudes of year-to-year variation are observed in VII, VIII, and IX. In all regions except III and X, the multiyear ice-coverage amplitudes in August are double the July amplitudes, and they are even greater in September. The multiyear amplitude of the general ice coverage for these regions over the 35-year period is large and nonuniformly distributed in longitude.

The nonuniform space distribution of the ice-coverage norms and the multiyear amplitudes is due to a variety of physico-chemical conditions which are different in each region and are determined by the particular features of the ice accumulation and deterioration processes. Numerous studies have shown that ice coverage provides an approximate index of the ice volume; therefore, the nonuniform distribution of the ice-coverage norms in summer is mainly determined by the nonuniform spatial distribution of the ice-cover thickness at the time of its maximum development. Regions with the thickest ice cover in winter are characterized by the highest specific ice coverage in summer, and regions with thin ice cover at the beginning of the melting season show a low ice coverage in summer.
The distribution of the specific ice coverage in the Arctic seas thus depends largely on the ice accumulation processes in autumn and winter, which in turn are determined by the climate and the system of air and sea currents. The nonuniformity of the ice-coverage amplitudes, on the other hand, is determined by anomalous hydrometeorological processes in the individual years.

Without going in detail into this question, we should note that in regions with diametrically opposite specific ice coverages, the range of amplitudes is $1/2$ to $1/3$ of that in other regions. The relatively small year-to-year variation of ice coverage in these regions is apparently associated with the predominant effect in each region of a certain group of factors which regulate ice coverage.

The specific ice-coverage norms thus can be treated as a background on which are superimposed the year-to-year cyclic fluctuations responsible for the ice-coverage anomalies at any given time. A study of the space and time structure of the ice-coverage cycles is therefore of definite interest as a first stage in exploring the relation of these cyclic changes to the observed general planetary periodic behavior of hydrometeorological and heliogeophysical processes.

"Two-year" Cycle

Recent studies have revealed a distinct cycle with a period of two to three years in all atmospheric and oceanic processes. Such cycles in ice coverage were noted by Betin and Preobrazhenskii [1]. Recently, two-year cyclic fluctuations in ice transfer were observed by Zakharov [5].

Analysis of the year-to-year variation of the ice coverage indicates that the two-year cycle not only is observed in each of the regions but also makes a significant contribution to the ice-coverage anomalies. The two-year cycle was observed to have its own characteristic features
in each region; in particular, the cycle amplitudes are different. Figure 3 shows the curves of the two-year ice-coverage cycles for all the regions. The geographical distribution of the two-year cycles in the Arctic falls into three groups:

1) western (IV and V), where the two-year cycle is the most active;
2) intermediate (VI, VII, VIII, IX), with the smallest amplitude of the two-year cycle;
3) eastern (X), where the two-year cycle again becomes more active.

![Figure 3. Two-year variations of the (reduced) ice coverage in September by regions.](image-url)
Statistical significance estimates of the two-year ice-coverage cycles were obtained by testing for autocorrelation anomalies from the same series with a time lag of one year. The check for autocorrelation was performed for each region separately, using ten-day ice-coverage data (Table 3). The significance estimate of the autocorrelation coefficient was computed from the relation $T = r\sqrt{n-1}$. For $T > 3$, the correlation is significant; for $T \leq 3$, the correlation is accidental. The results of this analysis are listed in Table 4, which + indicates significant correlation and - indicates accidental correlation.

According to Table 4, 40 of the 63 coefficients proved to be significant. The autocorrelation coefficients for region VI proved to be accidental. This result, however, does not necessarily mean that this region has no two-year ice-coverage cycles. It is probably due to the overstrictness of the significance test. Therefore, besides the test $T$, we also used the test $\rho$ (see Table 4). Table 4 shows that the test $T = r\sqrt{n-1} > 3$ is statistically equivalent to 0.1% significance level, which is a stricter test than the 5% significance level commonly used as the critical point in climatology.

### TABLE 3

AUTOCORRELATION COEFFICIENT OF THE ANOMALIES OF TWO-YEAR ICE-COVERAGE FLUCTUATIONS IN THE ARCTIC BY TEN-DAY PERIODS

<table>
<thead>
<tr>
<th>Region</th>
<th>July</th>
<th>August</th>
<th>September</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
<td>III</td>
</tr>
<tr>
<td>III</td>
<td>0.63</td>
<td>-0.65</td>
<td>-0.65</td>
</tr>
<tr>
<td>IV</td>
<td>-0.71</td>
<td>-0.80</td>
<td>-0.78</td>
</tr>
<tr>
<td>V</td>
<td>0.72</td>
<td>-0.74</td>
<td>-0.75</td>
</tr>
<tr>
<td>VI</td>
<td>-0.53</td>
<td>-0.56</td>
<td>-0.42</td>
</tr>
<tr>
<td>VII</td>
<td>0.62</td>
<td>0.70</td>
<td>-0.82</td>
</tr>
<tr>
<td>VIII-IX</td>
<td>0.73</td>
<td>-0.59</td>
<td>-0.49</td>
</tr>
<tr>
<td>X</td>
<td>0.54</td>
<td>0.65</td>
<td>-0.52</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
TABLE 4
SIGNIFICANCE ESTIMATES OF THE AUTOCORRELATION COEFFICIENTS
OF THE ANOMALIES OF THE TWO-YEAR ICE-COVERAGE FLUCTUATIONS
BY TEN-DAY PERIODS
(Top row, $T$; bottom row, $\rho$ in per cent.)

<table>
<thead>
<tr>
<th>Region</th>
<th>July</th>
<th>August</th>
<th>September</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
<td>III</td>
</tr>
<tr>
<td>III</td>
<td>$+$</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>IV</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>V</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>VI</td>
<td>$+$</td>
<td>1.0</td>
<td>0.5</td>
</tr>
<tr>
<td>VII</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>VIII-IX</td>
<td>$+$</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>X</td>
<td>0.5</td>
<td>0.1</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Student's distribution $t$ was also used to check the significance of the two-year ice-coverage cycle. The samples (ice-coverage anomalies) were taken from the same series and were correlated. Computations (Table 5) show that the two-year cycles are accidental in only a few of the regions. Only the two-year cycles in regions VII-IX may be treated as random fluctuations with 50% probability. The accidental

TABLE 5
PROBABILITY OF ACCIDENTAL TWO-YEAR FLUCTUATIONS OF ICE
COVERAGE BY TEN-DAY PERIODS (Student's $t$-distribution test)

<table>
<thead>
<tr>
<th>Region</th>
<th>July</th>
<th>August</th>
<th>September</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
<td>III</td>
</tr>
<tr>
<td>III</td>
<td>1.7</td>
<td>5.7</td>
<td>19.9</td>
</tr>
<tr>
<td>IV</td>
<td>0.1</td>
<td>0.03</td>
<td>0.04</td>
</tr>
<tr>
<td>V</td>
<td>9.3</td>
<td>12.7</td>
<td>8.0</td>
</tr>
<tr>
<td>VI</td>
<td>0.3</td>
<td>1.4</td>
<td>2.9</td>
</tr>
<tr>
<td>VII</td>
<td>42.2</td>
<td>24.9</td>
<td>9.3</td>
</tr>
<tr>
<td>VIII-IX</td>
<td>46.1</td>
<td>28.0</td>
<td>42.2</td>
</tr>
<tr>
<td>X</td>
<td>24.9</td>
<td>12.7</td>
<td>28.0</td>
</tr>
</tbody>
</table>
appearance of the two-year cycles in only these particular regions seems to be associated with multiyear changes in the two-year circumpolar ice-coverage wave.

The results of the analysis thus reveal a significant two-year cycle of ice coverage in most regions. It is important to study the variations of the two-year cycle in space. To this end, for each year we constructed the geographical distribution of the two-year anomalies in ice coverage. Reduced anomalies were used to ensure that all the data are comparable. The curves revealed an opposite trend in consecutive years. It therefore seems that the space distribution of ice coverage also shows a certain two-year cycle.

To prove the existence of two-year fluctuations of the space wave, we again checked for autocorrelation with a time lag of one year. The two-year anomalies of all regions for the year 1934 were correlated with the data for 1935, which in their turn were correlated with 1936 data, and so on. The series checked for correlation included ice-coverage data for the three navigable months (July, August, September). This approach was adopted in order to allow, first, for the changes in ice coverage during a single season and, second, for the similarity of these changes in consecutive years.

Indeed, the variation of the two-year ice-coverage cycle in all regions simultaneously during the entire navigable period may be represented as a variation of some fields in space and time coordinates. Successive statistical comparison of these fields by autocorrelation analysis of the data of one year with the data of another will reveal any similarity in variation.

In other words, the autocorrelation coefficient

\[ r = \frac{1}{n} \sum (z_i - \bar{z})(z_{i+1} - \bar{z}) \]

\[ = \frac{\sum z_1 z_2 - \bar{z}_1 \bar{z}_2}{\sqrt{\sum z_1^2 - \bar{z}_1^2} \sqrt{\sum z_2^2 - \bar{z}_2^2}} \]

where \( z_1 \), \( z_2 \) are the ice-coverage anomaly fields of the two-year cycle in space and time coordinates (the ordinate \( y \) is the ice-coverage anomaly for each month; the ordinate \( x \) is the number of seas), may provide a
measure of similarity of these fields. This test was applied to the entire observation series from 1934 to 1965.

Table 6 lists the autocorrelation coefficients and the results of the significance tests (the \( \rho \) test), which indicate that the fields of the spatial two-year ice-coverage wave for consecutive years are similar in the sense that one is essentially the reverse of the other. This proves the significance of the two-year spatial wave of ice coverage in all regions.

<table>
<thead>
<tr>
<th>Year</th>
<th>( r )</th>
<th>( p(%) )</th>
<th>Year</th>
<th>( r )</th>
<th>( p(%) )</th>
<th>Year</th>
<th>( r )</th>
<th>( p(%) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1934-1935</td>
<td>0.72</td>
<td>0.1</td>
<td>1944-1945</td>
<td>-0.88</td>
<td>0.1</td>
<td>1954-1955</td>
<td>-0.40</td>
<td>5.0</td>
</tr>
<tr>
<td>1935-1936</td>
<td>0.70</td>
<td>0.1</td>
<td>1945-1946</td>
<td>-0.92</td>
<td>0.1</td>
<td>1955-1956</td>
<td>-0.73</td>
<td>0.1</td>
</tr>
<tr>
<td>1936-1937</td>
<td>-0.58</td>
<td>1.0</td>
<td>1946-1947</td>
<td>-0.73</td>
<td>0.1</td>
<td>1956-1957</td>
<td>-0.76</td>
<td>0.1</td>
</tr>
<tr>
<td>1937-1938</td>
<td>-0.65</td>
<td>0.5</td>
<td>1947-1948</td>
<td>-0.57</td>
<td>0.5</td>
<td>1957-1958</td>
<td>-0.54</td>
<td>0.5</td>
</tr>
<tr>
<td>1938-1939</td>
<td>-0.74</td>
<td>0.1</td>
<td>1948-1949</td>
<td>-0.15</td>
<td>--</td>
<td>1958-1959</td>
<td>-0.84</td>
<td>0.1</td>
</tr>
<tr>
<td>1939-1940</td>
<td>-0.79</td>
<td>0.1</td>
<td>1949-1950</td>
<td>-0.75</td>
<td>0.1</td>
<td>1959-1960</td>
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<td>0.1</td>
</tr>
<tr>
<td>1940-1941</td>
<td>-0.80</td>
<td>0.1</td>
<td>1950-1951</td>
<td>-0.88</td>
<td>0.1</td>
<td>1960-1961</td>
<td>-0.60</td>
<td>0.5</td>
</tr>
<tr>
<td>1941-1942</td>
<td>0.02</td>
<td>--</td>
<td>1951-1952</td>
<td>-0.35</td>
<td>10.0</td>
<td>1961-1962</td>
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<td>0.1</td>
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<td>1942-1943</td>
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<td>1952-1953</td>
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<td>5.0</td>
<td>1962-1963</td>
<td>-0.14</td>
<td>--</td>
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<tr>
<td>1943-1944</td>
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<td>0.5</td>
<td>1953-1954</td>
<td>0.59</td>
<td>0.5</td>
<td>1963-1964</td>
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The space curves of the two-year cycle indicate that the anomalies for most of the series (1934-1965) have an opposite sign for odd and even years. The probability of sign ambivalence was high, and the monthly average ice-coverage anomalies were therefore computed for all regions from the ten-day data for each month for the odd and even years separately, between 1940 and 1966. The data were used to plot the multiyear average space curves of the two-year cycle for odd and even years, separately (Figure 4). The curves are seen to go in opposite directions. The opposed trend is very distinct, and apparently reflects a regular relationship. It was also established that, on the average, the even
years correspond to positive anomalies in the western regions and the odd years correspond to negative anomalies.

A reverse pattern is observed in the eastern regions. The general trend persists over the entire period. The sign in odd and even years changes in July for VIII-IX, in August for VII, and in September for VI.

Regions VI-IX may thus be regarded as the nodal region of the spatial circum-polar two-year wave; in even years, the sign of the anomaly changes in this region from positive in the western part to negative in the eastern part, and vice versa in odd years. The two-year fluctuations of the ice coverage in the nodal region proved to be relatively small and of a highly unstable sign. It is thus clear why the two-year cycle appears to be accidental in these regions.

Our results also lead to another important conclusion concerning the reverse ice state in the eastern and western regions. The hypothesis of Vize [3] was not confirmed by subsequent studies and was eventually dropped from active consideration. Our analysis of the two-year space-time cycles, however, definitely confirms the existence of opposed trends in the two regions.

The absence of a clearcut general trend for the significant values of ice coverage is due to the nonuniform annual contribution of the two-year anomalies.
As we have mentioned before, the amplitude of the two-year anomalies is not constant. In years of high east-west anomalies of the two-year cycle, the opposition of the significant ice-coverage values is quite distinct; in years when the anomalies differ markedly in magnitude (even though they may have the same sign) due to the contribution of other cyclic components, and especially when the anomalies have different signs, no opposition is observed in the significant ice-coverage values.

Our results thus indicate that the opposition in ice conditions apparent in the multiyear variations of the two-year space wave may be treated as a definite physical feature.

As we have noted before, the magnitude of the two-year ice-coverage fluctuations is quite substantial. It suffices to say that the maximum anomalies of the two-year fluctuations may reach 24-48% of the multiyear amplitudes in some regions. Since the anomalies may be either positive or negative, the contribution of the two-year cycle to the natural variation of the ice cover is thus quite substantial.

The magnitude of these ice-coverage fluctuations and their variability in time and in space can be estimated from the values of the standard deviations computed for each region for the years 1940-1965. The geographical distribution of the magnitudes of the two-year cycle emerges from Figure 5, which shows the lines of equal standard deviations. Two groups of regions clearly emerge from this figure: IV and V, with large standard deviations; and VII-IX, with small standard deviations. These two groups of regions in the Arctic are significantly different with regard to the magnitude of the two-year ice-coverage fluctuations and the degree of their variability. This result additionally corroborates the earlier conclusion about the space structure of the two-year cycle in multiyear variations of ice coverage.
Numerous authors have noted the existence of 4-5-year cycles in sea-ice coverage [5, 10, 11, 1]. Some attempts were made to explain the origin of these cycle [4, 8]. However, the exact nature of these ice-coverage fluctuations remains unclear at this stage, and no external forces are known which could produce this cycle. Observations nevertheless show that the five-year cycle is a permanent characteristic of various oceanic and atmospheric elements, the ice coverage included. However, while the identification of the two-year ice-coverage cycles is derived statistically and the seven-year cycles theoretically, the derivation of the five-year cycles is on less certain ground. The actual period of the five-year cycle is known to vary from four to six years. However, compared to the two-year and the seven-year cycles, the five-year ice-coverage cycles are much more random and substantially less definite (Figure 6).
Detailed studies of the cyclic ice-coverage curves fail to establish a strict dividing boundary between the two-year and five-year cycles, on the one hand, and the seven-year cycles on the other. Since 1950, for example, the five-year cyclic fluctuations have come very close to the seven-year cycle. In this period of highly developed nutation fluctuations in ice coverage, the five-year cycles are suppressed by the seven-year cycles. The five-year and two-year cycles reveal a certain likeness, which is particularly pronounced when the seven-year cycles are indistinct (1934-1950).

Comparison of the various cyclic ice-coverage curves indicates that the five-year cycles are probably the outcome of the superposition of the two- and seven-year cycles. To check this point, the two- and seven-year cycles obtained by the previous method were added and the resultant ice-coverage wave was compared with the five-year cycles (see Figure 6). A definite likeness was observed between the two curves. It therefore seems that the curve decomposition method is not sufficiently refined and does not quite separate all the cycles of close periods. This is quite obvious, since any smoothing and averaging technique suppresses short-period waves and isolates

![Graph showing cyclic ice-coverage trends](image-url)
the long-period components, but traces of the former nevertheless remain.

The five-year cycles shown in Figure 6 thus constitute a kind of artificial residual wave which results from the two-year and seven-year cycles and whose existence should be assigned to the particular properties of the filtration method used. The amplitudes of this wave are approximately 1/5 of the total two-year and seven-year wave. The decomposition of the ice-coverage curve into the two-year and seven-year cyclic components thus contains an a priori error of about 20 per cent, which is nonuniformly distributed between the cycles. The five-year cycles thus appear to be fictitious and of no forecasting value (as originally suggested by Vize).

To check these conclusions, we also plotted the lines of equal standard deviations for the five-year cycle (Figure 7) and compared them with the corresponding lines of the two- and seven-year cycles (see Figures 5, 9). The spatial distribution reveals many features in common. When the corresponding charts for the two- and seven-year cycles are superimposed, the resulting chart is quite similar to the five-year cycle distribution.

Fig. 7. Isopleths of standard deviations of the five-year ice-coverage cycles in the Arctic.
We have previously established that the opposition in ice-coverage conditions between the eastern and the western regions is observed in the two-year spatial wave. The opposition noted by Vize in the five-year cycle is no longer puzzling because of superposition effects. In this way we can also account for the relatively wide range of "periods" of the five-year cycle.

"Seven-year" Cycle

In 1954, Maksimov suggested that the free motion of the instantaneous pole of the Earth and the resultant circumpolar wave of the polar pressure tide should lead to seven-year beats in the sea and the atmosphere. Later, some researchers detected the effect of the movement of the Earth's pole in the variations of the ice coverage of certain seas.

These conclusions had to be verified for all regions, and it was necessary to establish all the characteristic features of the seven-year cycle of ice coverage. As in the analysis of the shorter cycles, the seven-year anomalies of ice coverage in all the regions were plotted in the form of isolines, which provided a graphic picture of the space and time distribution of the corresponding variations (Figure 8). A fairly clear alternation of positive and negative anomalies with a seven-year period was noted. This alternation was particularly obvious in the 1950's and 1960's, when the movement of the Earth's pole was very intense. Conversely, in the epoch of sluggish pole movement (the 1930's and 1940's), the sequence broke down.

We see from Figure 8 that the best defined anomaly nuclei of the seven-year wave appear in region VI. Here the anomalies are approximately a factor of 5-6 higher than the anomalies corresponding to the nutation periphery (regions I and II). Moreover, Figure 8 reveals a gradual shift of the nutation anomalies from east to west. This was originally noted by Nazarov [9] and later confirmed by Maksimov [7] and Kovalev [6].
Fig. 8. Isopleths of the (reduced) anomalies of the seven-year ice-coverage cycles in the Arctic in September.

The isoline chart thus confirmed almost all the basic features of the nutation changes in ice coverage previously noted by other authors. The space distribution of the variability of the seven-year cycles emerges from the lines of equal standard deviations of the nutation anomalies (Figure 9) computed from ten-day data between 1940 and 1965.

Fig. 9. Isopleths of standard deviations of the anomalies of the seven-year ice-coverage cycle in the Arctic.

Figure 9 indicates that the maximum variability of the seven-year fluctuations of ice coverage is noted in regions VI and VII. In other
regions, the nutation fluctuations of ice coverage are much less pronounced and their variability is almost three times less. This fact in itself is not new. It should be noted, however, that the macroscale region in which the seven-year ice-coverage cycle is the most active coincides with the region of conjugation of atmospheric processes in the Northern Hemisphere, where the node of the two-year circumpolar wave of ice coverage is located. Hence, the seven-year fluctuations of ice coverage are the most developed in those parts where the two-year fluctuations are weak. Comparison of the seven-year anomalies for the individual regions with the \( y \) component of the polar movement in February and March reveals a close correlation with a time lag of three to four years (Figure 10).

To proceed with a more detailed study of this relationship, we computed and plotted the correlation functions \( r_{L,y} \) and \( r_{L,x} \), which express the changes in the correlation between \( x \) and \( y \), the components of the pole's movement for January through December, and the seven-year anomalies of ice coverage with a time lag of from zero to four years.

One of the specimen correlation functions obtained for August ice-coverage data is shown in Figure 11. The curves of the correlation functions are close to sine curves, with amplitudes ranging from 0.4 for regions I and VIII to 0.8 for region VII. The correlation functions vary with a period of 14 months. Figure 11 also reveals a definite shift of the maxima and the minima in the general east-west direction. This shift, as we have noted before, reflects the time lag in the propagation of the ice-coverage anomalies in the western regions compared to those in the east. According to Gudkovich, this time lag is one year between central and eastern regions and 18 months between western and central regions.

Using the correlation functions, we can formalize these results and compute the respective time lags between all the regions. The computations were carried out for times corresponding to the maxima and minima of \( r_{L,x} \) and \( r_{L,y} \). The phase differences (in months) between
the corresponding extrema of the functions for the adjoining regions were then determined. The results were averaged using a time lag of from zero to four years. The time lag in the successive extrema of the functions was calculated relative to region VIII. The extreme of $r_L, y$ in region VII were found to lag 1.50 months relative to the corresponding extrema in IX; in VI the time lag reached 2.25 months, 3.50 months in IV, and 5.62 months in I and II. Similar results were
Fig. 11. Correlation functions \( r_{L,x} \) between the seven-year anomalies of ice coverage in August and the component \( x \) of the movement of the Earth's pole compared with time lag between 0 and 4 years for January through December.

obtained from the correlation functions, which gave 2.33, 3.19, 4.76, 6.62, and 6.90, respectively. Averaging the figures for both functions, we finally obtain the following phase differences (in months): 1.9, 2.7, 4.4, 6.1, 6.1, 6.3.

The angular velocity of the free movement of the pole is \( q = 2\pi/T \), where \( T \) is Chandler's period (14 months). In absolute figures, this velocity is 25.7°, and in angular units we have for the phase lag 49.1, 70.0, 112.6, 155.9, 157.4, 160.0°. This in turn corresponds to the following time lags of the nutation anomalies in years: 0.96, 1.36, 2.19, 3.03, 3.06, 3.11. The nutation anomalies in the western regions
thus lag 4.1 years behind those in the east. These time lags on the whole confirm the nutation origin of the east-west displacement of the ice-coverage anomalies.

Note that significant changes of the time lag are observed only in regions VII and VIII. To the west of these regions, the time lag on the whole remains constant. It thus seems that the deformation forces produced by the pole movement affect the ice coverage mainly in the central regions. It is also significant that in the extreme west and east regions the correlation functions do not exceed values of 0.2–0.3. Nutation anomalies in these regions, if at all present, are quite small.

The closest correlation between these phenomena is observed in the ice-coverage data for August. This effect is attributed to the seasonal variation of ice coverage and the degree of its variability in the navigable period. The nutation fluctuations of ice coverage reach their maximum in region VI in the second and third ten-day periods in August. The correlation functions $r_{L,y}$, $r_{L,x}$ computed for this region for the three ten-day periods of July, August, and September with time lags of from two to three years reveal a steady improvement of the correlation. The correlation coefficient $r_{L,y}$, which is $-0.596$ in the first ten-day period of July, increases to $-0.874$ in the second ten-day period of August, and then decreases to $-0.697$ in the third ten-day period of September. A similar seasonal variation is observed for the correlation coefficient $r_{L,x}$. The maximum magnitude of the correlation coefficient $r_{L,x} = -0.837$ according to the ice-coverage data for the second ten-day period of August is obtained for a time lag of three years.

Marking the values $(r_{L,y})_i$ along the $y$ axis and the values $(r_{L,x})_i$ along the $x$ axis, where $i$ is the month ($i_0 = $ January, $i_1 = $ February, $i_2 = $ December), we plot the cross-correlation functions in the form of circular diagrams. Note that the coordinate axes $x$ and $y$ were selected in accordance with the geographical system, which is used in geophysics.
for a fixed position of the pole. For the three ten-day periods of July and two ten-day periods of August, these diagrams constitute an unwinding spiral (Figure 12). Each loop of the spiral corresponds to 14 months, i.e., the period of the free movement of the pole.

![Figure 12. Spiral plotting the correlation data which characterize the change in correlation between ice coverage of region IV and the movement of the radius-vector of Earth's pole as function of the seasonal variations in ice coverage. I--July, first 10-day period; II--July, second 10-day period; III--July, third 10-day period; IV--August, first 10-day period; V--August, second 10-day period. Arabic numerals in the diagram identify the months.](image)

Let us now consider the amplitudes of the nutation cycles and their probable contribution to the general fluctuations of ice coverage in the Arctic. The seven-year cycle amplitudes were determined from ten-day data of the nutation anomalies for July, August, and September, and averaged over the number of cycles noted between 1940 and 1966 in each of the 19 samplings. As expected, the space distribution of the seven-year cycle amplitudes in the Arctic was identical to the corresponding distribution of the standard deviations. To eliminate the effect of seasonal factors, the amplitudes were expressed in per cent of the multiyear amplitudes of the natural series corresponding to ten-day ice-coverage data of all the months. According to Table 7, the amplitudes for each region are approximately of the same order of magnitude, so that averaging could be carried out. On the average, the nutation fluctuations of ice coverage in region VI are substantial, reaching 60 per cent of the multiyear amplitude, or about 20 per cent of the area of this region.
However, since this cycle is localized in a certain region, is most pronounced only in times of intense pole movement, and has a period of about seven years, it is less significant than the two-year cycle, which is of much greater magnitude in certain regions. It also should be noted that in regions VI and VII, where the contribution of the two-year cycle is small, the long-period cycles, particularly the seven-year cycle, significantly influence the multiyear fluctuations of ice coverage.

"Many-year" Cycles

To elucidate secular trends in the variation of ice coverage, we should concentrate on the "many-year" fluctuations (Figure 13). The curves in Figure 13 are based on studies of two-year and seven-year cycles.

Because of the limited period of fluctuations, we cannot conduct a statistically significant analysis of the many-year ice-coverage curves based on the monthly average data. It seems that the many-year fluctuations are composed of two cyclic components: a 19-22-year cycle and a residual background, or secular, cycle.

The anomaly data for the many-year ice-coverage wave for the years 1934-1964 were used to plot the space distribution curves.
Fig. 13. Curves of many-year fluctuations of (reduced) ice coverage in the Arctic for July (with two-year and seven-year cycles eliminated).

(Figure 14). The curves in the figure reveal definite regular features for all the months. Three main periods are evident for these curves over the years. The first period covers the years from 1934 to 1944 and coincides with the eleven-year solar cycle (No. 17, 1933–1944). The second period (1944–1954) may be assigned to solar cycle No. 18, and the third period (1954–1964) coincides with solar cycle No. 19. (The cycles are numbered according to the Zurich classification.)
During the odd cycles, Nos. 17 and 19, the many-year space curves of ice coverage show certain year-to-year changes such that in the successive even cycles Nos. 18 and 20, their trend is completely opposed to that previously observed in the minima of the odd eleven-year cycles. In the even cycle No. 18, the space distribution curves of the many-year ice-coverage wave undergo somewhat different changes with time. During the cycle, the space distribution curves vary consistently (see Figure 14).

Generalizing the preliminary results of our analysis, we conclude that the many-year space curves of ice coverage vary with a period which

Fig. 14. Year-to-year variation of the space curves (for July, August, Sept.) of the many-year fluctuations of ice coverage in the Arctic (with two-year and seven-year cycles eliminated).
is equal to double the eleven-year solar cycle. The statistical significance of this variation was checked by testing for autocorrelation. The monthly average anomalies of the many-year ice-coverage wave for all the arctic regions in one year were correlated with the corresponding anomalies of the following years. The data were tested for direct and reverse autocorrelation. The results of the autocorrelation tests shown in Tables 8, 9, and 10 and Figure 15 indicate that the autocorrelation curves are identical for solar cycles No. 17 and No. 19. Gradually dropping from high positive values (0.80-0.90) at the beginning of each cycle, the autocorrelation coefficients become negative and reach again comparatively large absolute values at the minima of the even cycles No. 18 and No. 20. The sign of the autocorrelation coefficients in odd cycles is reversed precisely at the maxima, i.e., in 1937-1938 and 1957-1958.

A different variation of the autocorrelation coefficients is observed during the even cycle No. 18. We see from Figure 15 that during the entire cycle the autocorrelation coefficients retain high positive values. This points to a fairly consistent variation of the curves.

TABLE 8

AUTOCORRELATION COEFFICIENT FOR THE MANY-YEAR SPACE WAVE OF THE (REDUCED) ICE COVERAGE IN THE ARCTIC. CYCLE No. 17 (1934-1944)

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TABLE 9

AUTOCORRELATION COEFFICIENTS FOR THE MANY-YEAR SPACE WAVE OF THE (REDUCED) ICE COVERAGE IN THE ARCTIC.
CYCLE No. 18 (1944-1954)

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TABLE 10

AUTOCORRELATION COEFFICIENTS FOR THE MANY-YEAR SPACE WAVE OF THE (REDUCED) ICE COVERAGE IN THE ARCTIC.
CYCLE No. 19 (1954-1964)

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</tbody>
</table>

We are thus led to the conclusion that the many-year ice-coverage wave has a 22-year cycle. However, the limited observation series, despite the encouraging initial results, are insufficient for making this conclusion final.
Fig. 15. Coefficients of direct and reverse autocorrelation of the many-year space wave of ice coverage in arctic regions during odd and even solar cycles.
Of considerable interest is the analysis of many-year ice-coverage fluctuations of the arctic seas in relation to the climate. These studies, besides having a theoretical value, also have a strictly practical importance, say, for long-range forecasting of the ice coverage of extensive arctic regions. Such data, however rough and tentative, are needed for long-range economic planning in the Arctic.

It seems to have been firmly established by now that the gradual warming of the climate in the high latitudes of the Northern Hemisphere which began in the mid-1920's reached a peak in the mid-1940's; the temperatures are now declining slowly but steadily.

There is ample evidence that the climatic changes were accompanied by definite changes in the hydrosphere. In particular, between 1925 and 1945 the total area of arctic ice shrank markedly, the water temperature increased in some regions of the near-Atlantic Arctic, etc. The warm nucleus moved from Greenland eastward, followed eventually by the cold nucleus [10].

Elimination of all the component cycles from the actual ice-coverage data for the different regions gives residual ice-coverage anomalies which are plotted as isopleths in Figure 16. These are essentially background, or secular, components of ice-cover fluctuations, which are apparently caused by the same factors as the secular climatic changes, since the ice cover may be treated as a product of the climate.

If the zero isopleth of background anomalies is assumed to identify the beginning of the period of steady decrease in ice coverage (a better policy would be to take the isopleth of maximum positive anomalies, but the limited observation series makes it impossible to pinpoint the exact time of sign change), a regular motion of the background wave from west to east is clearly observed. The displacement of the nucleus of minimum ice-coverage values is particularly noticeable.

In the west, the maximum of negative anomalies was observed in the mid-1940's; in the central regions, it was attained in the 1950's;
and in the extreme east, it was observed in the late 1960's. The initial, the maximal, and the final phases of the background low thus moved eastward with the same velocity. This indicates a definite regular behavior of this phenomenon and enables us to extrapolate the time of future background high for tentative ice forecasting.

Fig. 16. Isopleths of the anomalies of background fluctuations of ice coverage in August (in thous. km²) by regions.

The wave of low ice coverage passed over the Soviet arctic seas in twenty-five years. Seeing that a substantial increase of the background ice coverage (transition to positive anomalies) in the west occurred in the early 1950's, and that in the late 1960's the anomalies had already reached values which were approximately double the absolute values before the warming period (the early 1920's), we can expect another sign reversal of the anomalies in this region in the next decade.

In the extreme east, where the negative ice-coverage anomalies have apparently reached their maximum, a decrease of anomalies is to be expected in the next few years, to be followed in the mid-1970's by a transition to positive anomalies which will reach a maximum in the
early 1980's. This prediction applies to background ice coverage only. The actual year-to-year variations will not necessarily correspond to this scheme.

In view of the considerable contribution of the low-order cyclic components to the actual ice coverage, the particular combination of the cycles in different years may produce substantial (positive or negative) anomalies, regardless of the general background. Such effects were indeed observed in the previous epoch. Nevertheless, in the forthcoming epoch of a general background high, which is expected in the next decade to extend over the eastern seas as well, the frequency of years with high ice coverage will apparently also increase in this region. Positive ice-coverage anomalies, with allowance for the expected increase in the background, will apparently exceed the maximum figures for the last forty years. In some regions this will result in a shorter navigable period and poorer sailing conditions than existed during the low ice years of the test period.

BIBLIOGRAPHY


TESTING A NUMERICAL MODEL OF SPRING-SUMMER REDISTRIBUTION OF SEA ICE

Yu. P. Doronin, N. A. Zhukovskaya, and A. V. Smetannikova

Considerable emphasis has been placed on developing ways to compute ice thickness and ice drift, which are determined separately. However, ice compaction has not been taken into consideration in developing theoretical methods of ice drift computation. It is only in empirical formulas that the so-called "wind coefficients" were calculated in a number of cases with allowance for ice compaction.

A mathematical model combining these elements into one whole was described in [2]. The model deals with steady-state drift produced by tangential wind stresses (wind drag). The interaction between ice floes was introduced in the form of viscous drag $R$. The general balance equation of the forces acting on ice may be written as

$$F_1 + F_2 + K + R = 0,$$

where $F_1$ is the air drag, $F_2$ is the water drag, $K$ is the Coriolis force, and $R$ is the drag associated with the interaction of floes.

The tangential shearing stresses acting on the top and the bottom surface of ice were determined from the geostrophic wind. A logarithmic wind speed or current velocity profile was assumed for the quasi-stationary sublayer $h$; Ekman spiral distribution was assumed outside this sublayer. The forces $F$ are thus expressed by the relation

$$F_j = \rho_j \frac{k_j \sqrt{2a_j \xi}}{1 + D_j \sqrt{2a_j \xi}} (G - w),$$

where

$w = u + iv$;

$G = u_g + iv_g$;
\[ D = \frac{h}{\ln \frac{k}{\chi}}; \]

\[ a\sqrt{2\mu}; \]

\[ i = \sqrt{-1}; \]

\[ u, v \quad \text{— the drift velocity components along the axes } ox \text{ and } oy; \]

\[ u_g, v_g \quad \text{— the geostrophic wind components along these axes;} \]

\[ \kappa, \chi \quad \text{— the eddy viscosity and the molecular viscosity, respectively;} \]

\[ l \quad \text{— the Coriolis parameter}; \]

\[ \rho \quad \text{— the density}; \]

\[ j=1 \text{ for air parameters and } j=2 \text{ for water parameters.} \]

The Coriolis force in this notation may be written in the form

\[ K = -2ip\mu h, \quad (3) \]

where \( p \) is the density and \( h \) is the thickness of ice.

The interaction of ice floes in the model was expressed in terms of changes in momentum with an effective drag coefficient \( k_{I} \), which is a linear function of the compaction \( N \),

\[ k_{I} = \alpha N; \quad (4) \]

\[ R = \alpha \left[ \frac{\partial}{\partial x} \left( N \frac{\partial W}{\partial x} \right) + \frac{\partial}{\partial y} \left( N \frac{\partial W}{\partial y} \right) \right]. \quad (5) \]

Simulation tests gave \( \alpha = 3 \cdot 10^3 \text{ ton/sec}; \) for this coefficient, the transition from the zone of close ice to the zone of open ice more or less corresponds to the true picture. This choice, however, leads to opening patterns in compact ice which do not correspond to the true picture. The actual dependence of \( k_{I} \) on \( N \) is possibly nonlinear. Unfortunately, no experimental material is available for establishing a firm relation between floe drag and ice compaction. It is hoped that numerical experiments in the future will enable us to select a more fitting expression for the effective drag coefficient.

In principle, if \( N \) were known, equations (1) through (5) would make it possible to determine the ice drift components. However, ice compaction is inevitably variable under drift conditions (partly because of nonuniform
wind and current fields), so that another equation is needed to express the changes in ice compaction. In particular, if the nonuniform drift velocity is treated as the only factor responsible for the variation of $N$, we may use the simple relation

$$\delta N_{\text{dyn}} = -\text{div}(N V) \tau,$$  \hspace{1cm} (6)

where $V$ is the drift velocity vector and $\tau$ is the time interval.

In summer, ice melting markedly affects the thickness and the compaction of ice. An expression was derived in [3] which determines the change in ice compaction as a result of progressive melting from the top, the bottom, and the side surfaces. In our model at this stage we used only the approximate estimate of the variation in $N$ due to thermal factors according to Zubov's relation [4]

$$\delta N_{\text{therm}} = (N - 1) \left[ \exp \left( \frac{N \Phi \tau}{L \rho H} \right) - 1 \right],$$  \hspace{1cm} (7)

where $L$ is the heat of fusion of ice and $\Phi$ is the flux of radiant and turbulent heat absorbed by water.

We improved somewhat on relation (7) by modifying Zubov's original assumption that the entire heat $\Phi$ is used in lateral melting of ice: in our treatment, only a fraction proportional to the ice compaction was taken as the source of lateral melting. The remainder was used to heat the water and was consequently ignored in our calculations. The heat flux $\Phi$ was determined from total radiation data, the known albedo of water (taken equal to 0.1), and given air temperature and wind speed.

The change in ice thickness due to melting from the top surface was derived using the relation from [1] and the same initial data used in computing $\Phi$. The albedo was assumed to vary with time but to remain constant over the entire sea surface. This is a crude assumption which is justified as the first approximation only, when we are trying to assess the effect of thermal factors alone on ice compaction and ice drift. In further studies, the ice albedo should be taken into consideration depending on the state of ice and its distribution over the surface of the sea.
In our model, the ice thickness was determined, with allowance for its movement, from the relation

\[ H(t + \tau) = H(t) - \delta H, \]

where \( \delta H \) is the ice melting in a time \( \tau \), \( H(t) \) is the temperature of ice which reached a given point at the end of the time \( \tau \);

\[ H(t) = H_{nr} \left[ 1 - \frac{|u| \tau}{s} - \frac{|v| \tau}{s} \right] + \frac{|u| \tau}{s} H_{n \pm 1, r} + \frac{|v| \tau}{s} H_{n, r \pm 1}. \]

Here, \( s \) is the grid spacing; \( n \) and \( r \) are the grid point indices in the direction of the two axes \( ox \) and \( oy \).

Thus, to compute the ice thickness from the drift velocity, we had to identify the point from which the floe started before arriving at the grid point \( nr \) and then apply linear interpolation to compute \( H(t) \) at this point. Since the time interval \( \tau \) was relatively small (one day), the distances traversed by the ice floes did not exceed a few kilometers. We could thus ignore the changes in the meteorological elements along the ice trajectory and compute the heat fluxes from the data for the grid points \( nr \).

The drift velocity and the ice compaction were computed by iteration using the implicit scheme. The computed drift velocity was regarded as acceptable only if the computed ice compaction did not exceed 10. If the computations gave \( N > 1 \), the drift velocity was corrected in proportion to the components of the difference \((1 - N)\) along the coordinate axes. This analysis was performed for each iteration for all the grid points with ice.

The fast ice was allowed for by the logic of the program: it was assumed to remain stationary until its thickness had been reduced to a certain preset critical value in the process of melting, after which it was regarded as drift ice.

The program for computing the ice compaction and drift at the points of a regular grid in one of the regions of the Arctic basin was first written for the URAL-2 computer and then modified for the M-220. The computations were carried out using the results of a 22-year observation series.
The regular grid had a 75 km spacing along the x and y axes; it contained 23 rows, with 15 points in each row—i.e., a total of 345 points. The grid points corresponding to dry land were appropriately distinguished in the computer memory, which omitted them from the actual computations; in the printout they were crosshatched by overprinting with the letter W. For each sea point, the following hydrological data were specified: ice thickness (in cm), ice compaction (in tenths of a whole), and the initial drift velocity (taken equal to zero).

The meteorological data (atmospheric pressure, air temperature, and total radiation) were specified in the form of ten-day averages for 14 meteorological stations. The program was divided into three parts.

1. **Preparatory routine.** This routine included preparation of the input, its translation into machine language, and allocation of storage for the fields \( u, v, \delta N, \delta H \). All these fields, as well as the \( H \) and \( N \) fields, were stored on a magnetic drum. Pressure, temperature, and total radiation data were interpolated by the least squares method to the regular grid points. The approximating polynomial for the pressure \( P \) had the form

\[
P_i = a_0 + a_0 x_i + a_2 x_i^2 + a_0 y_i + a_0 y_i^2 + a_1 x y + a_2 x^2 y^2 + a_2 x y^2 + a_2 x^2 y^2.
\]

(10)

The expansion coefficients obtained by this method were assumed to remain constant during each ten-day period and were stored in the computer until the transition to the next ten-day period.

2. **Dynamic routine.** The hydrological field data for each sea point were accessed on the drum and computations were made of the drift velocity and the resulting change in compaction. The drift velocity was improved by repeated computations if \( (N_{xt} + \delta N_{xt}) > 1 \). This routine also determined the maximum residues in the drift velocity components between two successive iterations. The permissible residue was taken as 0.5 km/day. Computations in some years were carried out with residues of 0.05 km/day. The reduction of the permissible residue increased the number of iterations, but the final accuracy remained the same.
When convergence of the iteration process had been established, the fields \( N \) and \( \delta N \) were added and the result was stored in the positions allocated to the field \( N \); the field \( \delta N \) was then cleared.

3. **Thermal routine.** This part of the program was designed to determine the change in ice thickness and its compaction due to thermal factors. In the interpolation formula (9), when considering advection into the grid point \( nr \), the values of \( H_{n+1,r} \) and \( H_{nr+1} \) were selected using the following quadrant notation based on different sign combinations of \( u \) and \( v \) (see below).

<table>
<thead>
<tr>
<th>( u )</th>
<th>( v )</th>
<th>Quadrant</th>
<th>Grid point selection</th>
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<td>+</td>
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<td>I</td>
<td>( nr, nr+1, n+1, r )</td>
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<tr>
<td>+</td>
<td>-</td>
<td>IV</td>
<td>( nr, nr+1, n+1, r )</td>
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<td>-</td>
<td>-</td>
<td>III</td>
<td>( nr, nr-1, n-1, r )</td>
</tr>
<tr>
<td>-</td>
<td>+</td>
<td>II</td>
<td>( nr, nr+1, n+1, r )</td>
</tr>
</tbody>
</table>

For \( H_{n+1,r} = 0 \), \( u_{nr} \) was taken equal to zero; for \( H_{nr+1}, v_{nr} \) was also taken equal to zero.

If the calculation of ice melting gave \( \delta H \leq 0 \) (that is, no melting), the program assumed \( \delta H = 0 \). Then \( H(t+\tau) \) was analyzed. If it turned out to be less than or equal to zero, \( H(t+\tau) \) and \( N_{nr} \) were also taken equal to zero. Otherwise, \( \delta N_{\text{therm}} \) was computed and added to the previously obtained \( N_{nr} \).

The new values of \( H \) and \( N \) obtained by this method were stored in the \( H \) and \( N \) fields and used as initial data for the next time interval \( \tau \). The results for each ten-day period were printed out. No attempt was made to determine the drift velocity of a particular ice floe, so that the drift velocity components were recorded only for the printout times, i.e., for each tenth day. For ice thickness and compaction, resultant figures were given. In Figures 1a and 1b these elements are given for each point in the following sequence: \( u, v, H, N \). If there was no ice near the grid point, a plain dot was printed to identify the corresponding grid position.

In the absence of regular ice thickness surveys in the open sea, the initial estimate of this element at each grid point was highly approximate. It was based on all the ice thickness data for each year: the measured ice thickness data of meteorological stations, the results of
computations from degree-days of frost at points selected in the open sea, and occasional ice thickness surveys in different parts of the sea, sometimes obtained in conjunction with radio buoy installation. These data were approximately corrected for ice compaction and age, which were taken from visual charts. The ice albedo used in computations varied from month to month (from 75% to 60%). The albedo of the water surface was assumed to remain constant over the entire period (10%).

Fig. 1. The position of the computed (dash-dot) and the actual (solid curve) ice edge at the beginning of the computation period (a) and at the end of the computation period (b).

The results of these computations were presented in the form of charts on which the grid points carried the numerical data for ice thickness and compaction at the end of each ten-day period in the spring-summer season. The position of the ice edge was plotted using these numbers. For purposes of comparison, the actual position of the true ice edge was also marked on these charts. A total of 240 charts were analyzed. The
Ice coverage was determined separately for the northeast and the southwest parts of the test region, and the likelihood of a fit between the computed and the actual ice coverage to within ±20% amplitude was calculated.

The computed compaction values at some grid points were compared with the actual figures. Comparisons of the computed and the actual ice edge position indicate that in most cases the difference does not exceed one or two grid spacings (75-150 km). The best fit for almost the entire series is observed between the ice edge position in June and its position in the first half of July, and in the period between the second half of August and the end of September. Figure 1 shows the computed and the actual ice edge position for two different cases in one year.

In most cases (with the exception of three years), the computations for the southwest part between the second half of July and the second half of August indicate a somewhat faster clearing of ice than was actually observed, so that the computed ice edge was located farther to the north than the true ice edge. Between the end of August and the end of September, the computed ice edge recedes northward at a slower rate than the actual ice edge does. The computation scheme thus yields a constant error which apparently can be suppressed by further improvement of the computation coefficients.

The accuracy of these computations is clearly affected by the accuracy of the initial data. The ice thickness was specified quite crudely, since no systematic measurements of this element for the open sea are available. The inaccuracy in ice thickness measurements (which may reach 20 cm and higher) is particularly significant for the computation of the rate of ice clearing from the sea areas covered by relatively thin ice, which is mainly concentrated in the southwest part of the test area and melts at the fastest rate in July and the beginning of August.

The ice thickness in spring in the open sea can be obtained with higher accuracy by two methods: by organizing systematic measurements of ice thickness in the spring over the entire sea area; or by developing a computation model for predicting the maximum growth of ice thickness in winter, with allowance for ice dynamics in the sea.
Our numerical model ignored the effect of constant currents which apparently make a significant contribution in the southwest region. This factor will have to be introduced with further improvement of the model. Errors in the initial data also stem from the inaccuracy of the albedo, which was assumed to be constant over the entire sea surface. It is known that the albedo changes markedly in time and in space during the ice melting period. In the southern part, the melting process begins earlier and has a larger annual amplitude than in the north, so that the surface albedo will be different depending on the distribution of ice according to age, hummocking, snow coverage, and other elements.

The proposed model ignores all these fine features, but a provision is made for introducing the space and time variation of the albedo.

Comparison of the computed ice coverage figures with the actual data indicates that the highest likelihood is obtained for the northeast part (Table 1); the likelihood is somewhat lower for the southwest part.

**TABLE 1.**

<table>
<thead>
<tr>
<th>Region</th>
<th>June</th>
<th>July</th>
<th>August</th>
<th>September</th>
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<tr>
<td></td>
<td>III</td>
<td>I</td>
<td>II</td>
<td>III</td>
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<tr>
<td>Southwest</td>
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</tr>
<tr>
<td>Northeast</td>
<td>95</td>
<td>90</td>
<td>90</td>
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Comparison of the computed and the actual ice compaction values at the grid points reveals a difference not exceeding 1/10 to 2/10 in most cases. Our tests based on a large volume of observation material thus fully justify the application of the model for computations of ice distribution over the particular area. The model yields acceptable data on the distribution of ice thickness and compaction at the grid points.

The development of a mathematical model simulating a particular natural phenomenon opens new possibilities of further detailed study of the various component elements.
Our numerical model permits analyzing the formation of the ice cover in the spring-summer period as a result of the combined effect of thermal and dynamic factors. In particular, it permits assessing the separate contribution from the dynamic and the thermal factors to ice formation over a long period. We computed the ice edge position with and without the thermal factors. In the latter case, the ice area was found to be substantially larger than the actual area. Introduction of the thermal factors is particularly important for determining the state of ice in the southern regions.

The numerical model makes it possible to estimate the effect of atmospheric processes on changes in the state of ice. A number of alternative computations carried out to assess the effect of various atmospheric factors on ice distribution reveal the great potential of this simulation approach, especially for purposes of forecasting: it now becomes possible to assess separately the effect of errors in different parameters on the ice forecast. The relevant parameters for the spring-summer period include the distribution of ice albedo over the sea surface, the time variation of the ice albedo, the initial state of ice, air temperatures, distribution of atmospheric pressure, and the state of the fast ice.

In this paper we considered individual examples which illustrated the effect of the above parameters on the thickness and the compaction of ice. For three years, the computations were carried out using a 5 per cent higher albedo for June–August. As a result, the ice melting by the end of July increased to 25–30 cm, which affected the ice compaction by two- to three-tenths.

As we have noted before, the presence of fast ice was allowed for in our computations; as long as its thickness exceeded some critical value, the fast ice did not respond to wind stresses, and for subcritical thicknesses it was treated as drift ice driven by wind. The critical thickness for this transition point was assumed to be 100 cm.

To elucidate the effect of fast ice on the distribution of ice compaction, we repeated our computations, treating the fast ice as wind-driven drift ice over the entire test period. Comparison of the charts
obtained by this method reveals excessively fast opening of ice and clearing of the sea.

The effect of year-to-year fluctuations in air temperature on ice formation in the spring-summer period is of considerable importance for the development of numerical forecasts. In the numerical model, it is an index of turbulent heat and moisture transfer processes.

Seeing that the temperature difference between the ice surface and the air in the spring-summer period is small (close to zero), the contribution of these processes to ice melting is negligible compared to the effect of the radiant energy. It should be expected that the results of computations using many-year averages for the air temperature will be close to the actual data. These computations were indeed carried out with the ten-year series. In all cases, perfect fit of ice edge position and ice compaction was observed, i.e., the year-to-year fluctuations of air temperature over the sea did not affect the results of our computations. The differences were noticeable only for the ice thickness: they reached 20-30 cm in the southern part (for June-August) and 5-10 cm in the northern part.

The effect of other factors on ice formation in spring-summer, such as the initial ice thickness and the total radiation, can also be estimated, but detailed studies of the combined influence of the various factors deserve a separate study, which cannot be undertaken until after the numerical model has been appropriately modified.

The aim of this paper was to derive the basic numerical schemes for the computation of ice distribution in a particular region of the Arctic and to test their validity using many-year observation results.

BIBLIOGRAPHY

MANY-YEAR VARIATIONS OF THE ICE COVERAGE OF THE GREENLAND SEA
AND METHODS OF FORECASTING IT

A. A. Kirillov and M. S. Khromtsova

Most of the ice drifting south from the Arctic basin into the Greenland Sea through the Fram Strait along the east coast of Greenland is known as the East Greenland Pack. A substantially smaller fraction of drift ice reaching the Greenland Sea from the Arctic basin is deflected to the Spitsbergen area, where the ice is rapidly melted by the warm branch of the Atlantic current washing the west coast of the archipelago. The melting is particularly rapid in summer. In winter, an ice-free bay is generally observed, whereas in particularly favorable summers, ice-free water extends eastward along the northwestern coast of Spitsbergen, sometimes as far as 20°E.

Most of the researchers who study the ice conditions in the Greenland Sea have measured the ice area of the East Greenland Pack only; i.e., they ignore the Greenland Sea ice near West Spitsbergen. This situation is apparently an outgrowth of earlier studies, which did not include the entire area of the Greenland Sea. No air surveys were possible at that time, and the ice data supplied by expedition vessels, commercial ships, and coastal stations were insufficient. Ships seldom ventured into the northern parts of the Greenland Sea, and on-board observations mainly covered the central and southern parts, near the east coast of Greenland.

The present analysis is based on the following sources and data:

1. The Danish ice yearbooks, Isforholdene, for 1924-1939 and 1946-1962.
The various data were processed to yield ice distribution charts for the Greenland Sea in 1924-1939 and 1946-1968, inclusive. (No data were available for 1940-1945.) The ice coverage of the Greenland Sea for 1924-1968 was determined by planimetry in per cent of the total sea area within its official boundaries (see Figure 3). The results are listed in Table 1.

### Table 1.

**Ice Coverage of the Greenland Sea (in Per Cent).**

<table>
<thead>
<tr>
<th>Year</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
<th>VIII</th>
<th>IX</th>
<th>X</th>
<th>Average for IV-VIII</th>
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<tr>
<td>1924</td>
<td>63</td>
<td>53</td>
<td>(44)</td>
<td>35</td>
<td>38</td>
<td>47</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>1925</td>
<td>(52)</td>
<td>(48)</td>
<td>(43)</td>
<td>(35)</td>
<td>(34)</td>
<td>(34)</td>
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<tr>
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<tr>
<td>1927</td>
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<td>53</td>
<td>(52)</td>
<td>(38)</td>
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<tr>
<td>1934</td>
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<td>(62)</td>
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<td>1936</td>
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<td>43</td>
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</tbody>
</table>

**Average for 1924-1939**

- Average: 61
- Maximum: 77
- Minimum: 43

**1946**

- Average: 61
- Maximum: 75
- Minimum: 48

**1947**

- Average: 75
- Maximum: 65
- Minimum: 48

**1948**

- Average: 73
- Maximum: 63
- Minimum: 49

**1949**

- Average: 68
- Maximum: 63
- Minimum: 49

**1950**

- Average: 69
- Maximum: 63
- Minimum: 49

**1951**

- Average: (70)
- Maximum: 65
- Minimum: 49

**1952**

- Average: 63
- Maximum: (62)
- Minimum: 43

**1953**

- Average: 62
- Maximum: 62
- Minimum: 43

**1954**

- Average: 70
- Maximum: (62)
- Minimum: 44

**1955**

- Average: 67
- Maximum: (62)
- Minimum: 43

**1956**

- Average: 60
- Maximum: 57
- Minimum: 40

**1957**

- Average: 43
- Maximum: 40
- Minimum: 39

**1958**

- Average: 43
- Maximum: 40
- Minimum: 39

**1959**

- Average: 61
- Maximum: 66
- Minimum: 48

**1960**

- Average: 43
- Maximum: 44
- Minimum: 33

**1961**

- Average: 65
- Maximum: 70
- Minimum: 45

**1962**

- Average: 52
- Maximum: 61
- Minimum: 43

**1963**

- Average: 83
- Maximum: (70)
- Minimum: 43

**1964**

- Average: 55
- Maximum: (58)
- Minimum: 48

**1965**

- Average: 92
- Maximum: 87
- Minimum: 48

**1966**

- Average: 89
- Maximum: 92
- Minimum: 49

**1967**

- Average: 74
- Maximum: 70
- Minimum: 49

**1968**

- Average: 84
- Maximum: (82)
- Minimum: 47

**1946-1968**

<table>
<thead>
<tr>
<th>Year</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
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<tr>
<td>1951</td>
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<td>19</td>
<td>37</td>
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<td></td>
</tr>
</tbody>
</table>

**1946-1968 rr.**

- Average: 63
- Maximum: 79
- Minimum: 43

**1924-1968 rr.**

- Average: 63
- Maximum: 79
- Minimum: 43
The new ice data (see Table 1 and Figures 1, 2, 3) make it possible to consider the year-to-year and the seasonal changes in ice coverage of the Greenland Sea and to identify its dependence on the basic factors. The conclusions obtained in this study could be used to verify earlier findings and to improve the existing methods of ice forecasting for the Greenland Sea.

We see from Figure 2 and Table 1 that the maximum ice coverage $F$ of the Greenland Sea is observed in the winter months (March and April). In most cases, the ice cover starts to decrease by April and reaches its minimum generally in August, less often in September, and only rarely in July. In certain years (1930, 1947-1950, 1963, 1965), the decrease of ice coverage from April through August is a continuous and regular process. However,
it changes abruptly in other years. In 1927, 1928, 1932, 1961, and 1968, the ice cover decreases markedly at first; then the rate of decrease slows down before it drops steeply to minimum. In 1925, 1937, 1952, and 1964, this abrupt decrease is observed in a different manner: the ice cover at first decreases very little or remains constant; then it drops sharply from May-June through August or July, slowing somewhat from July to August. In some years (1933-1935, 1956, 1959), a second spring maximum is observed, usually in May but occasionally in June. In 1962, the ice area increased substantially—by 12 per cent—between May and June. In the entire 39-year series of observations, the largest drop in ice coverage is observed usually between June and July, regardless of the average level of ice coverage during the year.

We see from Table 1 that the average ice coverage of the Greenland Sea from April through August in 1924-1939 (50 per cent) is somewhat less than in 1946-1968 (52 per cent). This latter period was colder than the earlier period, an observation which is also supported by the maximum and minimum ice coverage figures for the Greenland Sea. Thus, between 1924 and 1939 the average ice coverage for April-August reached the norm once (which constitutes 6 per cent of the 16-year series), exceeded the norm in seven cases (44 per cent), and was less than the norm in eight cases (50 per cent). In the years 1946-1968, it reached the norm twice (8 per cent of the 23-year series), exceeded the norm in thirteen cases (57 per cent), and was less than the norm in eight cases (35 per cent).

Karelin [3] showed that the total ice area near the east coast of Greenland in 1921-1939 was substantially less than in 1898-1920. This decrease during the warm period is attributed by the author to a decrease in the formation of local ice.

The many-year ice distribution charts of the Greenland Sea enabled us to determine the probability of an encounter with ice (the position of the ice edge) for winter (April) and summer (August) months (Figures 3a and 3b).

The 0 per cent isopleth in Figures 3a and 3b indicates that no ice was observed southeast of this isopleth in that particular month. The
50 per cent isopleth corresponds to the many-year average position of the drift ice edge. The 100 per cent isopleth indicates that ice was always present in that particular month north and west of the isopleth. We see from the figures that the position of the ice edge in the Greenland Sea underwent considerable changes from April to August and in each month separately.

![Map showing ice distribution](image)

**Fig. 3. Probability of ice encounter in April (a) and August (b).**

1—100%; 2—50%; 3—0%;
4—Greenland Sea boundary

Let us examine the new observation series (see Tables 1 and 2) in relation to ice inertia of the Greenland Sea from year to year and from month to month.

To bring out the year-to-year or the seasonal ice inertia, the ice coverage figures for the current year or month were compared with the corresponding data for the preceding period. Analysis of the correlation coefficients shows that the month-to-month ice inertia (during one year) is fairly pronounced in the Greenland Sea. The winter coefficients are
higher than the coefficients in summer:

- March to April: 0.92
- April to May: 0.70
- May to June: 0.82
- June to July: 0.52
- July to August: 0.64

April to June: 0.52
April to July: 0.60
April to August: 0.34
May to August: 0.29

The year-to-year ice inertia in the Greenland Sea is very weak. The autocorrelation coefficients for the April-August mean ice coverage and for the ice coverage figures for the individual months do not exceed 0.15-0.20.

TABLE 2.

CHANGES OF THE ICE COVERAGE OF THE GREENLAND SEA FROM APRIL TO AUGUST (IN PER CENT).

<table>
<thead>
<tr>
<th>Year</th>
<th>April-May</th>
<th>May-June</th>
<th>June-July</th>
<th>July-August</th>
</tr>
</thead>
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<tr>
<td>1924</td>
<td>-10</td>
<td>-11</td>
<td>-9</td>
<td>-3</td>
</tr>
<tr>
<td>1925</td>
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</tr>
<tr>
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<td>-8</td>
<td>-1</td>
<td>+5</td>
<td>-7</td>
</tr>
<tr>
<td>1927</td>
<td>-10</td>
<td>-2</td>
<td>-1</td>
<td>-14</td>
</tr>
<tr>
<td>1928</td>
<td>-8</td>
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<td>-2</td>
</tr>
<tr>
<td>1930</td>
<td>-8</td>
<td>-3</td>
<td>-8</td>
<td>-6</td>
</tr>
<tr>
<td>1931</td>
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<td>-1</td>
<td>-10</td>
<td>+4</td>
</tr>
<tr>
<td>1932</td>
<td>-9</td>
<td>-2</td>
<td>-14</td>
<td>-13</td>
</tr>
<tr>
<td>1933</td>
<td>-1</td>
<td>-8</td>
<td>-3</td>
<td>-10</td>
</tr>
<tr>
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<td>+8</td>
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<tr>
<td>1942</td>
<td>-6</td>
<td>-6</td>
<td>-5</td>
<td>-10</td>
</tr>
</tbody>
</table>

The dependence of the ice coverage of the Greenland Sea on various factors was considered by numerous authors [1,2,3,4,5,7,9]. In earlier studies it was shown that the ice conditions in the Greenland Sea are influenced by changes in the atmosphere and the hydrosphere, which in their turn influence the thermal and the dynamic state of water and ice. Thus
Vize [1,2] showed that the state of ice in the North Atlantic and the adjoining seas, including the Greenland Sea, is influenced by the past state of the atmosphere and itself influences the current and future state of the atmosphere. He established a relationship between the ice conditions in the Greenland Sea and the atmospheric circulation over the North Atlantic and the adjoining arctic seas (Barents, Kara).

Lebedev [5,6] studied the ice conditions of the Greenland Sea and concluded that the ice coverage can be determined from (1) a thermal factor, i.e., the air temperature in Barentsburg (in October-February) and (2) a dynamic factor, i.e., the pressure difference between Barentsburg and Jan Mayen for the same period.

Some authors have tried, without success, to correlate the ice coverage of the Greenland Sea or the area of the East Greenland Pack with various solar activity indices [6,8]. To check the earlier conclusions and relations by using the new data, we established the dependence of the ice coverage on the following factors: air temperature in Barentsburg; quantity of warm Atlantic water reaching the Arctic basin from the Greenland Sea through the Fram Strait; ice transfer from the Arctic basin to the Greenland Sea; frequency of E-type circulation; solar activity; and ice coverage of the Greenland Sea for the preceding year (average during the navigable period or for the individual months).

We see from Table 3 that the correlation between the ice coverage $F$ and the solar activity $W$ or the atmospheric circulation $E$ is quite negligible, and is virtually nil in some months. Thus the ice coverage in June shows a direct, though weak, correlation with the solar activity; but there is no correlation in July, and in August a small correlation of opposite sign is observed. These correlations are thus of no forecasting value.

Relatively low correlation coefficients were also obtained between the ice coverage, on the one hand, and the transfer of Atlantic waters $V$ and the transport of ice from the Arctic basin $D$, on the other (see Table 3). The correlation between the ice coverage of the Greenland Sea and the air temperature in Barentsburg is characterized by relatively high correlation
### TABLE 3.
CORRELATION COEFFICIENTS BETWEEN THE ICE COVERAGE OF THE GREENLAND SEA AND VARIOUS HYDROMETEOROLOGICAL FACTORS.

<table>
<thead>
<tr>
<th>Months</th>
<th>$T_1$ X-II</th>
<th>$T_1$ X-IV</th>
<th>$D$ X-II</th>
<th>$D$ X-IV</th>
<th>$V$ X-II</th>
<th>$E$ X-II</th>
<th>$W$ X-II</th>
<th>$T_2$ VIII-IX previous year</th>
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</thead>
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<td>0.26</td>
<td></td>
<td>0.08</td>
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<td></td>
<td>-0.07</td>
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<td>-0.10</td>
<td>0.01</td>
<td>0.16</td>
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<tr>
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<td>0.36</td>
<td>0.38</td>
<td>-0.38</td>
<td>-0.05</td>
<td>0.04</td>
<td>-0.21</td>
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<tr>
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<td>-0.50</td>
<td>0.39</td>
<td>0.34</td>
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<td>-0.24</td>
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<td></td>
<td></td>
<td></td>
<td>-0.07</td>
</tr>
</tbody>
</table>

Remarks: $T_1$ and $T_2$ are the total monthly average air temperatures in Barentsburg for the corresponding periods; $D$ is ice transport from the Arctic basin into the Greenland Sea through the Fram Strait (calculated by Gudkovich and Nikolaeva); $V$ is the quantity of Atlantic water reaching the Arctic basin through the Fram Strait (average for the preceding year, according to Shpaikher); $E$ is the number of days with E-type circulation (according to Vangengeim); $W$ is the Wolf sunspot number, yearly average taken with a two-year phase shift relative to the ice coverage data.

coefficients, but only in summer months. The values of these correlation coefficients are close to the earlier results of Lebedev and Uralov.

It should be noted that when Lebedev compared the mean ice coverage of the Greenland Sea for April-August with the pressure difference between Barentsburg and Jan Mayen for October-February, he obtained higher correlation coefficients than those in Table 3 (for ice transport). Pressure difference between these two points thus provides a more reliable index of dynamic processes affecting the sea ice conditions than the computed ice transport from the Arctic basin.

Simultaneous analysis of the three arguments mentioned above yielded a number of regression equations which can be applied for ice forecasts for the Greenland Sea in summer months. Thus,
Equations (1)-(3) correspond to general correlation coefficients of 0.72, 0.78, and 0.71, respectively; the reliability of these relations is 82 to 88 per cent to within ±0.8σ or ±20 per cent amplitude.

Using equations (1)-(3), we can forecast the ice coverage of the Greenland Sea in summer two to six months in advance. This forecast is based entirely on actual observation data.

Our study leads to the following basic conclusions:

1. The ice coverage of the Greenland Sea is subject to substantial year-to-year and seasonal variations; the maximum ice coverage is mainly observed in March-April, the minimum in August. The highest rate of ice clearing is noted in July; between June and July the ice coverage on the average decreases by 10 per cent.

2. Seasonal ice inertia (stability of the sign of ice anomalies) is very pronounced in the Greenland Sea, whereas the year-to-year inertia is very weak.

3. An extensive observation series confirms the existence of close correlations between the ice coverage of the Greenland Sea and various indices of thermal and dynamic processes of the preceding autumn-winter period. The most effective index of thermal processes is provided by the air temperature in Barentsburg in October-February. Introduction of the temperatures for April and May hardly improves the correlation. The correlation between the monthly average ice coverage and air temperature in Barentsburg improves from May to August, the correlation coefficient increasing from -0.34 (May) to -0.62 (August).

The correlation between the ice coverage of the Greenland Sea and the quantity of Atlantic water reaching the northern part of the sea and farther north through the Fram Strait into the Arctic basin also improves from spring to summer. Thus the correlation coefficient in June is as
low as -0.10, whereas in July and August it increases to -0.38 and -0.39, respectively.

4. The new data based on an extensive series of observations yielded higher correlation coefficients (than before) between the ice coverage of the Greenland Sea and ice transport from the Arctic basin into the Greenland Sea.

5. Regression equations were obtained for forecasting the ice coverage of the Greenland Sea in summer months two to six months in advance with a reliability of more than 80 per cent.

BIBLIOGRAPHY


MANIFESTATION OF ATMOSPHERIC CYCLES IN ICE-COVER COEFFICIENT

V. F. Zakharov

The main feature of the year-to-year variations in the ice coverage of the Arctic seas is the presence of cyclic changes [1-4, 6-10, et al.]. The variation in ice coverage reveals cycles of 2-3, 3-4, 4-5, 6-7, 7-9, 9-10, 11, 18-19 years, and longer. Not all these cycles are significant, since ice observations in most polar seas are greatly restricted in time and are insufficient for drawing definite conclusions concerning the nature of these oscillations. It is therefore somewhat premature to attribute the observed year-to-year variations in the ice coverage mainly to the interaction of various cyclic processes in nature. The results of multiyear ice studies in the Arctic reveal the great importance of random factors; after all, repeated attempts to confine long-range ice forecasts to cyclic factors have never led to the expected results. Yet whenever the cyclic processes are adopted as one of the arguments in the forecasting equations, the results prove quite successful.[6]. Year-to-year changes in the ice coverage thus may be treated as the combined result of irregular and cyclic factors.

It has been firmly established that meteorological factors are directly responsible for the changes in ice coverage of the Arctic seas. These are air temperature and surface wind, which is generally expressed in terms of the pressure gradient. The air temperature determines the rate of ice melting, whereas the pressure gradient determines the trend and the rate of ice transfer between adjoining water areas. Both the temperature and the atmospheric pressure undergo cyclic variations in time, which are transmitted to hydrospheric phenomena, particularly to the ice cover. The observed cyclic patterns in the ice-cover variations thus reflect the atmospheric cyclic processes. The purpose of our
study was to determine to what extent these variations correspond to the atmospheric cycles.

Cyclic variation in a given element does not always reflect exactly the cyclic process which produced the variation. The effect of solar activity on the ice cover is generally transmitted indirectly, down a chain of interacting factors. This transmission naturally involves a certain distortion, which becomes more pronounced as the chain grows longer. Cycle transmission, however, is not the most important factor. More important is the mode of transmission from one medium to another, from one sphere of phenomena to another: from solar to atmospheric, from atmospheric to hydrospheric, ice phenomena included. The transmission mechanism in these cases introduces certain changes and modifications. These modifications are of the greatest interest, and we shall consider a number of examples of cyclic processes as they are reflected in the ice-cover coefficient.

The ice-cover coefficient is related to the atmospheric pressure through the pressure gradient, which is an index of the rate and the direction of air transport processes. According to Vize [2], the increase of atmospheric pressure in the Arctic involves a decrease in pressure gradients, attenuation of atmospheric circulation, and hence an increase in ice coverage. But it is entirely possible that the increase of pressure will increase the gradients and reduce the ice coverage. This might correspond to the marked increase in pressure in the eastern Arctic, with the pressure remaining constant or possibly increasing slightly in the western Arctic. Hence it is clear that the fluctuations of the pressure field, even if they are synchronized over large distances, may lead to different ice conditions in two apparently similar cases. This is the basic feature of the manifestation of pressure cycles in the ice-cover coefficient.

The fact that the ice-cover coefficient of a sea is determined by the pressure gradient, and not directly by the pressure, is of considerable significance. As an example, consider the Laptev Sea. Suppose
that the pressure at Arctic Cape and Kotel'nyi Island (between which the Laptev Sea exchanges ice with the Arctic basin) undergoes cyclic fluctuations with identical periods, initial phases, and amplitudes. Figure 1a shows the pressure curves for the two reference points; for simplicity, perfectly periodic curves are assumed. In this case, the gradient, or the pressure difference, remains constant despite the well-formed pressure variations. The ice transfer between the sea and the Arctic Ocean thus remains unchanged, and the ice coverage of the sea remains unaffected (assuming that it is entirely determined by ice transfer). Thus no cyclic variations in the ice coverage are observed unless the parameters of the cyclic pressure wave change in space. It is shown in [4] that the 5-6-year pressure cycle which is prominent on Arctic Cape and on Kotel'nyi Island involves insignificant changes in gradient, so that ice transfer and ice coverage should not display any cyclic features associated with this factor.

The effect of pressure cycles on ice coverage is thus determined not by the extent of each individual cycle but by the nonuniform distribution of the cycle amplitude in space. The higher the nonuniformity, the greater the pressure gradient and the more pronounced the cyclic pattern in the ice cover, and vice versa. Assuming that there are sources of cyclic processes in the atmosphere from which the disturbance propagates over a finite distance, dying off eventually, the effect of these disturbances on the ice coverage will be more pronounced as the intensity of the disturbance at source increases and the damping distance diminishes.

Let us consider the case in which a relatively inconspicuous cyclic variation in atmospheric pressure has a very pronounced effect on pressure gradients and ice-cover coefficient. The analysis will be based on actual observation data. Ice transfer between the Laptev Sea and the Arctic basin is subject to sharp year-to-year fluctuations. An important feature of these fluctuations, at least over the last thirty years, is the presence of 2-3-year cyclic variations. Harmonic analysis of the monthly ice transfer figures based on 1937-1968 data made it possible
to establish the exact value of the cycle period as 27 months. These variations may be identified with the so-called "over-two-year" cycle, which is identified in numerous meteorological elements, particularly pressure and temperature.

Figure 2a shows the distribution of the periods for the difference in atmospheric pressure between Arctic Cape and Kotel'nyi Island, which can be identified with the distribution of the ice transfer periods. We see from Figure 2a that the peak corresponds to a period of 27 months.
Figure 2b shows the period distribution curves for the pressure at the two points, which do not reveal any 27-month variations. The observed regular features in the variation of ice transfer between the Laptev Sea and the Arctic basin and of the ice coverage thus cannot be regarded as a reflection of regular variations of the pressure field.

However, regular variations in the pressure gradient do not arise on their own. They result from the actual variations of pressure at two points. The variations at each point do not necessarily have to follow a regular two-year cycle. It is sufficient that the two-year cycle be apparent for one of the two points only or that the pressure variation at each point contain two-year components (Fig. 1b). We see from Figure 1b that periods with well-formed oscillations alternate with periods without any oscillations, whereas the pressure difference curve between the two points reveals a regular two-year periodicity.

![Period distribution curves for differences in atmospheric pressure between Arctic Cape and Kotel'ny Island](image)

**Fig. 2.** Period distribution curves for differences in atmospheric pressure between Arctic Cape and Kotel'ny Island (a) and for the pressure at the two points (b).

1--Arctic Cape, 2--Kotel'ny Island.

Let us now turn to Figure 1c. Suppose that the difference in pressure curve at Kotel'ny Island and the Arctic Cape amounts to a certain difference in the pressure amplitudes. The oscillations
at Kotel'nyi Island fall off near the middle of the relevant time interval and then increase again. The situation at Arctic Cape is the reverse. What will the changes in the pressure gradient and hence in the ice transfer be under these conditions? Figure 1c shows the variation of the pressure difference \( \Delta p \) between the two points for this case. Analysis of the results leads to the following conclusions. If the pressure amplitude is greater at Kotel'nyi Island than at Arctic Cape, the pressure gradient varies in phase with the pressure. Conversely, if the amplitude is greater at Arctic Cape, a reverse picture is observed: the pressure gradient varies in counterphase with the pressure field. The transition from in-phase oscillations to counterphase oscillations corresponds to the time during which the pressure amplitudes at Arctic Cape and Kotel'nyi Island are equal.

This case is of obvious interest since it explains the change in sign of the correlation between the cyclic pressure variations and the ice-cover variations. This may be due to pronounced variation in the pressure amplitude, the pressure amplitude at Arctic Cape becoming larger than that at Kotel'nyi Island. Such changes are often observed in the field. This is consistent with the results of observations of cyclic fluctuations in various hydrometeorological elements, including pressure, which revealed a pronounced lack of constancy, in both time and space, of the basic parameters of the cyclic process. Under these conditions, the relationship between the amplitudes, periods, and phases of cyclic processes at two distant points will always change. The cases considered explain why well-formed cyclic variation of atmospheric pressure has at best an insignificant effect on ice coverage, how cyclic variations in ice coverage are produced by slight pressure changes, and why in certain cases the sign of the correlation between cyclic pressure and ice-cover variations is reversed.

While dealing with sign reversal of the correlation between the cyclic variations in atmospheric pressure and ice coverage, we inevitably touch on the more general topic of instability of the Sun-Earth relations.
Sleptsov-Shelevich [11] maintains that this instability is only apparent, being solely due to the time lag between the solar activity phenomena and their terrestrial manifestations. Figure 1c shows, however, that in a number of cases the instability is quite real. Strong changes in the amplitude, which suddenly become greater in one region than in another, may be the cause of the change in sign.

Our discussion of the manifestation of pressure cycles in ice transfer and ice coverage provides an indication of the changes which occur when the cyclic phenomena are transmitted from one medium to another. The resultant cycle may often be completely different from the originating cycle. A mere likeness of cycle periods between atmospheric and ice phenomena does not prove a causal relationship between them. The only exception is the ice cycles associated with air temperature. Unlike air pressure, temperature has a direct effect on the ice coverage. Cyclic changes in the ice coverage produced by air temperature fluctuations constitute a fairly accurate reflection of the atmospheric cyclic process. In those regions in which the year-to-year variations in ice coverage are mainly determined by the air temperature, the cyclic changes in the ice cover are a mirror reflection of the temperature cycle.

BIBLIOGRAPHY


APPLICATION OF DISCRIMINANT ANALYSIS TO LONG-RANGE ICE FORECASTING FOR THE ARCTIC SEAS

Yu. V. Nikolaev and E. G. Kovalov

The main factors determining the formation of the ice cover in Arctic seas have been elucidated by numerous studies, and effective methods have been developed for ice forecasting based on knowledge of these factors.

Long-range ice forecasting for Arctic seas relies on physical and statistical relations originally derived from field observations of the ice and on certain principles of hydrometeorological processes. In recent years various statistical methods have become popular in hydrometeorological forecasting, as they yield the best forecasts without going into a tedious and painstaking analysis of all the factors which determine a particular hydrometeorological phenomenon.

A statistical method based on discriminant analysis [2] can be applied to ice forecasting. The essential features of this method follow.

The forecast phenomena are divided into a number of classes or gradations. The initial information, available in the form of fields of a particular hydrometeorological element, is also divided into corresponding classes or gradations. Then the covariance matrices $R^*$ between the classes and $R$ over all the initial fields are computed. The principal vectors of the bundle of forms $R^*(u, u) - \lambda R(u, u)$ enable us to pass from a system of primary indices $P_{i\ell}^l$, $i = 1, 2, \ldots, n$, $\ell = 1, 2, \ldots, M$, where $n$ is the number of grid points and $M$ the number of terms in the initial set, to a system of secondary indices $P_{a\ell}^l$.
In the initial set of fields contains useful information for forecast preparation, the indices derived by transformation (1) will adequately forecast the required phenomenon; i.e., they will describe the forecast phenomenon with high probability. This follows from the extremum properties of a bundle of forms, according to which

\[ \frac{R^s(u, u)}{R(u, u)} \max \]

if the variables of the forms \( R^s(u, u) \) and \( R(u, u) \) are the principal vectors of the bundle of forms. The maximum of this ratio corresponds to maximum differences between the classes of the initial information. If the classes are distinguishable, the information content of the indices \( P^l_s \) should also differ, since the classes of the initial fields by definition correspond to the classes of the forecast phenomena.

The highest information content is apparently obtained for those \( P^l_s \) which correspond to the first maximum eigenvalues of the bundle of forms. If the number of classes of the forecast conditions is 2, we should select only one of the \( n \) possible indices, the one which corresponds to the first largest eigenvalue of the bundle of forms.

Let us consider the application of this method to forecasting the total ice coverage of Arctic seas. Studies of the reasons for changes in the total ice coverage establish the general features which govern the formation of ice conditions. The total ice coverage reflects the most general features in changes of the ice coverage of the Arctic seas. This is shown by decomposing the ice coverage according to the natural orthogonal components. The decomposition of the fields in natural orthogonal components may be treated as a linear transformation of the initial information, represented as a set of \( n \)-dimensional vectors.
\[ P_l(P_{1l}, P_{2l}, \ldots, P_{nl}), \ l = 1, 2, \ldots, M \]

It can be shown that the transition from the system of primary indices, \( P_{il}, \ i = 1, 2, \ldots, n, \ l = 1, 2, \ldots, M \), to the system of secondary indices

\[ P_{il}' = \sum_{i=1}^{n} u_{is} P_{il} \quad s = 1, 2, \ldots, n, \ l = 1, 2, \ldots, M \]

is optimal if the coefficients of the linear form \( u_{isl} \) are the coordinates of the eigenvectors of the covariance matrix of the initial set. The variance of the sampling indices along the coordinate axes of the \( n \)-dimensional space then takes on extremal values which are equal to the corresponding eigenvalues of the covariance matrix.

Ice data for the Kara, Laptev, East Siberian, and Chukchi seas were used for the decomposition. The results of the analysis show that the overall ice coverage of the Arctic seas in August is closely correlated with the first coefficient of the expansion \((r = 0.93)\). The coordinates of the first eigenvector \( u_1 \) enable us to estimate the contribution of the individual regions of the Arctic seas to the total ice coverage. The higher the value of \( u_1 \), the greater the contribution. We see from Table 1 that this contribution varies between wide limits. The maximum contribution to the total ice coverage comes from the central regions and the least contribution from western and eastern parts. This conclusion is further supported by the values of the correlation coefficients between the total ice coverage and that of individual regions.

Following these results, the authors applied discriminant analysis to the problem of total ice forecasting of Arctic seas. The initial hydrometeorological fields for the analysis were the monthly average pressure and air temperatures at 12 grid points for the period between 1937 and 1961. We see from Figure 1 that the grid to some extent covers the main centers of atmospheric activity and all the Arctic seas.
The results obtained at the Ice Forecasting Department of the Arctic and Antarctic Institute indicate that the summer ice coverage is the outcome of ice accumulation (growth and drift of ice in winter) and deterioration processes (melting and drift in the spring-summer period). Ice accumulation and deterioration in the Arctic seas were found to be directly determined by processes which take place over these seas.

### Table 1

<table>
<thead>
<tr>
<th>Region</th>
<th>( u_1 )</th>
<th>( r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kara Sea</td>
<td></td>
<td></td>
</tr>
<tr>
<td>southwest</td>
<td>0.43</td>
<td>0.32</td>
</tr>
<tr>
<td>northeast</td>
<td>0.30</td>
<td>0.49</td>
</tr>
<tr>
<td>Laptev Sea</td>
<td></td>
<td></td>
</tr>
<tr>
<td>west</td>
<td>0.54</td>
<td>0.65</td>
</tr>
<tr>
<td>east</td>
<td>0.48</td>
<td>0.72</td>
</tr>
<tr>
<td>East Siberian Sea</td>
<td></td>
<td></td>
</tr>
<tr>
<td>west</td>
<td>0.32</td>
<td>0.50</td>
</tr>
<tr>
<td>east</td>
<td>0.23</td>
<td>0.31</td>
</tr>
<tr>
<td>Chukchi Sea</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.16</td>
<td>0.25</td>
</tr>
</tbody>
</table>

The air pressure and air temperature fields corresponding to the grid points in Figure 1 therefore appear to carry a redundancy of information in individual seasons. The air temperature in the Greenland region may have no significant effect on ice formation or melting in the Arctic seas. We should thus try to identify optimal regions—that is, those regions whose atmospheric activity most affects the ice conditions in the Arctic seas.

When constructing the bundle of forms for discriminant analysis, we should take care to ensure that the sample volume is greater than
the number of grid points. The limited series of ice data (only 32 years) makes it impossible to select a sufficient number of cases with ice coverage below average (class A) and above average (class B). Almost half of these cases are observed in years with ice coverage close to the average, so that the number of years classified as A and B is only slightly greater than the number of grid points. The relationship between the sample volume and the number of grid points can be improved by reducing the total number of grid points. This approach is permissible for the optimal regions.

One of the methods for identifying the optimal regions [2] calls for the analysis of the parameter

\[ d_s^2 = \bar{P}_s^2(A) + \bar{P}_s^2(B) - 2\bar{P}_s(A)\bar{P}_s(B) \]

\[ s=1, 2, \ldots, n, \]

where \( d_s^2 \) is the mean square distance between the two classes in the \( s \)th grid point.

By partitioning the initial pressure and temperature field into classes in accordance with ice coverage classes using maximum values of \( d_s^2 \) as the criterion, we can identify the grid points with the maximum
information content. To eliminate the effect of seasonal factors, we replaced the parameter $d_s^2$ in actual analysis by the ratio $d_s^2/\sigma_s^2$, where $\sigma_s^2$ is the variance over the entire set at the $s$th grid point.

To reduce the volume of information and avoid random perturbations and disturbances, the optimal regions were identified by selecting the first six grid points with maximum values for the seasonal-average pressure and temperature fields, i.e., for summer (June-August), spring (April-May), winter (December-February), and autumn (September-November). The other grid points were ignored.

Analysis of Figure 2 leads to definite conclusions about the effect of air pressure and temperature distribution in different seasons on the ice coverage of the Arctic seas. In the spring-summer period, the regions of maximum atmospheric activity are situated over the Arctic seas. This conforms to the results of earlier ice studies of the individual seas. In autumn, the optimal temperature region is situated in the eastern part of the Arctic basin, whereas the optimal pressure regions are in the north and west. It thus seems that the air temperature in autumn and the particular pressure distribution have no direct influence on the ice conditions of the following summer. However, since these regions are characterized by a sufficiently high information content, we may conclude that the autumnal processes influence the subsequent ice conditions through modification of atmospheric circulation.

Having identified the optimal regions by this method, we constructed the bundles of forms for pressure and temperature for the period from August back through September of the year before. The eigenvalues and the principal vectors of the bundles were then determined for each month. Since only two ice coverage classes (above average and below average) were used in constructing the covariance matrices, further analysis was confined to investigating the first eigenvalue and the corresponding first principal vector of the bundles. Then the pressure and temperature indices $P_i$ and $T_i$ were computed for
Fig. 2. Optimal regions for air pressure (a) and air temperature (b) fields for the different seasons.

I—summer, II—spring, III—winter, IV—autumn.
each month and correlated with the total ice coverage (Table 2). An analysis of Table 2 reveals a definite correlation between the total ice coverage and the forecast indices of certain months. This correlation is the most pronounced for autumn processes (September-November).

### TABLE 2

**CORRELATION COEFFICIENTS BETWEEN THE INDICES $P'_i$ AND $T'_i$ AND THE TOTAL ICE COVERAGE**

<table>
<thead>
<tr>
<th>Index</th>
<th>VIII</th>
<th>VII</th>
<th>VI</th>
<th>V</th>
<th>IV</th>
<th>III</th>
<th>II</th>
<th>I</th>
<th>XII</th>
<th>XI</th>
<th>X</th>
<th>IX</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P'_i$</td>
<td>0.39</td>
<td>0.26</td>
<td>0.44</td>
<td>0.11</td>
<td>0.37</td>
<td>0.39</td>
<td>0.21</td>
<td>0.32</td>
<td>0.32</td>
<td>0.52</td>
<td>0.50</td>
<td>0.11</td>
</tr>
<tr>
<td>$T'_i$</td>
<td>0.37</td>
<td>0.34</td>
<td>0.22</td>
<td>0.30</td>
<td>0.30</td>
<td>0.34</td>
<td>0.37</td>
<td>0.24</td>
<td>0.30</td>
<td>0.24</td>
<td>0.50</td>
<td>0.54</td>
</tr>
</tbody>
</table>

(*) months of preceding year.

Regression equations can be constructed using our results. These equations are effective for forecasting, however, only if the arguments satisfy the condition of completeness of all the indices. It is therefore unreasonable to construct separate regression equations for each month. On the other hand, construction of a regression equation which includes a large number of indices $P'_i$ and $T'_i$ involves considerable difficulty, since the individual indices are generally correlated so that the matrix of the system of normal equations may turn out to be poorly conditioned.

These difficulties can be avoided by averaging the indices over several months. The optimum averaging procedure is similar to the calculations of the moving average. Besides the moving averages, we also compute the multiple correlation coefficients between the set of the indices and the ice coverage in order to estimate the significance of the individual periods. Our tests show that averaging over three-month periods is the most effective (Table 3).
The data of Table 3 make it possible to identify the optimal periods. There are four such periods for the index $P^I_i$: June-August; April-June; January-March; and, in the preceding year, October-December. Three periods are identified for the index $T^I_i$: April-July; January-March; and, in the preceding year, September-November. Using the optimal periods, one can prepare regression equations for forecasts of various durations (Table 4). We see from Table 4 that the resulting regression equations forecast the total ice coverage of the Arctic seas.

### Table 3

**Correlation Coefficients Between Total Ice Coverage of Arctic Seas and the Indices $P^I_i$ and $T^I_i$ Averaged Over Three-Month Periods**

<table>
<thead>
<tr>
<th>Index</th>
<th>VI-VIII</th>
<th>V-VII</th>
<th>IV-VI</th>
<th>VII-V</th>
<th>II-IV</th>
<th>I-III</th>
<th>XII-II</th>
<th>XI-I</th>
<th>X-XII</th>
<th>IX-XI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T^I_i$</td>
<td>0.45</td>
<td>0.47</td>
<td>0.48</td>
<td>0.43</td>
<td>0.49</td>
<td>0.49</td>
<td>0.45</td>
<td>0.38</td>
<td>0.51</td>
<td>0.66</td>
</tr>
<tr>
<td>$P^I_i$</td>
<td>0.56</td>
<td>0.46</td>
<td>0.55</td>
<td>0.48</td>
<td>0.48</td>
<td>0.51</td>
<td>0.41</td>
<td>0.59</td>
<td>0.65</td>
<td>0.63</td>
</tr>
</tbody>
</table>

### Table 4

**Forecasting Regression Equations for Various Durations**

<table>
<thead>
<tr>
<th>Regression equation</th>
<th>Range of forecast (months)</th>
<th>Overall correlation coefficient</th>
<th>Accuracy of forecast</th>
<th>A = 25%</th>
<th>A = 20%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>equation</td>
<td>norm</td>
<td>equation</td>
<td>norm</td>
</tr>
<tr>
<td>$F = 0.83P_{IX-XII} + 0.46T_{IX-XI} + 57$</td>
<td>seven</td>
<td>0.72</td>
<td>0.70</td>
<td>0.81</td>
<td>0.65</td>
</tr>
<tr>
<td>$F = 0.72P_{IX-XII} + 0.33T_{IX-XI} + 0.34P_{I-III} + 31T_{I-III} + 52$</td>
<td>four</td>
<td>0.77</td>
<td>0.62</td>
<td>0.87</td>
<td>65</td>
</tr>
<tr>
<td>$F = 0.62P_{IX-XII} + 0.28T_{IX-XI} + 0.31P_{I-III} + 0.29T_{I-III} + 0.36P_{IV-VII} + 0.04T_{IV-VI} + 52$</td>
<td>one</td>
<td>0.79</td>
<td>0.61</td>
<td>0.97</td>
<td>0.81</td>
</tr>
</tbody>
</table>

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one to seven months in advance, with reliability substantially higher than the norm.

A comprehensive analysis of our forecasting relations should be conducted using independent data. A short series of observations is inadequate for this purpose. We did perform a certain check, however. All calculations were performed twice: once using the complete sample (1937-1968), and once using a partial sample (1937-1963). Comparison of the results revealed an adequate fit. The results indicate that autumn-early winter atmospheric processes have a significant effect on the formation of the total ice coverage (see Table 3).

Let us consider the physical effects produced by the processes of this period. Kovalev [1] showed that the total ice coverage of Arctic seas is related in a definite way to the character of the atmospheric processes. Table 5 lists the correlation coefficients between the total ice coverage of the Arctic seas and the frequency of the atmospheric circulation forms (E, W, and C).

**TABLE 5**

CORRELATION COEFFICIENTS BETWEEN TOTAL ICE COVERAGES AND ATMOSPHERIC CIRCULATION FORMS (E, W, C)

<table>
<thead>
<tr>
<th>Type</th>
<th>Months</th>
<th>X</th>
<th>XI</th>
<th>XII</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
<th>VIII</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td></td>
<td>-0.50</td>
<td>-0.44</td>
<td>-0.35</td>
<td>-0.18</td>
<td>0.52</td>
<td>0.21</td>
<td>0.42</td>
<td>0.24</td>
<td>-0.14</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>W</td>
<td></td>
<td>0.43</td>
<td>0.27</td>
<td>0.30</td>
<td>0.24</td>
<td>-0.14</td>
<td>0.30</td>
<td>-0.11</td>
<td>0.30</td>
<td>-0.31</td>
<td>-0.02</td>
<td>-0.06</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td>0.08</td>
<td>0.30</td>
<td>0.46</td>
<td>0.09</td>
<td>0.45</td>
<td>-0.15</td>
<td>-0.18</td>
<td>-0.25</td>
<td>0.03</td>
<td>0.14</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Table 5 reveals, on the one hand, a definite correlation between the total ice coverage and the frequency of the E form in October-November of the preceding year, in accordance with the data in Table 2. We thus conclude that the index \( P' \) for these periods corresponds to E-type circulation processes. Comparison of the frequency of E-type circulation in October-December with the index \( P \) for the same period...
(determined for the optimal regions) indicates that a correlation exists between the two factors \( r = -0.49 \). A substantially closer correlation \( r = -0.84 \) is observed for the index taken over the entire grid (12 points), since the circulation forms characterize atmospheric processes over a large territory of the Northern Hemisphere.

On the other hand, the correlation between the total ice coverage and the frequency of the E-type circulation in October-December is -0.58; the correlation with the index \( P_{X-XII} \) based on the entire grid is -0.48; and that with the index \( P_{X-XII} \) based on the optimal region is 0.65. The correlation between the total ice coverage and the index \( P_{X-XII} \) of the optimal region is higher than the correlation with the atmospheric circulation types, since ice forms in Arctic seas as a result of the direct interaction of the seas with the atmospheric processes over them.

The pressure distribution for E-type circulation suggests that the index \( P_{X-XII} \) characterizes the direction and intensity of air transfer. High frequency of E-type circulation corresponds to negative values of \( P \) and enhanced western and southwestern transfer over the Barents and Kara seas. This transfer pattern from October through December favors low ice coverage in the western Arctic, as it involves injection of relatively warm air and water masses. A reverse pattern is observed for low frequency of E-type processes (positive \( P_{X-XII} \)).

E-type processes predominant in October-December transform into W-type circulation in spring. As a result, western and southwestern transfer patterns predominate over the Laptev Sea, driving ice out of this region.

The pressure distribution in the autumn-winter period, determining the direction and intensity of air transfer, thus influences the total ice coverage directly and indirectly, through transformation of atmospheric circulation between winter and summer.

Studies of the dependence of ice coverage on autumn air temperature in the Arctic seas fail to reveal any significant correlation \( r = 0.15 \). However, a fairly close correlation is observed between the total ice coverage and the index \( T \) based on the temperature field extrinsic to the Arctic seas \( r = 0.66 \). This indicates that the air temperature for this period should be treated as an index of some other processes, rather than as a factor directly influencing the formation of the ice cover.
Let us consider the temperature distribution in September-November in an optimal region for the two ice coverage classes (above average and below average). We computed the average representative air temperatures in September-November for each of the six points in years preceding class-A years (below average) and class-B years (above average). A chart of temperature differences was then constructed to bring out the different features of the two distributions (A and B). The chart in Figure 3 shows that if the eastern Arctic is substantially colder than the western Arctic in September-November, the following year will have above-average total ice coverage. If the preceding September-November is substantially warmer, the total ice coverage for a given year will be below average.

Fig. 3. Temperature difference between classes A and B in September-November.

It thus follows that the index $T$ obtained by discriminant analysis constitutes a characteristic of the temperature distribution over a given water area. According to [1], the air temperature distribution in autumn reflects the distribution of ice in summer and is closely related to atmospheric circulation types. Thus in years when it is colder in the east than in the west, E-type processes are poorly
developed in autumn, while in years when the west is colder than the east, E-type processes are enhanced.

It is clear from the above that the pressure and temperature indices have a definite physical interpretation, as they reflect the development of the atmospheric processes in the autumn and early winter period. The processes of this period affect the formation of the ice cover in the Arctic seas for the subsequent period both directly, through ice formation in autumn and winter, and indirectly, through transformation of atmospheric processes between winter and summer. Our positive results indicate that indices can be selected for preparation of ice forecasts for individual seas, also.

BIBLIOGRAPHY


A number of studies have been carried out which deal with the long-period tides and the related "astronomical" currents in the Arctic basin and adjoining seas [1, 2, 6, 7]. However, while the results obtained for the long-period tidal fluctuations of the sea level are quite reliable, the astronomical currents have been studied insufficiently. The main reason for this is the absence of continuous long observation series and rigorous methodology for the separation of the astronomical currents from the total current. Nevertheless, the work of Maksimov [6] and Vorob'ev [2] made it possible to estimate, albeit quite approximately, the order of magnitude of the velocities of the semimonthly and the monthly tidal currents in the Arctic basin and the adjoining seas. Thus Maksimov, using the observation data of the Severnyi poljus high-latitude stations, established average velocities of 3.1 cm/sec for the semimonthly and 4.5 cm/sec for the monthly cycle in the central part of the Arctic Ocean. Vorob'ev analyzed the current data of buoy stations and the drift data of radio buoys in the Arctic seas and obtained maximum velocities of 6.5 and 5.7 cm/sec for the semimonthly and the monthly tidal currents, respectively.

The analytical method applied by these authors to the observation results, however, does not eliminate the effect of the wind component. And yet it is wind that largely determines the drift of ice and the sea currents in the surface layers, especially over short time intervals [4, 5, 8]. Therefore, if the wind is subject to random or regular fluctuations which are close in frequency to the long-period tides, they will inevitably affect the apparent characteristics of the astronomical currents [3].
Analysis of the *Severnyi polus*-15 data led Gudkovich and Evdokimov to the conclusion that the quasi-periodic components of ice drift with frequencies close to those of the long-period tides contained two distinct parts, one associated with the corresponding wind speed fluctuations and the other attributed to the effect of the astronomical currents. Their method permits eliminating the effect of the wind component and isolating the wind-independent characteristics of the semimonthly and the monthly tidal drift in the Arctic basin. The true maximum velocity of the astronomical current was found to be approximately half the corresponding periodic components of the total drift, reaching 1.5 cm/sec for the semimonthly tidal drift and about 1.0 cm/sec for the monthly cycle. Allowing for the general nature of the long-period tides in the World Ocean, we conclude that the velocities of the tidal currents in the peripheral seas may be higher than the corresponding velocities in the Arctic basin. Indeed, Vorob'ev's results based on the drift data of the *Lenin* and the *G. Sedov* give figures for the semimonthly and monthly astronomical drift of ice in the continental shelf which are as high as two-thirds of the corresponding periodic component of the total drift and are comparable to the ten-day average velocity of the wind drift. It is therefore desirable to elucidate the role of the astronomical drift in the formation and changes of the ice conditions in the Arctic seas.

Taking 4.0 cm/sec for the average velocity of the semimonthly and the monthly astronomical drift, we find that the displacement of ice during a complete tidal cycle will be on the average about 25 km and 50 km, respectively. Allowing for the direction of propagation and the frontal length of the tidal wave, we find that the effect of the semimonthly and the monthly tidal current will change the ice coverage of the Laptev Sea by 3 per cent and 6 per cent, respectively. This change is hardly of any practical significance. However, the effect of the astronomical drift on ice compaction and hence navigability in certain regions of the Northern Seaway may be quite substantial at times.

As an example, let us consider a region $b$ km wide and $l$ km long (the front of the tidal wave is parallel to $b$). Let the ice compaction in this
region at the end of the tidal cycle be $n_0$ (in proper fractions) and the ice compaction in an adjoining northern region be $n_1$. The ice compaction in the test region at the end of the tidal cycle will then be

$$n = \frac{b_0 n_0 + b_1 n_1}{b_2} = \frac{b_0 n_0 + b_1 n_1}{l}$$

where $s$ is the displacement of ice during one tidal cycle.

This relation may be rewritten as

$$n = n_0 + n_1 \frac{s}{l}$$

or, for $n_0 = n_1$,

$$n = n_0 \left(1 + \frac{s}{l}\right)$$

The last expression is valid for

$$\frac{s}{l} \leq \frac{1 - n_0}{n_0}$$

since ice compaction is never greater than unity ($n \leq 1$). If the left and right members of this relation are equal, the ice compaction is maximum ($n = 1$) and closing or hummocking of ice may occur.

Figure 1 plots the ratio $s/l$ as a function of the initial ice compaction $n_0$ in the test region and also shows the dimensions $l$ of the close ice zones at the end of the tidal cycle as a function of the initial compaction or, more precisely, the ice compaction at the end of the tidal cycle for the chosen velocity of the semimonthly and monthly astronomical drift. It follows from Figure 1 that the dimensions of the close ice zones increase markedly as the initial compaction approaches 1.

We thus conclude that the effect of the astronomical drift on ice conditions may be quite significant for regions with a high ice-cover coefficient. Conversely, in regions with open sailing ice and large expanses of clear water, the effect of the long-period tidal currents is virtually unnoticeable.
Fig. 1. The ratio $s/l$ vs. initial ice compaction $n_0$ (1) and the dimensions of close ice zones at the end of the semimonthly (2) and the monthly (3) tidal cycle.

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RESULTS OF THE STUDY OF NONUNIFORM ICE DRIFT IN THE ARCTIC BASIN

N. A. Volkov, Z. M. Guškovich, and V. D. Uglov

Nonuniform ice drift is associated with time and space changes in the drift-producing forces. As it is difficult to establish the instantaneous drift velocity, inertial effects connected with the time-dependence of drift have been little studied. In this paper we consider some characteristics of nonuniform drift in the steady-state case with nonuniform distribution of forces in space.

Under these conditions, the projections of the drift velocity \((u, v)\) on the coordinate axes \((x, y)\) may be treated as functions of position: \(u = u(x, y)\), \(v = v(x, y)\). Taking the origin at some point \(x_0, y_0\), we expand these functions in Taylor series:

\[
\begin{align*}
  u &= u_0 + \left( \frac{\partial u}{\partial x} \right)_0 x + \left( \frac{\partial u}{\partial y} \right)_0 y \\
  v &= v_0 + \left( \frac{\partial v}{\partial x} \right)_0 x + \left( \frac{\partial v}{\partial y} \right)_0 y
\end{align*}
\]  

(1)

Terms of higher orders are omitted, as they are immaterial for short time intervals.

The four differentials entering (1) can be combined to give the following characteristics of nonuniform drift:

\[
\begin{align*}
  \text{div} \vec{V} &= \frac{\partial u}{\partial y} + \frac{\partial u}{\partial x} \\
  \text{rot} \vec{V} &= \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} \\
  \text{def}_1 \vec{V} &= \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \\
  \text{def}_2 \vec{V} &= \frac{\partial u}{\partial y} - \frac{\partial u}{\partial x}
\end{align*}
\]

(2)  

(3)  

(4)  

(5)

Here \(\vec{V}\) is the drift velocity vector.
Equation (2) characterizes the velocity of divergence or convergence of the floes, i.e., the rate of compaction or opening of the ice cover. The effect of drift divergence on changes of ice compaction were considered by a number of authors [7, 10, 12, etc.]. In "isobaric drift," which is characterized by the relations

\[ u = -K \frac{\partial p}{\partial y} \quad v = K \frac{\partial p}{\partial x} \]

where \( K \) is the isobaric coefficient and \( p \) is atmospheric pressure, the drift divergence is zero. Changes in ice compaction are thus the result of deviations of drift velocity or direction from the isobaric values. According to [1, 5], the direction of ice drift is deflected right from the isobars in summer and left in winter. Therefore, in summer the ice becomes more open in cyclonic systems and more compact in anticyclonic systems. In winter, the reverse pattern is observed.

Equation (3) characterizes the rotational velocity of the ice cover. The direction and the velocity of rotation of the floes [1, 3, 9] are highly variable, but lengthy observations reveal a general clockwise rotation in some regions and counterclockwise rotation in others. Buinitskii [1] suggested that the fast short-time changes in ice rotation are determined by the rotational character of the wind fields, whereas the general turning motion is largely determined by the transverse non-uniformity (curl) of the field of steady currents. The first of these two assumptions was confirmed by comparing the rotation of floes with the divergence of the pressure gradient and the wind curl [3]. The observed relations had a correlation coefficient of 0.75-0.83. The wind-independent component of the turning motion reached 6-10 minutes of arc per day.

Equations (4) and (5) characterize the rate of deformation of a surface element of the ice cover. Equation (4) corresponds to angular deformation (i.e., the change in the angle between the sides of a square surface element), and equation (5) represents the nonuniformity of the linear deformation along two perpendicular axes. Unlike the expressions
for the divergence and the curl, which are invariant under coordinate transformations, the expression for the deformation is significantly dependent on the particular choice of the coordinate axes: when the axes are turned through 45°, the two relations are interchanged. The expression

\[ \text{def} \bar{V} = \sqrt{(\text{def}_1 \bar{V})^2 + (\text{def}_2 \bar{V})^2} \] (6)

however, is invariant under coordinate transformations, and it therefore provides a quite general expression of the deformation of an ice surface element.

American scientists [13] attach considerable importance to ice deformation, which in their opinion is the main cause of debacle (cracking and hummocking of ice). However, they consider only the angular deformation, which in our interpretation relates to the total deformation as one projection of a vector relates to the entire vector.

The first studies of debacle were made by N. N. Zubov, who analyzed the drift of close ice in moving pressure systems [8]. Interesting results on the frequency of debacle in various pressure systems were obtained by Buinitskii, who analyzed the observation data of the R/V G. Sedov [1]. According to these data, the great majority of debacles (more than 68 per cent of all the cases) occurred with a pressure trough extending over the drift region. It is significant that the amount of deformation (6) has its maximum near the hyperbolic point of the pressure fields.

Investigation of nonuniform drift and related processes under actual conditions requires simultaneous observations at several points. Such observations were first carried out in 1961 in a polar region. Some results of their processing were published in [3]. A similar experiment was repeated over a long time period during the 1962 high-latitude aerological expedition.

Four portable drift stations were set up at the corners of an ice polygon with sides 70 km long in the drift area of the Severnyi polyus-10.
station to the northeast of the De Long Islands. The observation program for these stations included a combination of frequency astronomical, meteorological, and hydrological measurements.

Astronomical observations were carried out using a special program developed by Bushuev [2]. The analysis was based on 46 virtually simultaneous determinations which differed by no more than ±0.5 hours in time and 14 determinations for which the time difference did not exceed ±2.5 hours. In the latter case, the observations were reduced to a common time using ice drift data of Severnyi polyus-10.

Thus 60 synchronous measurements at four points made between 26 March and 16 May (50 days) provided the basic material for the investigation of ice drift and its nonuniformity. Baryshev's computer program was used to derive the right coordinates, the magnitude, the direction, and the velocity of drift between the astronomical points for each of the four stations, as well as the distance between the stations, the polygon area, the orientation of the sides, and the diagonals of the quadrangle at the observation times.

The space derivatives in (2)-(5) were computed using equations (1). Given the coordinates and the projections of the drift velocity at the four points, we can obtain eight equations of the form (1) which, besides the four derivatives, contains the four unknowns \( x_0, y_0, u_0, \) and \( v_0. \) Eliminating these unknowns, we obtain for the sought derivatives

\[
\begin{align*}
\frac{\partial y}{\partial x} &= \frac{(u_2 - u_4) (x_1 - x_3) - (u_1 - u_3) (x_2 - x_4)}{(y_2 - y_4) (x_1 - x_3) - (y_1 - y_3) (x_2 - x_4)} \\
\frac{\partial y}{\partial y} &= \frac{(v_2 - v_4) (x_1 - x_3) - (v_1 - v_3) (x_2 - x_4)}{(y_2 - y_4) (x_1 - x_3) - (y_1 - y_3) (x_2 - x_4)} \\
\frac{\partial v}{\partial x} &= \frac{(u_1 - u_3) (y_2 - y_4) - (u_2 - u_4) (y_1 - y_3)}{(y_2 - y_4) (x_1 - x_3) - (y_1 - y_3) (x_2 - x_4)} \\
\frac{\partial v}{\partial y} &= \frac{(v_1 - v_3) (y_2 - y_4) - (v_2 - v_4) (y_1 - y_3)}{(y_2 - y_4) (x_1 - x_3) - (y_1 - y_3) (x_2 - x_4)}
\end{align*}
\]

(7)

*The number subscripts in these equations identify the different points (subscript 4 is the Severnyi polyus-10 station).
A similar method was used to compute the corresponding wind nonuniformity characteristics.

Figure 1 is a drift chart of the four points (including the Severnyi polyus-10 station) between 25 March and 15 May 1962. Note the marked similarity of the four drift paths. Between 26 March and 11 April, each point described two counterclockwise loops; then the drift assumed a northeasterly course (until 15 April), which changed to a northerly course. Between 20 and 28 April, each point again described one

Fig. 1. Schematic diagram showing the drift of the four points between 25 March and 15 May 1962. Dashed lines show the polygon outline at the beginning and the end of the drift period.
counterclockwise loop; until 4-5 May, easterly drift was maintained; and from 5 May to the end of the test period, the drift changed to north-westerly. The resultant drift over the 50 days was directed almost precisely to the north. Some data on the drift of the four points are listed in Table 1.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Severnýi polýus-10</th>
<th>No. 1</th>
<th>No. 2</th>
<th>No. 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direction of general drift</td>
<td>289°</td>
<td>293°</td>
<td>291°</td>
<td>290°</td>
</tr>
<tr>
<td>(relative to 90°)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Resultant displacement (km)</td>
<td>114.8</td>
<td>109.3</td>
<td>105.6</td>
<td>107.4</td>
</tr>
<tr>
<td>Velocity of resultant drift</td>
<td>2.30</td>
<td>2.18</td>
<td>2.11</td>
<td>2.15</td>
</tr>
<tr>
<td>(km/day)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total displacement (km)</td>
<td>349.8</td>
<td>334.7</td>
<td>316.6</td>
<td>349.8</td>
</tr>
<tr>
<td>Average drift velocity (km/day)</td>
<td>7.00</td>
<td>6.70</td>
<td>6.33</td>
<td>7.00</td>
</tr>
<tr>
<td>Tortuosity</td>
<td>3.04</td>
<td>3.07</td>
<td>3.00</td>
<td>3.26</td>
</tr>
</tbody>
</table>

The data of the table confirm that the drift characteristics of the four points are close to one another. A slight decrease of drift velocity is noted from south (Severnýi polýus-10) to north (point No. 2).

Drift and wind data for each point were processed by the correlation method [6], which yields the average values of the wind coefficient \( k \), the angle \( \alpha \) between the drift direction and the wind, and the direction \( D \) (relative to 90°E) and velocity \( C \) of the wind-independent quasi-steady current (Table 2).

Table 2 lists the relevant data for each point for the two periods comprising the entire observation series. Despite slight differences in \( k \) and \( \alpha \), their average values are characteristic of the entire Arctic basin [1, 5]. Note that the wind coefficient and the drift deflection angle decrease from the first period to the second. The quasi-steady current also markedly changes in time and in space; its average direction is west-northwest.
TABLE 2
AVERAGE VALUES OF DRIFT CHARACTERISTICS FOR THE FOUR POINTS INDEPENDENT OF WIND.

<table>
<thead>
<tr>
<th>Points</th>
<th>From 4 to 24 April</th>
<th>From 24 April to 15 May</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>k</td>
<td>α</td>
</tr>
<tr>
<td>Sev pol-10</td>
<td>0.019</td>
<td>31°</td>
</tr>
<tr>
<td>No. 1</td>
<td>0.022</td>
<td>21°</td>
</tr>
<tr>
<td>No. 2</td>
<td>0.020</td>
<td>25°</td>
</tr>
<tr>
<td>No. 3</td>
<td>0.250</td>
<td>32°</td>
</tr>
<tr>
<td>Average</td>
<td>0.021</td>
<td>27°</td>
</tr>
</tbody>
</table>

Calculations of the polygon area (Figure 2) reveal marked changes during the observation period. The maximum area is 5910 km² and the minimum is 5137 km², the average being 5364 km². Thus the maximum change in area was 773 km², i.e., about 14 per cent of the average value. The prevailing values, however, were within ±5 per cent of the average polygon area. The rms error $E$ was ±0.10-0.15 of meridian arc (mile). The error in the distances between points for simultaneous observations [1] was thus $m_k = E \sqrt{2} = 0.2$ mile. The drift velocity at the time of observation as a rule did not exceed 0.2 knots, so that the error in the distance

---

**Fig. 2.** Time variation of polygon area.
due to imperfect simultaneity of observations (±0.5 hr difference) was $m_g = ±0.1$ mile. Adding up the above errors, we find for the rms error in the distance between the points $m = ± \sqrt{m_k^2 + m_g^2} = ±0.2$ mile = ±0.4 km.

The error in the area of a nearly square polygon thus can be obtained from the equation

$$\Delta S = 2l \Delta l$$

where $l \approx 70$ km is the side of the square,

$$\Delta l = m$$

Thus $\Delta S = 56$ km$^2$, i.e., slightly over one per cent of the polygon area, which is significantly less than the observed changes.

If we assume that the drift changes linearly in space within the polygon area, the prevailing changes in ice compaction are also ±5 per cent, which is double the corresponding changes in the polar region covered by the 1961 high-latitude aerological expedition.

Of considerable interest is the relation between the drift divergence, which represents the changes in ice compaction, and the wind field characteristics, in particular its divergence. Since the surface wind is deflected left from the isobar (its deflection from the pressure gradient is always less than 90°), wind divergence is always negative in cyclonic fields and positive in anticyclonic fields. In light of the particular features of the wind drift, the wind divergence should have the same sign as the drift divergence in winter and an opposite sign in summer.

The indistinct relation between wind divergence and drift divergence is due to the fact that the expedition operated in the transition period between winter and summer conditions. Nevertheless, the expedition findings established a change in the sign of the correlation between the two divergences. The correlation coefficient was $+0.32±0.13$ between 4 April and 24 April and $-0.38±0.12$ between 24 April and 15 May. The relation
was thus characteristic of winter conditions at the beginning of the observation period, changing to the summer-type relation at the end of the period.

To form some idea of floe rotation, measurements were taken of the changes in the orientation of the two diagonals of the ice polygon, from point No. 1 to point No. 3 and from point No. 2 to Severmyi polynya-10. Despite occasional discrepancies in the results, the orientation of the two diagonals on the whole changed in phase. The floe rotation was therefore characterized by the mean change in rotation as obtained by averaging the results for the two diagonals (Fig. 3, curve 1).

We see from Figure 3 that between 7 and 15 April the rotation was mostly counterclockwise, whereas between 16 and 25 April clockwise rotation was observed, which then reverted to counterclockwise (until 5 May) and changed again to clockwise (to the end of the observation period). The resultant rotation between 4 April and 15 May was 1.5° counterclockwise.

The rotation velocity is related to the drift curl by the equality

$$\omega = \frac{\Delta \alpha}{\Delta t} = - \frac{1}{2} \text{rot} \ U$$

(8)

The minus sign indicates that the floe rotates counterclockwise when the curl is positive. If the rotation velocity is expressed in minutes of arc per hour and the curl in one per hour, relation (8) takes the form

$$\frac{\Delta \alpha}{\Delta t} = - 1720 \text{ rot} \ U$$

(9)

The linear relation between drift and wind suggests that the rotation velocity of the ice cover should depend on the curl of the wind field. In our case, this relation has a correlation coefficient \( r = -0.71 \pm 0.05 \). The regression equation has the form

$$\frac{\Delta \alpha}{\Delta t} = - 30 \cdot \text{rot} \ V - 0.50$$

(10)
Here the rotation velocity is in minutes of arc per hour and the wind curl is in 1/hr.

The regression coefficient in (10) is higher by more than a factor of 2.5 than the coefficient in the corresponding equation for the polar region [3]. This is partly due to the higher mobility of the ice (a higher wind coefficient) and partly to the effect of the near islands and fixed ice. The constant term in (10) indicates that the wind-independent rotation of ice is counterclockwise, as opposed to the situation observed in the polar region; the velocity of this counterclockwise rotation is 0.5 minutes of arc per hour, or 12 minutes per day.

Fig. 3. Polygon rotation between 4 April and 14 May. 1--actual data; 2--calculated from (10); 3--calculated from (10), but ignoring the constant rotation component.
Figure 3 (curve 2) plots the floe rotation computed from equation (10). It adequately fits the curve of the true rotation (curve 1). Note that the resultant counterclockwise rotation of the floe occurred in a time of prevailing anticyclonic winds (negative wind curl), when the wind component of the rotation should be clockwise. This is evident from curve 3 (see Fig. 3), which is derived from equation (10) ignoring the constant component of rotation.

Let us consider to what extent the nonuniformity of the quasi-steady currents may cause the "wind-independent" component of floe rotation. The current data listed in Table 2 may be used in conjunction with equations (3) to compute the drift curl associated with the currents. It was found to be $+54 \times 10^{-6} \text{ l/hr}$ for the first period and $+302 \times 10^{-6} \text{ l/hr}$ for the second period. Using (9), we readily find the drift curl corresponding to the wind-independent rotation velocity (which is 0.5 minutes of arc per hour). It is found to be equal to $+290 \times 10^{-6} \text{ l/hr}$. This figure is consistent with the above results for the curl of the quasi-steady current field. Buinitskii's hypothesis concerning the effect of nonuniform surface currents on floe rotation has thus been confirmed for the first time by observation results.

Log records of the Severnyi polyus-10 station and the portable stations establish that debacle (cracking and hummocking) mostly occurs when the drift deformation values computed from (6) are high. With positive drift divergence, formation of cracks and ice lanes prevailed, whereas with negative divergence hummocking generally occurred. Comparison of the drift and the wind deformations (the wind deformations were computed using the same formula and for the same time intervals as the drift deformations) reveals a definite synchronism in the time variation of the two quantities, although no quantitative relationship was detected. These topics will be further treated in future studies.

The availability of series of virtually simultaneous coordinate determinations for the corner points of the ice polygon makes it possible to compute an important characteristic of nonuniform drift associated
with random (turbulent) motion of ice. This is the so-called coefficient of horizontal diffusion, defined by the relation [11]

$$K = \frac{\Delta L^2}{2\Delta t}$$  \hspace{1cm} (11)

where $\Delta L$ is the change in distance between two ice floes in a time $\Delta t$ (superior bar signifies averaging of the square of the distance increment in time or in space).

Let us estimate the error in the diffusion coefficient calculated from (11). As we have seen before, the distance between two points is determined with a rms error $m = 0.4$ km. Hence

$$\Delta L^2 = (\Delta L_0 + m)^2 = \Delta L_0^2 + 2\Delta L_0 m + m^2.$$  

Here $\Delta L_0$ is the sought distance increment. The product $2\Delta L_0 m$ is close to zero (as a covariance of random variable), and $m^2$ is the error variance. Thus, because of the error in $\Delta L$, the diffusion coefficient is too high by an additive term $K'$, which is

$$K' = \frac{m^2}{2\Delta t} \approx 2 \cdot 10^4 \text{ cm}^2\cdot\text{sec}^{-1} \text{ for } \Delta t = 1/2 \text{ day}$$

$$K' = 10^4 \text{ cm}^2\cdot\text{sec}^{-1} \text{ for } \Delta t = 1 \text{ day}$$

Since the time intervals $\Delta t$ were different, the diffusion coefficient was calculated from (11) separately for the two cases $\Delta t \approx 12$ hrs (37 cases) and $\Delta t \approx 24$ hrs (28 cases), as well as for the entire observation series ($\Delta t = 21.4$ hrs). We know [11] that the coefficient of horizontal diffusion depends on the scale of motion. This dependence is generally represented by a power relation of the form

$$K (L) = CL^\alpha$$  \hspace{1cm} (12)

According to the theory of locally isotropic turbulence, the constant $C$ has the dimensions of energy flux (cm$^2$/sec), and the power exponent is $\alpha = 4/3$ (the Richardson-Obukhov "four-thirds" law). $K$ was thus computed separately for the four sides of the square ($\Delta L_{1-2}$, $\Delta L_{2-3}$, ...
\(\Delta L_{3-4}, \Delta L_{4-1}\), each 76 km long on an average, and for the two diagonals \(\Delta L_{1-3}\) and \(\Delta L_{2-4}\), each about 100 km on an average. The results of these calculations are listed in Table 3.

<table>
<thead>
<tr>
<th>(\Delta t) (hrs)</th>
<th>(\bar{L} = 70) km</th>
<th>(\bar{L} = 100) km</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(K \cdot 10^{-5})</td>
<td>number of observations</td>
</tr>
<tr>
<td>12.0</td>
<td>1.08</td>
<td>148</td>
</tr>
<tr>
<td>24.0</td>
<td>1.58</td>
<td>112</td>
</tr>
<tr>
<td>21.4*</td>
<td>1.33</td>
<td>240</td>
</tr>
</tbody>
</table>

*Average for time intervals ranging between 9 and 114 hrs.

We see from Table 3 that the coefficient of horizontal diffusion is one order of magnitude higher than the error value. The coefficient increases both with the increase of the time averaging scale and with the increase of the spatial scale of the phenomenon. The differences in the spatial scales, unfortunately, are too small to allow a reliable estimation of the values of \(C\) and \(\alpha\) in (12). According to [4], the dependence of the diffusion coefficient on the floe separation \(\bar{L}\) is adequately approximated with a power function with \(\alpha = 5/8\) for scales of the order of \(10^4-10^5\) cm (100-1000 m). It is significant that our data corresponding to scales of the order of \(10^7\) cm fit the curve constructed by Gorbunov and Timokhov. This is clear from Figure 4, where the average values of \(K\) are plotted against \(\bar{L}\) on a logarithmic scale.

It should be noted that the data in [4] correspond to a different time-averaging scale (\(\Delta t = 66\) min) and characterize the motion of more open and smaller floes. The validity of the analogy between the horizontal turbulent exchange and the corresponding processes in the ice cover should also be checked.
$K = \frac{\Delta l^2}{2\Delta t}$ vs. floe separation $l$.

1--according to [4];  
2--from our calculations.  
The straight line corresponds to the power exponent $\alpha = 5/8$. 

The various aspects of nonuniform drift and turbulent motion considered in this paper require further theoretical and experimental clarification.

BIBLIOGRAPHY


Thermal effects can be ignored when drift of ice is studied over relatively short time intervals. The ice cover is then considered as an assembly of mechanical objects, floating ice floes of various shapes and sizes.

The external forces which affect the drift of ice include the shearing wind stresses on the ice surface $\tau_x$, shearing stresses at the ice-water interface $\tau_y$, and pressure gradients associated with sea-level changes [10]. Moving ice is also subjected to Coriolis forces, reaction from fixed rigid boundaries (the coastline), and internal stresses due to the interaction of individual floes.

Momentum is directly transmitted between interacting floes, and it is also dissipated as a result of inelastic collisions and relative displacement of floes in the process of equipartition of velocities. In a loose drifting pack, momentum is partly transmitted through interaction of local perturbations of hydrodynamic pressure and velocity fields produced by moving isolated floes [1,5].

The distribution of ice over the sea surface is characterized by a surface density $\rho(x,y)$, i.e., mass of ice per unit surface area. Introducing a nondimensional compaction function $S(x,y)$, we find [9]

$$\rho = \rho_i h S$$

where $\rho_i$ is the density of ice and $h$ is the floe thickness.

Taking $h = \text{const}$ everywhere for simplicity, we replace $\rho$ by the compaction $S$ in what follows.

The characteristic time and length scale are needed in order to describe the drift of ice. The microscopic length scale characterizing
the local structure of the compaction field is fixed by the mean floe size \( r \) and the mean floe spacing \( \lambda \), i.e., the "mean free path" of the ice floe. In drifting ice, \( r \) and \( \lambda \) are of the same order of magnitude. In compact ice, with \( S > 0.7 \), the mean free path is zero. The macroscopic length scale is generally fixed by the linear extent of the water body \( L >> r, \lambda \).

The microscopic time scale is determined by the collision time \( t_c \) and the interaction time \( t_{\text{int}} > t_c \), that is, the time it takes the ice floes to reach a uniform velocity distribution. A characteristic collision time for drifting ice is \( t_\lambda = \lambda / v_{\text{av}} \), where \( v_{\text{av}} \) is the mean velocity of the ice floes.

The macroscopic time scale is fixed by the time of macroscopic relaxation \( \tau_m \), that is, the time for the ice structure to approach the equilibrium state without external forces [6]. In this state the floes are stationary, and the interaction forces between the floes and with the coastline are zero. A variety of equilibrium states are clearly possible, the compaction being a function of the coordinates \( x \) and \( y \).

Suppose that initial state of the ice cover is characterized by an initial equilibrium distribution \( S(x, y, 0) = S_0(x, y) \). External forces acting on the ice cover produce an initial transfer of momentum and leveling-out of velocities, resulting in a local equilibrium of velocities. This so-called kinetic stage of motion is characterized by the length scales \( r, \lambda \) and the time scales \( t_c, t_{\text{int}}, \) and \( t_\lambda \). The kinetic stage of ice drift was studied in [6,7] by methods of statistical mechanics.

The description of the subsequent stages of motion depends on the space and time scales of the perturbations producing the drift. In every particular problem, these scales can be determined from the wave number spectra and the frequency characteristics of the external perturbations. If these scales are sufficiently large, further evolution of motion, after the initial leveling-out of velocities, is determined on the whole by the variation of the macroscopic variables, which may be selected, say, as the compaction \( S \) and the drift velocity \( V \). By
suitable averaging, a compact ice cover may be reduced to a continuous
and locally isotropic medium [9], and we can proceed with hydrodynamic
description of this stage of motion. The dimensions of the averaging
region clearly impose certain restrictions on the grid spacing for
which the continuous drift model is valid. The law of ice mass conserva-
tion for this model has the form [4]
\[ \frac{1}{S} \frac{dS}{dt} = -\text{div} \, \mathbf{V} \]  
and the equation of motion of the ice cover in a system of axes fixed
to the rotating earth is written as
\[ \rho_1 h \frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} + \rho_1 h \mathbf{\Omega} \times \mathbf{V} + \mathbf{F} = \mathbf{f} \]  
where the first term represents the inertial force of the ice mass, the
second is the Coriolis force, \( \mathbf{F} \) is the internal forces acting on the ice
cover, and \( \mathbf{f} \) is the external forces. All vectors and vector operators
are two-dimensional here.

Equation (3) can be simplified for particular problems, since the
relative magnitude of each term is different for different cases. For
example, the advective term \((\mathbf{V} \cdot \nabla) \mathbf{V}\) is significant only for local pro-
cesses, and it can be omitted from studies of the general circulation
of ice in the reservoir.

As an example, let us consider the tidal drift of ice in a marginal
sea with time scale \( \tau_t = 6 \text{ hrs} = 2.16 \cdot 10^4 \text{ sec} \) and length scale\( L = 1000 \text{ km} = 10^8 \text{ cm} \). The drift velocity is \( \mathbf{v} \approx 10 \text{ cm/sec} \), and the
compaction is \( S \approx 1 \). Then
\[ \left| \frac{\partial \mathbf{V}}{\partial t} \right| \approx 5 \cdot 10^{-4} \text{ cm/sec}^2, \quad |(\mathbf{V} \cdot \nabla) \mathbf{V}| \approx 10^{-6} \text{ cm/sec}^2 \]

Let us estimate the magnitude of the inertial term in equation (3),
setting \( h = 100 \text{ cm} \). In this case,
\[ |\rho_1 hS| \approx 0.1 \text{ g} \cdot \text{sec}^2/\text{cm}^3 \quad \text{and} \quad |\rho_1 h \frac{\partial \mathbf{V}}{\partial t}| \approx 0.5 \cdot 10^{-4} \text{ g/cm}^2 \]
The Coriolis force for these scales is of the same order of magnitude as the inertial force

\[ |\gamma hS\mathbf{\Omega} \times \mathbf{v}| = \frac{0.1\pi \cdot 10}{2 \cdot 2.16 \cdot 10^4} \approx 0.7 \cdot 10^{-4} \text{ g/cm}^2 \]  

(5)

The internal forces \( F \) can be defined as the divergence of some plane tensor \( p_{ij} \) characterizing the internal stresses in the ice cover. Thus multiplying the equation of continuity (2) by \( V \) and adding it to the equation of motion (4), we obtain an equation of momentum transfer. In tensor notation, we have

\[ \rho \frac{\partial}{\partial t} (SV_i) + \frac{\partial}{\partial x_j} (p_h S V_i V_j - p_{ij}) + \rho h \xi_{ijk} \mathbf{\Omega} \cdot \mathbf{v}_k = f_i \]

\[ (i = x, y) \]  

(6)

The first term of this equation is the momentum rate of change, and the second term is the divergence of the momentum flux. The stress tensor \( p_{ij} \) accounts for both the internal pressure forces in the ice cover and the irreversible momentum transfer from high-velocity to low-velocity regions, i.e., the internal friction. The term \( - \partial p_{ij} / \partial x_{i,j} \) represents that part of the momentum flux which is not associated with direct momentum transfer with the mass of the moving ice.

Assuming internal friction in the ice cover which is a linear function of the velocity gradient, the internal stress tensor can be written in the usual form for a viscous fluid [2]

\[ p_{ij} = [-p_S + (\zeta - \eta) \text{ div } \mathbf{v}] \delta_{ij} + 2\eta e_{ij} \]  

(7)

where the parameters \( \eta \) and \( \zeta \) have the dimensions of dynamic viscosity per unit ice layer thickness and in general depend on the thickness and compaction of ice. The rate of deformation tensor \( e_{ij} \) describes the kinematics of the continuous drift model in the hydrodynamic stage of motion,

\[ e_{ij} = \frac{1}{2} \left[ \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right] \]  

(8)
If the overall motion of ice floes is regarded as a sum of mean motion and fluctuating motion, equation (3) or (6) for the mean motion will include additional terms which represent the mean momentum flux per unit area transported by the fluctuating velocities, i.e., the Reynolds stresses. This additional momentum flux can be assessed as follows:

\[ \Phi_i = \frac{\partial \tau_{ij}}{\partial x_j}, \quad \text{where} \quad \tau_{ij} = \rho_i h \mathbf{S} \nu_i \nu_j \]

In our case the fluctuating velocities are of the same order of magnitude as the mean motion velocity, so that for \( v \sim 10 \text{ cm/sec} \) we have \( |\tau_{ij}| \sim 10 \text{ g/cm} \). Hence \( |\Phi| \sim 10^{-7} \text{ g/cm} \), which fits the estimate for the advective component. The Reynolds stresses are thus of limited local significance and in no way determine the overall motion.

The stresses of ice compression are characterized by the tensor \( p_{ij} \). For the normal components of these stresses we have

\[
\begin{align*}
p_{xx} &= -p_s + \zeta \text{div} \mathbf{v} + \eta \left( \frac{\partial u_x}{\partial x} - \frac{\partial v_y}{\partial y} \right) \\
p_{yy} &= -p_s + \zeta \text{div} \mathbf{v} - \eta \left( \frac{\partial u_x}{\partial x} - \frac{\partial v_y}{\partial y} \right)
\end{align*}
\]  

The pressure of ice compression will be defined as minus the mean value of the normal compression-stress components,

\[ p = -\frac{p_{xx} + p_{yy}}{2} = p_s - \zeta \text{div} \mathbf{v} \]  

If the equation of motion is taken with the advective component, the ice compression pressure should contain an additional dynamic term \( \rho_i h S(\nu^2/2) \) and the term \( \rho_i h \mathbf{S} \mathbf{Q} \times \mathbf{v} \) should be replaced with the total rotational force \( \rho_i h \mathbf{S}(\mathbf{Q} + \mathbf{rot} \mathbf{v}) \times \mathbf{v} \) [8]. In our case, the dynamic term is of the same order of magnitude as the Reynolds stresses and can be omitted from the analysis of the general circulation of ice in the reservoir.
The term $p_S$, representing the direct pressure exerted by the ice floes on one another, does not depend explicitly on drift velocity and should be related to changes in ice compaction. This relation will close the system of equations (2), (3) for the macroscopic variables $S$, $v$, $p_S$ and enable us to determine the amount of ice compression independent of the motion of ice, i.e., compression in a stationary ice pack pressed against the coast. The second term in (10) is of pure dynamic origin and is determined by the divergence of the drift velocity. It is related to energy dissipation when ice compaction changes.

Let us consider the drift of compact ice, when the deviations of compaction from the equilibrium value are small and irreversible energy losses due to floe interaction are large. Let us first find the change in compaction $\delta S = S - S_0$. The continuity equation (2) for $\delta S \ll S_0$ gives

$$\frac{d(\delta S)}{dt} = -S_0 \text{ div } v$$

(11)

For tidal drift, when the time dependence of all the variables is represented by the exponential factor $\exp(-i\omega t)$, we have

$$\delta S = \frac{S_0}{i\omega} \text{ div } v$$

(12)

where $v$ and $\delta S$ are complex quantities.

For $\delta S$ we obtain the estimate

$$|\delta S| = \left| \frac{\eta_0 S_0}{\pi} \left( \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \right) \right| \sim 10^{-3}$$

(13)

For small changes in compaction, the parameters $\eta$ and $\zeta$ may be regarded as constant (with $S_0 = \text{ const}$). Then ignoring the advective transport and Reynolds stresses, we write the equation of motion of the ice cover in the form

$$\rho_l h_S \frac{\partial v}{\partial t} + \rho_l h_S \Omega \times v + \text{grad } \rho - \eta \nabla^2 v = f$$

(14)
Applying the divergence operator to this equation, we get

\[ \rho h S \frac{\partial}{\partial t} \text{div} \nu + \nabla^2 p = \rho h S (\Omega \cdot \text{rot} \nu) + \text{div} T \]  

(15)

where

\[ T = f + \eta \nabla^2 \nu \]

We expand the ice compression pressure \( p \), which is a function of compaction and drift velocity, in a Taylor series. In equilibrium, \( p = 0 \). Then, dropping higher-order derivatives, we obtain to first approximation

\[ p = \left( \frac{\partial p}{\partial S} \right)_{S_0} \delta S + \left( \frac{\partial p}{\partial \nu} \right)_{S_0} \delta \nu \]

(16)

The first term in this expression corresponds to the static component \( p_S \) and the second term to the dynamic component \( p_D \).

We thus obtain the following closing equation:

\[ p_S = \left( \frac{\partial p}{\partial S} \right)_{S_0} \delta S \]

(17)

The component \( p_D \) of ice compression is written in the form

\[ p_D = \left( \frac{\partial p}{\partial S} \right)_{S_0} \frac{dS}{dt} \nabla \nu \frac{dS}{dt} \]

Using (11) and taking for tidal drift \( \nu = \delta \exp (-i\omega t) \), we find

\[ p_D = \frac{S_0}{i\omega} \left( \frac{\partial p}{\partial S} \right)_{S_0} \text{div} \nu \]

(18)

Equating the coefficients before \( \text{div} \nu \) in (10) and (18), we get

\[ \zeta = \frac{iS_0}{\omega} \left( \frac{\partial p}{\partial S} \right)_{S_0} \]

(19)
which coincides with the bulk viscosity for periodic processes with characteristic times substantially less than the time of macroscopic relaxation [2].

We see from (17) and (19) that the derivative $(\partial p/\partial S)_{S_0}$ should be estimated before $p_S$ and $\zeta$ can be found. To this end, consider equation (15). Inserting $p$ and $p_S$ from (10) and (17) and using the continuity equation (11) without the advective term, we find

$$-\mu hS_0 \frac{\partial}{\partial x} \frac{\partial (hS)}{\partial t} + \left[ \left( \frac{\partial p}{\partial S} \right)_{S_0} + \frac{\zeta}{S_0} \cdot \frac{\partial}{\partial t} \right] \eta^2 (hS) =$$

$$= \rho_i hS (\Omega \cdot \text{rot} \mathbf{v}) + \text{div} \mathbf{T}$$  \hspace{1cm} (20)

The left-hand side of this expression is the Stokes equation for the propagation of damped compression waves in a continuous medium with internal friction. The right-hand side of (20) may be interpreted as representing distributed sources which excite compressive stresses in the ice cover. In a certain sense, this is analogous to the problem of sound generated by turbulence [11].

The viscous forces $\eta \nabla^2 \mathbf{v}$ [3, 10] constitute a relatively weak source of compression and account for local effects only. The previously dropped terms, including the Reynolds stresses and the component $\rho_i hS \text{rot} \mathbf{v} \times \mathbf{v}$ of the total rotational force, are also of limited local significance. The main sources of excitation of compressive stresses in ice covering a substantial water area are represented by the terms with the divergence of the external force vector and the component $\rho_i hS \Omega \times \mathbf{v}$ of the total rotational force.

The excitation of ice compression is thus determined not only by the boundary conditions of the problem, i.e., the coastline configuration. The physical processes mentioned above will excite compression in offshore ice in open water bodies. The compression in coastal areas is predominantly static, whereas the compression in offshore ice is a dynamic wave process. The compression wave is unstable, so that discontinuities are formed at right angles to the direction of motion. In
nature this corresponds to the formation of cracks and ice lanes alternated with strips of compact ice.

For periodic excitation of frequency \( \omega \), equation (20) without its right member describes the propagation of a damped compression wave which travels with the phase velocity

\[
c_c = \left[ \frac{1}{H} \frac{\partial \rho}{\partial S} \right]^{1/2}
\]

(21)

The propagation velocity of the compression wave in compact ice may also be determined using the relation between the velocity and the change in compaction, which follows from the condition of momentum conservation:

\[
\sigma_c = v \frac{\dot{S}}{\dot{S}}
\]

Using (13) in our case \( (v \sim 10 \text{ cm/sec}) \), we find

\[
\sigma_c \sim 10^4 \text{ cm/sec}
\]

(22)

which is much higher than the drift velocity and is equal, to orders of magnitude, to the velocity of the tidal wave. Using (4) and (22), we obtain from equation (21)

\[
\left( \frac{\partial \rho}{\partial S} \right)_{S_h} = \rho h S_0 c_c^2 \sim 10^7 \text{ g/cm}
\]

From (13) and (17) we have \( p_S \sim 10^4 \text{ g/cm} \). The corresponding term in the equation of motion (14) is of the same order of magnitude as the inertial and the Coriolis terms

\[
\text{grad } \rho_S \sim \frac{10^4}{10^8} = 10^{-4} \text{ g/cm}^2
\]

Local values of ice compression may be quite high, as is evident from the phenomenon of ice reefing. The reefing pressure \( p_r(h) \) may be estimated if the reefing process is treated as loss of stability of a compressed floating ice slab. Then
\[ p_r = \left( \frac{\rho_2 g E h^3}{12(1 - \nu^2)} \right)^{1/2} \approx 0.6 \cdot 10^6 \text{ g/cm} \]

where \( E = 4 \cdot 10^6 \text{ g/cm}^2 \) is the normal elasticity modulus of ice, \( \rho_2 g = 1.0 \text{ g/cm}^3 \) is the specific weight of water, \( \mu = 0.34 \) is the Poisson ratio for ice, and \( h = 100 \text{ cm} \) is the mean thickness of the ice fields.

The compression pressure enters the equation of motion (14) as \( \text{grad} \, p \). In case of reefing, this term may markedly exceed the local value of the external force vector. Therefore, during reefing, the local direction of drift of compact ice does not necessarily coincide with the direction of overall drift. These local perturbations, however, have an insignificant effect on the general circulation of ice in the reservoir.

If reefing involves interaction of large ice floes, the change in compaction \( \delta S \) is very small. The propagation velocity of the compression wave in this case may be quite high. Indeed, from (13) and (23) we have

\[ \left( \frac{\partial p}{\partial S} \right) \approx \frac{p_r}{\delta S} \approx 0.6 \cdot 10^6 \text{ g/cm} \]

From (21) we then have \( \sigma_c = 10^5 \text{ cm/sec} \), which is close in order of magnitude to the propagation velocity of a longitudinal elastic compression wave in a continuous ice slab. This accounts for the fact that ice reefing may occur almost simultaneously over large parts of the water body. The corresponding drift velocity is

\[ v = \frac{p_r}{\rho_1 h c_c} \approx 10^2 \text{ cm/sec} \]

which fits the results of observations.

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THE EFFECT OF LONG-PERIOD TIDES ON ICE DRIFT IN THE ARCTIC BASIN

Z. M. Gudkovich and S. N. Evdokimov

The series expansion of the lunisolar tidal potential contains a group of terms with periods ranging from a few days to a few years. The corresponding components of the World Ocean tides are known as long-period tides. Considerable attention has been paid recently paid to studies of long-period tides [3,4,7,8,9]. Of particular interest for application in hydrologic and ice forecasts is the "monthly" group of long-period tides, which includes the monthly, semimonthly, nine-day, and seven-day components.

Maksimov [7] and Vorob'ev [3,4] observed marked fluctuations of sea level in the Arctic with monthly and semimonthly periods. Analysis of ice drift observations in the Arctic basin led Maksimov to the conclusion that the tidal, or "astronomical," drift in the Central Arctic constitutes one of the components of the total ice drift and is responsible for the loop movement of ice, especially in low wind periods. The existence of semimonthly and monthly variations in currents and drift was established by an analysis of buoy station observations and of radio beacon drift in the Arctic seas [4].

The amplitude of current and drift velocities due to the long-period tides is relatively small, and the particular methodological aspects of measurements acquire special significance. Shuster's version of the harmonic period analysis is generally used. The wind component generally is not excluded from the observation data. However, wind changes associated directly with synoptic processes may also reveal a certain quasi-periodicity, with periods which are (accidentally or regularly) close to the periods of the long-wave tides [10]. Therefore, the components associated with synoptic processes in the atmosphere, and in the particular the wind component, should be eliminated from the observation data when studying the long-period tidal effects in the World Ocean.
Our purpose was to develop the relevant methodological aspects for the case of semimonthly and monthly tides and their effect on ice drift in the Arctic basin. The basic data were provided by the Severnyi poliyus-15 [North Pole-15] observations for the 140-day period between 10 May and 27 September 1966. Sufficiently detailed and accurate observations of ice drift and wind were carried out in this period. The main ice drift observations consisted of astronomical determination of the station coordinates by means of theodolites, according to Bushuev's program [2], which calls for twice-daily observations using four stars with azimuths differing by approximately 90°. When meteorological conditions precluded star sightings, the coordinates were determined relative to the Sun; the position lines were reduced to a common time with allowance for the wind data and the average relations between drift and wind.

The wind data were provided by the Severnyi poliyus-15 anemograph recordings; the anemograph sensor was mounted on a standard 6 m mast. The astronomical observations were processed on a URAL-2 computer, which computed the coordinates of the astronomical points and the characteristics of the resultant drift between these points: direction and magnitude of drift, drift velocity, and velocity projections on two perpendicular axes. The ordinate axis was identified with the northerly direction of the 90°E meridian, and the easterly axis perpendicular to this meridian was chosen as the abscissa. The computer program was compiled by I. V. Korshakov and O. M. Tolshina.

Since the time intervals between the observations did not coincide with calendar days and were of different lengths, the daily average drift data needed for our analysis were obtained by the following method.

Each component of drift velocity between the astronomical points was plotted on a graph in the form of a stepped line. The abscissa axis gave the time and the ordinate the projection of the drift velocity. The hour-by-hour values of the velocity projections read off the graph were used to

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*The time interval between successive observations varied from 8 to 76 hours, with an average of 23 hours.
determine the daily average drift-velocity components \((U_x, U_y)\). The daily average wind-velocity components \((V_x, V_y)\) were obtained by a similar method. The hourly wind values read off the anemograph tapes were first tentatively used to compute the direction and the velocity of the resultant wind for the time intervals between successive observations. The wind direction was reduced to the meridian 90°E, and the corresponding wind projections were determined.

Before we proceed with a description of the actual procedures and results, we would like to give a brief description of the general drift of the Severnyi polus-15 station during the relevant observation period. The initial position of the station was 79°09'N and 168°34'E; the final position was 80°49'N and 173°00'E. The resultant NNW displacement was 208 km, but the actual drift of the station followed a fairly complex pattern: the velocity and the direction of drift changed repeatedly. The overall distance traveled by the floe on which the station was located was 920 km; i.e., the daily mean velocity of the floe was 6.6 km/day (7.6 cm/sec), whereas the tortuosity (the ratio of resultant velocity to mean velocity) for the entire period was only 0.23. A considerable part of the total distance was accounted for by loop-shaped evolutions. The maximum drift velocity between astronomical points reached 24.6 cm/sec, but in 75 percent of the cases it did not exceed 10 cm/sec.

The average wind speed for the entire period was 4.0 m/sec, the maximum speed reaching some 11 m/sec. In all cases, strong winds coincided with maximum drift velocities. During the periods of nearly zero resultant wind, the station moved in a northerly direction with a speed of about 2 cm/sec. This drift is due to the steady Trans-Arctic Current.

Harmonic analysis of the 140-day series of the drift components \(U_x\) and \(U_y\) yielded the characteristics of the three astronomical drift components: the nine-day component \(M_9\), the fortnightly or semimonthly component \(M_f\), and the four-week or monthly component \(M_m\). These characteristics are listed in Table 1.

The data of Table 1 were used to construct the ellipses of the horizontal components of these waves (Figure 1). We see from Figure 1 and
### TABLE 1.
"ASTRONOMICAL" DRIFT COMPONENTS OF THE SEVERNYI POLYUS-15 STATION.

$(\phi_{av} = 79^\circ 42'N, \lambda_{av} = 174^\circ 00'E)$. Angles $\phi$ and $\tau$ are cited relative to first day of observations, angle $\gamma$ relative to $90^\circ E$.

<table>
<thead>
<tr>
<th>Wave Period</th>
<th>$\phi_y$ (cm/sec)</th>
<th>$\phi_x$ (cm/sec)</th>
<th>$R_y$ (cm/sec)</th>
<th>$R_x$ (cm/sec)</th>
<th>$\tau$</th>
<th>$\gamma$</th>
<th>$U_{max}$ (cm/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9 days</td>
<td>330°</td>
<td>0.88</td>
<td>272°</td>
<td>1.80</td>
<td>280°</td>
<td>70°</td>
<td>1.85</td>
</tr>
<tr>
<td>14 days</td>
<td>36</td>
<td>3.00</td>
<td>304</td>
<td>1.20</td>
<td>38</td>
<td>358</td>
<td>3.00</td>
</tr>
<tr>
<td>28 days</td>
<td>125</td>
<td>1.60</td>
<td>222</td>
<td>0.17</td>
<td>128</td>
<td>357</td>
<td>1.60</td>
</tr>
</tbody>
</table>

Table 1 that the largest amplitude is that of the semimonthly drift wave, which reaches 3 cm/sec. The nine-day and the monthly components are significantly smaller. The orientation of the semimonthly and monthly drift ellipses is almost coincident, whereas the major axis of the nine-day component is turned through an angle of 72-73° compared to the other two. The drift vector in the nine-day and semimonthly cycles turns counterclockwise, whereas the monthly drift vector rotates clockwise.

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**Fig. 1.** Ellipses of periodic ice drift components:
- a) monthly,
- b) semimonthly,
- c) nine-day.

Roman numerals on this and following figures identify the successive days of each wave.
On the whole, the direction and the magnitude of the maximum velocity of the three components coincide with Maksimov's results [7] for this region; the observed differences do not exceed the natural variability of the characteristics. The data of Table 1 thus appear to indicate that the maximum speed of the three astronomical drift components is between 20 per cent (for the monthly drift) and 40 per cent (for the semimonthly drift) of the daily average total drift, and if the three components are in phase their resultant may reach the daily average total drift value.

Ice drift in the Arctic basin is made up of two principal components [1,5]: the wind component, associated with the direct instantaneous action of the wind in the particular region; and the "wind-independent" component, associated mainly with the effect of currents which are independent of the local instantaneous wind. The first component is decisive for relatively short-time intervals (days, ten-day periods), whereas the second component is significant for the general drift of ice over long periods (season, year, etc.). The astronomical drift component associated with the long-period tidal currents is essentially wind-independent, and it should therefore produce corresponding variations in the wind-independent drift component. Wind variations, in their turn, should affect the characteristics of the astronomical drift.

To explore this question, the daily average wind speed components for the relevant time period were subjected to harmonic analysis. The results are listed in Table 2.

<table>
<thead>
<tr>
<th>Wave Period</th>
<th>$\phi_y$</th>
<th>$R_y$ (m/sec)</th>
<th>$\phi_x$</th>
<th>$R_x$ (m/sec)</th>
<th>$\tau$</th>
<th>$\gamma$</th>
<th>$V_{max}$ (m/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9 days</td>
<td>272°</td>
<td>0.76</td>
<td>231°</td>
<td>0.39</td>
<td>260°</td>
<td>25°</td>
<td>0.80</td>
</tr>
<tr>
<td>14 days</td>
<td>45</td>
<td>0.70</td>
<td>269°</td>
<td>0.72</td>
<td>64</td>
<td>315</td>
<td>0.90</td>
</tr>
<tr>
<td>28 days</td>
<td>105</td>
<td>0.90</td>
<td>318°</td>
<td>0.32</td>
<td>109</td>
<td>342</td>
<td>0.94</td>
</tr>
</tbody>
</table>
The ellipses of these wind components are shown in Figure 2. There is a marked likeness to the corresponding drift ellipses: the maximum speed phases almost coincide, the direction of the maximum drift velocities is deflected to the right from the direction of the maximum wind speeds through an angle which is close to the average angle between the directions of wind drift and wind. The ratio of the maximum and the minimum drift and wind velocities corresponds to the usual values of the wind coefficient of ice drift in the Arctic basin in summer. A comparison of these characteristics is given in Table 3. In two of the three cases, the drift and wind vectors rotate in the same sense (counterclockwise); they rotate in opposite senses for the monthly wave only. For the monthly component, however, the ellipse is extremely flattened, and the drift is virtually reduced to reversing movement. These factors indicate that much of the astronomical drift should

![Fig. 2. Ellipses of the periodic wind components. (a) monthly, (b) semimonthly, (c) nine-day.](image)

TABLE 3.

<table>
<thead>
<tr>
<th>Wave Period</th>
<th>$T_{U-V}$</th>
<th>$Y_{U-Y}$</th>
<th>$U_{max}$</th>
<th>$U_{min}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>9 days</td>
<td>20°</td>
<td>45°</td>
<td>0.023</td>
<td>0.028</td>
</tr>
<tr>
<td>14 days</td>
<td>-26°</td>
<td>43°</td>
<td>0.034</td>
<td>0.030</td>
</tr>
<tr>
<td>28 days</td>
<td>19°</td>
<td>15°</td>
<td>0.017</td>
<td>0.012</td>
</tr>
<tr>
<td>Average</td>
<td>4°</td>
<td>34°</td>
<td>0.025</td>
<td>0.023</td>
</tr>
</tbody>
</table>
be attributed to the corresponding quasi-periodicity of the wind, which clearly deserves special study.

To elucidate the true characteristics of long-period tidal currents in the Arctic basin, we used the correlation method for the computation of the wind-independent drift component [6] for various tide phases, followed by harmonic analysis of the results. In this method, the series of drift and wind components \((U_x, U_y, V_x, V_y)\) are partitioned into segments covering half the wave period. The phases of the odd and even segments differ by 180°. Separate correlation analysis of the combined odd and even series thus yields the characteristics of wind drift and wind-independent current for two opposite tide phases. These characteristics correspond to the middle of each half-period. Successively shifting the partition points for the even and odd half-periods, we can find the sought characteristics for each day.

The method was applied to compute the drift and current characteristics on a day-to-day basis for the semimonthly and monthly tidal waves. The computations were performed on a URAL-2 computer. The results of the computations for the semimonthly wave are presented in Tables 4 and 5. Table 4 gives the wind drift characteristics: the wind coefficients \(K\) and the angle \(\alpha\) between the wind drift and the wind for both drift components. The correlation coefficient \(r\) in the table expresses the closeness of the correlation between the corresponding drift and wind components. It follows from Table 4 that the wind drift characteristics are virtually independent of the tide phase. The mean value of the wind coefficient is 0.018; the average value of \(\alpha\) is 40°.

Table 5 lists the components \(C_x\) and \(C_y\) and the magnitude \(C\) of the current speed; the direction \(D\) relative to 90°E is also given. We see from the table that the wind-independent drift components undergo fairly distinct oscillations during the wave period: the current speed varies from 1.3 to 3.2 cm/sec, and its direction changes from 256° to 290°. The mean current speed is 2.2 cm/sec, and the average direction is 277°, which is consistent with the standard data on the permanent currents in the observation region.
### TABLE 4

DAY-BY-DAY WIND DRIFT CHARACTERISTICS OF THE SEMIMONTHLY TIDAL WAVE

<table>
<thead>
<tr>
<th>Day</th>
<th>$K_x$</th>
<th>$K_y$</th>
<th>$\alpha_x$</th>
<th>$\alpha_y$</th>
<th>$r_x$</th>
<th>$r_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.018</td>
<td>0.018</td>
<td>36°</td>
<td>40°</td>
<td>0.88</td>
<td>0.89</td>
</tr>
<tr>
<td>1-e</td>
<td>0.019</td>
<td>0.019</td>
<td>47</td>
<td>39</td>
<td>0.89</td>
<td>0.88</td>
</tr>
<tr>
<td>2-e</td>
<td>0.018</td>
<td>0.018</td>
<td>44</td>
<td>40</td>
<td>0.89</td>
<td>0.85</td>
</tr>
<tr>
<td>3-e</td>
<td>0.018</td>
<td>0.018</td>
<td>44</td>
<td>39</td>
<td>0.88</td>
<td>0.86</td>
</tr>
<tr>
<td>4-e</td>
<td>0.018</td>
<td>0.017</td>
<td>42</td>
<td>40</td>
<td>0.88</td>
<td>0.84</td>
</tr>
<tr>
<td>5-e</td>
<td>0.019</td>
<td>0.018</td>
<td>38</td>
<td>40</td>
<td>0.90</td>
<td>0.88</td>
</tr>
<tr>
<td>6-e</td>
<td>0.019</td>
<td>0.018</td>
<td>38</td>
<td>40</td>
<td>0.88</td>
<td>0.88</td>
</tr>
<tr>
<td>7-e</td>
<td>0.018</td>
<td>0.018</td>
<td>45</td>
<td>40</td>
<td>0.88</td>
<td>0.91</td>
</tr>
<tr>
<td>8-e</td>
<td>0.019</td>
<td>0.016</td>
<td>36</td>
<td>43</td>
<td>0.88</td>
<td>0.90</td>
</tr>
<tr>
<td>9-e</td>
<td>0.018</td>
<td>0.017</td>
<td>39</td>
<td>42</td>
<td>0.86</td>
<td>0.94</td>
</tr>
<tr>
<td>10-e</td>
<td>0.019</td>
<td>0.018</td>
<td>39</td>
<td>40</td>
<td>0.88</td>
<td>0.89</td>
</tr>
<tr>
<td>11-e</td>
<td>0.018</td>
<td>0.017</td>
<td>37</td>
<td>41</td>
<td>0.84</td>
<td>0.92</td>
</tr>
<tr>
<td>12-e</td>
<td>0.018</td>
<td>0.017</td>
<td>40</td>
<td>41</td>
<td>0.83</td>
<td>0.88</td>
</tr>
<tr>
<td>13-e</td>
<td>0.018</td>
<td>0.018</td>
<td>41</td>
<td>40</td>
<td>0.86</td>
<td>0.92</td>
</tr>
</tbody>
</table>

### TABLE 5

DAY-BY-DAY CHARACTERISTICS OF THE WIND-INDEPENDENT DRIFT FOR THE SEMIMONTHLY TIDAL WAVE

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>0</th>
<th>1-e</th>
<th>2-e</th>
<th>3-e</th>
<th>4-e</th>
<th>5-e</th>
<th>6-e</th>
<th>7-e</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_x$, cm/sec</td>
<td>-3.0</td>
<td>-3.0</td>
<td>-3.1</td>
<td>-2.8</td>
<td>-2.4</td>
<td>-1.9</td>
<td>-1.7</td>
<td>-1.5</td>
</tr>
<tr>
<td>$C_y$, cm/sec</td>
<td>0.4</td>
<td>0.7</td>
<td>0.9</td>
<td>1.0</td>
<td>0.8</td>
<td>0.3</td>
<td>0.3</td>
<td>0.0</td>
</tr>
<tr>
<td>$C_z$, cm/sec</td>
<td>3.0</td>
<td>3.1</td>
<td>3.2</td>
<td>3.0</td>
<td>2.5</td>
<td>1.9</td>
<td>1.7</td>
<td>1.5</td>
</tr>
<tr>
<td>$D$, degree</td>
<td>277</td>
<td>283</td>
<td>286</td>
<td>290</td>
<td>288</td>
<td>278</td>
<td>280</td>
<td>279</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>8-e</th>
<th>9-e</th>
<th>10-e</th>
<th>11-e</th>
<th>12-e</th>
<th>13-e</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_x$, cm/sec</td>
<td>-1.6</td>
<td>-1.3</td>
<td>-1.5</td>
<td>-2.0</td>
<td>-2.3</td>
<td>-2.4</td>
<td>-2.2</td>
</tr>
<tr>
<td>$C_y$, cm/sec</td>
<td>-0.2</td>
<td>-0.3</td>
<td>-0.4</td>
<td>-0.1</td>
<td>-0.1</td>
<td>-0.4</td>
<td>-0.3</td>
</tr>
<tr>
<td>$C_z$, cm/sec</td>
<td>1.6</td>
<td>1.6</td>
<td>1.6</td>
<td>2.0</td>
<td>2.3</td>
<td>2.4</td>
<td>2.2</td>
</tr>
<tr>
<td>$D$, degree</td>
<td>263</td>
<td>257</td>
<td>256</td>
<td>273</td>
<td>272</td>
<td>273</td>
<td>277/2</td>
</tr>
</tbody>
</table>

The observation variation of the current points to a significant effect of the semimonthly and monthly tides. The characteristics of the monthly and semimonthly tidal currents derived by harmonic analysis of the current velocity components for each day are listed in Table 6. Figure 3 presents the current ellipses.
TABLE 6
CHARACTERISTICS OF SEMIMONTHLY (M) AND MONTHLY (Mm) TIDAL CURRENTS

<table>
<thead>
<tr>
<th>Wave</th>
<th>$\phi_x$ (deg)</th>
<th>$P_x$ (cm/sec)</th>
<th>$\phi_y$ (deg)</th>
<th>$P_y$ (cm/sec)</th>
<th>$\tau$ (deg)</th>
<th>$K$ (cm/sec)</th>
<th>$Y$ (cm/sec)</th>
<th>$\overline{C_{max}}$ (cm/sec)</th>
<th>$C_{max}$ (cm/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>215</td>
<td>0.85</td>
<td>46</td>
<td>0.53</td>
<td>38</td>
<td>270</td>
<td>302</td>
<td>1.0</td>
<td>1.5</td>
</tr>
<tr>
<td>Mm</td>
<td>275</td>
<td>0.58</td>
<td>144</td>
<td>0.31</td>
<td>103</td>
<td>244</td>
<td>292</td>
<td>0.6</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Note that this method does not yield directly the speed of the maximum tidal current $C_{max}$. It only gives the average velocity for one half-period, covering a high or a low of a tidal cycle, i.e.,

$$\overline{C_{max}} = \frac{1}{\pi} \int_0^\pi C_{max} \sin x$$

where $x=qt$ ($q$ is the angular wave velocity, $t$ is time), whence

$$C_{max} = \frac{\pi}{2} \overline{C_{max}} \approx 1.5 \overline{C_{max}}$$

The true maximum velocity of the tidal semimonthly current is 1.5 cm/sec, and that of the monthly current is 0.9 cm/sec; i.e., it is approximately half the corresponding periodic components of the total drift. The resultant displacement of a floe in one direction due to the monthly or the semimonthly tides thus should not exceed 10-12 km, and the maximum average daily velocity of the total monthly or semimonthly drift hardly attains one-third of the average daily velocity of the wind drift.

We see from Table 6 and Figure 3 that the major axes of the ellipses are oriented close to the average direction of the Trans-Arctic Current, markedly deviating from the orientation of the major axes of the corresponding general drift ellipses. Unlike the semimonthly component of the general drift, the semimonthly (and also the monthly) tidal current turns clockwise during the tidal cycle. This is more characteristic of such
phenomena in the Northern Hemisphere. Table 6 also lists the position angles $K$ reduced to common astronomical time, corresponding to tidal maxima:

For the semimonthly wave $M_f$ . . . . . . . . . $2S = 0^\circ$

For the monthly wave $M_m$ . . . . . . . . . . . $S - P = 0^\circ$

Here $S$ is the mean longitude of the Moon, $P$ is the longitude of the perigee of the lunar orbit.

![Fig. 3. Ellipses of long-period tidal currents: a) monthly $M_m$, b) semimonthly $M_f$.](image)

According to this theory, the long-period tides have the form of a standing wave with antinodes on the Equator and in the polar regions, and nodes around $35^\circ$N and $35^\circ$S. The phase of water-level oscillations at high latitudes shows a $180^\circ$ phase shift relative to the tidal force phase. Maximum speeds of tidal currents are observed in the region of the nodal lines, whereas near the poles the speeds are low. The phase of maximum currents flowing from the Equator to the poles lags $90^\circ$ behind the force, and the phase difference for the reverse currents is $270^\circ$.

Although the real phenomenon may differ markedly from the idealized theoretical scheme due to the effect of large land masses and complex bottom topography, the observed current patterns, in our opinion, confirm the theoretical scheme. We see from Table 6 that the phase of the maximum currents flowing toward the Atlantic is close to $270^\circ$ in both cases.
Quasi-periodic variation of the wind may thus introduce significant distortions in the characteristics of the astronomical drift as derived by harmonic analysis. The proposed method almost completely eliminates the effect of wind on ice drift.

BIBLIOGRAPHY


ICE DRIFT IN AN INHOMOGENEOUS PRESSURE FIELD

K. L. Egorov

Observations of polar expeditions reveal that the drift of an ice floe surrounded by other floes is markedly different from the pure wind-driven drift of an isolated floe. The difference stems from the interaction of floes which move with a certain relative velocity. The interaction is caused by the inhomogeneous field of wind speeds, the nonuniform distribution of the ice floes according to horizontal extent and vertical thickness, and differences in ice surface roughness. Various authors who have worked on the theory of ice drift have had to advance certain hypotheses to account for and quantitatively estimate the effect of floe interaction on the total drift.

The floe interaction factor was first introduced by Sverdrup [5--
reference not included in the Russian bibliography], who described this interaction force as a certain function of the ice drift speed. Sverdrup postulated that the drag induced by the interaction between the ice floes is proportional to the ice drift velocity and is directed against the motion of ice. Using his function, Sverdrup showed that the introduction of collisions between ice floes may reduce the angle of deviation of ice drift from the direction of the wind near the surface. Gevorkyan [1] noted that the frictional forces between floes constitute a highly uncertain factor which is difficult to determine experimentally and tried to establish the theoretical form of the function characterizing the interaction of ice. Assuming that part of the kinetic energy acquired by the floe through interaction with the moving air stream is lost in collisions with other ice floes, the author derived a quadratic dependence of the frictional forces on the drift velocity, with a coefficient dependent on the physical parameters of the ice cover.

Gevorkyan's treatment of ice floe interaction, like Sverdrup's approach, links the interaction forces with the mean drift velocity and does not reflect drift inhomogeneity. The hypothesis advanced by Laikhtman [3] seems
more satisfactory in this respect. Laikhtman treats the ice cover as a film of viscous fluid, with groups of floes or individual floes simulating the fluid molecules. The lateral friction between the floes is introduced by additional terms in the equations of motion of ice, which are analogous to the viscous terms in a turbulent flow. By [3], the equations of motions of ice, without gradient currents in the sea, have the form

\[
\begin{align*}
\lambda m v_0 + \tau_x + \tau_y + k_0 m \left( \frac{\partial u_0}{\partial x^2} + \frac{\partial u_0}{\partial y^2} \right) &= 0; \\
-\lambda m u_0 + \tau_y + \tau_x + k_0 m \left( \frac{\partial v_0}{\partial x^2} + \frac{\partial v_0}{\partial y^2} \right) &= 0,
\end{align*}
\]

(1) (2)

where \( \lambda = 2\omega z \) is the Coriolis parameter,

\( m \) is the mass of ice per unit surface area,

\( k_0 \) is the coefficient of lateral friction of ice,

\( u_0, v_0 \) are the drift velocity components of ice,

\( \tau_x, \tau_y, \tilde{\tau}_x, \tilde{\tau}_y \) are the components of the tangential stresses at the air-ice and ice-water interfaces, respectively.

Having determined the tangential stresses from the equations of motion in the boundary layers of the atmosphere and the sea under the ice, for vertically constant turbulence coefficients \( k \) and \( \tilde{k} \), Laikhtman obtained an equation for the complex velocity \( \bar{W}_0 = u_0 + i v_0 \):

\[
A W_0 - k_0 \left( \frac{\partial W_0}{\partial x^2} + \frac{\partial W_0}{\partial y^2} \right) = B \bar{G},
\]

(3)

where

\[
\begin{align*}
A &= \frac{1}{m} \left[ \kappa \rho \alpha + i \left( m \lambda + \kappa \tilde{\rho} \alpha \right) \right]; \\
B &= \frac{k \rho \alpha}{m} (1 + i); \\
\bar{G} &= U + iV.
\end{align*}
\]

(4)

Here

\[
\begin{align*}
a &= \sqrt{\frac{\omega_z}{k}}; \\
\tilde{a} &= \sqrt{\frac{\omega_z}{\tilde{k}}}.
\end{align*}
\]

\( U, V \) are the geostrophic wind components.
Ruzin [4] obtained a solution of this equation for ice drift in an arbitrary wind field in the open sea:

\[ W_0 = \frac{B}{A} \sum_{n=0}^{\infty} \left( \frac{k_0^n}{A^n} \right) \Delta^n G(x, y), \]  

(5)

where

\[ \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}. \]

We will use this solution to analyze the effect of floe interaction on ice drift for a typical pressure system of the Central Arctic, which is described by the following relation [2] (Figure 1):

\[ \rho = P_0 \sin \frac{2\pi x}{a} \cdot \sin \frac{2\pi y}{b}. \]  

(6)

![Figure 1](image)

Then

\[ U = \frac{1}{k} \cdot \frac{\partial P}{\partial y} = \frac{P_0}{k} \cdot \frac{2\pi}{b} \sin \frac{2\pi}{a} x \cdot \cos \frac{2\pi}{b} y; \]

\[ V = \frac{1}{k} \cdot \frac{\partial P}{\partial x} = \frac{P_0}{k} \cdot \frac{2\pi}{a} \cos \frac{2\pi}{a} x \cdot \sin \frac{2\pi}{b} y. \]
Separating solution (5) into the real and the imaginary part, we find

\[ u_0 = \sum_{n=0}^{\infty} \frac{k_0^n}{A^{2(n+1)}} (M_{n+1} \Delta^n U - N_{n+1} \Delta^n V); \]  

\[ v_0 = \sum_{n=0}^{\infty} \frac{k_0^n}{A^{2(n+1)}} (M_{n+1} \Delta^n V + N_{n+1} \Delta^n U), \]

where

\[ |A|^2 = (\text{Re} \ A)^2 + (\text{Im} \ A)^2; \]

\[ M_n = \text{Re} B; \quad N_n = \text{Im} B; \]

\[ M_{n+1} = \text{Re} A + N_n \text{Im} A; \]

\[ N_{n+1} = \text{Re} A - M_n \text{Im} A. \]

Since

\[ \Delta^n U = (-1)^n \left( \frac{2\pi}{ab} \right)^{2n} (a^2 + b^2)^n U; \]

\[ \Delta^n V = (-1)^n \left( \frac{2\pi}{ab} \right)^{2n} (a^2 + b^2)^n V, \]

we replace (7) and (8) with

\[ u_0 = \frac{1}{|A|^2} \sum_{n=0}^{\infty} (-1)^n \left( \frac{2\pi}{|A| ab} \right)^{2n} (a^2 + b^2)^n k_0^n (M_{n+1} U - N_{n+1} V); \]

\[ v_0 = \frac{1}{|A|^2} \sum_{n=0}^{\infty} (-1)^n \left( \frac{2\pi}{|A| ab} \right)^{2n} (a^2 + b^2)^n k_0^n (M_{n+1} V + N_{n+1} U). \]

Here \( k_0/|A| ab << 1 \), so that the series converge very rapidly, as \( (k_0/|A| ab)^n \), and with good accuracy we need only retain the first two terms of the expansion. The first terms in the series constitute the components of the pure wind-driven drift for noninteracting floes.

Using expressions (10) and (11), let us now analyze the effect of floe interaction forces on the angle between the direction of drift and the direction of the geostrophic wind (the isobar). We will use only the first two terms of the expansions (10) and (11). Introducing the notation

\[ \mu = 2k_0 \left( \frac{2\pi}{|A| ab} \right)^2 (a^2 + b^2), \]

we obtain

\[ \tan \varphi = \frac{v_0}{u_0} = \frac{V}{U} \times \]

\[ \frac{M_1 + \frac{U}{V} N_1 - \mu \frac{k_0 a}{m} \left( \frac{\text{Re} \ A)^2 - \frac{13}{2}}{2} - \frac{U}{V} \left( \frac{\text{Im} \ A)^2 - \frac{12}{2}}{2} \right) \right)}{M_1 - \frac{V}{U} N_1 - \mu \frac{k_0 a}{m} \left( \frac{\text{Re} \ A)^2 - \frac{13}{2}}{2} + \frac{V}{U} \left( \frac{\text{Im} \ A)^2 - \frac{12}{2} \right) \right)}. \]
Separating solution (5) into the real and the imaginary part, we find
\[ \tan \alpha = \frac{V}{U} = \pm 1; \quad \tan \alpha = 0; \quad \cot \alpha = 0. \] (13)

a. For \( \tan \alpha = 1, \)
\[ \tan \varphi = \frac{\text{Re} A}{\text{Im} A} \cdot \frac{1 + \mu \left( \frac{\lambda}{2 \text{Re} A} + 1 \right)}{1 - \mu \text{Re} A} = \tan \varphi_0 \beta_1, \]
where \( \tan \varphi_0 = \frac{\text{Re} A}{\text{Im} A} ; \ \varphi_0 \) is the value of \( \phi \) for noninteracting floes; \( \text{Re} A > 0, \)
so that \( \beta_1 > 1, \ \tan \varphi > \tan \varphi_0. \)

b. For \( \tan \alpha = -1, \)
\[ \tan \varphi = - \frac{\text{Im} A}{\text{Re} A} \cdot \frac{1}{\beta_2}; \quad |\tan \varphi| < |\tan \varphi_0|. \]

c. For \( \tan \alpha = 0, \)
\[ \tan \varphi = \frac{\text{Re} A - \text{Im} A}{\text{Re} A + \text{Im} A} \cdot \frac{1 + \mu \left( \frac{(\text{Im} A)^2 - \frac{\lambda^2}{2}}{(\text{Re} A)^2 - \frac{\lambda^2}{2}} \right)}{1 - \mu \left( \frac{(\text{Re} A)^2 - \frac{\lambda^2}{2}}{(\text{Re} A)^2 - \frac{\lambda^2}{2}} \right)} = \tan \varphi_0 \beta_2, \]
where \( \tan \varphi_0 = \frac{\text{Re} A - \text{Im} A}{\text{Re} A + \text{Im} A} ; \)
\( \beta_2 < 1, \) so that \( |\tan \varphi| < |\tan \varphi_0|. \)

d. For \( \cot \alpha = 0, \)
\[ \cot \varphi = \cot \varphi_0 \beta_2; \quad |\cot \varphi| < |\cot \varphi_0|. \)

According to the results of [3], ice drift is deflected to the right from the direction of the geostrophic wind. Conditions a, b, c, d indicate that in all four cases the interaction forces act to reduce the angle of deflection of ice drift from the isobar. Calculations carried out for \( \lambda = 1.4 \times 10^{-6} \ \text{sec}^{-1}, m = 2 \times 10^2 \ \text{g/cm}, k = 17 \times 10^6 \ \text{cm}^2/\text{sec}, k = 2 \times 10^2 \ \text{cm}^2/\text{sec}, \)
\( k_0 = 3 \times 10^{10} \ \text{cm}^2/\text{sec}, a = 2 \times 10^8 \ \text{cm} \) show that the angle between the direction of ice drift and the isobar decreases from 6° to 2° when the interaction
forces are introduced. Since \( \tan \alpha \) has no singularities at the intermediate points, we conclude that at all the points of the pressure field (6) lateral floe friction acts to bring the direction of ice drift closer to the isobar.

Note that the angle between the pure wind-driven drift and the isobar, in accordance with the boundary layer model used in this analysis, is independent of the geostrophic wind speed. Our results therefore provide only an indication of the qualitative effect of the lateral ice friction on the total drift.

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5. [Not given in the original.]
WAVE DRIFT OF AN ISOLATED FLOE

A. I. Arikainen

The various aspects of the theory of ice-cover drift have not been thoroughly studied. A number of papers in Soviet and Western literature cover this topic (F. Nansen, H. Sverdrup, R. G. Gevorkyan, V. V. Shuleikin, M. E. Shvets, D. L. Lakhtman, and others). The most recent publications include the work of Gudkovich and Nikiforov [1] and Timokhov [6].

A common assumption of these authors is that sea ice is made to drift by wind and sea currents. This, however, is valid for fairly close ice only. In the case of an isolated floe near the ice edge or in clear water, the above forces, in our opinion, are reinforced by a definite contribution from sea waves.

Our aim in this paper is to allow for the possible effect of the hydrodynamic impact of sea waves on an isolated ice floe, i.e., to derive a formula which would represent both qualitatively and quantitatively the action of waves on an ice floe. The resultant drift of a floe is assumed to consist of two components: wave and wind. This treatment of two separate components implies that the wave drift component is independent of the wind drift and vice versa.

Waves apparently have a twofold effect on the displacement of the floe:

1. Displacement in the wave current, whose velocity is expressed by the standard relation [2]

\[ V_{wc} = r_0^2 \left( \frac{2\pi}{\lambda} \right)^2 C, \]  

(1)

where \( r_0 \) is the radius of the orbit of a particle on the sea surface (clearly, \( r_0 \) is equal to half the wave height \( h \) on the surface), \( \lambda \) is the wavelength, and \( C \) is the phase velocity of the waves. Computations based
on relation (1) are plotted in Figure 1.

![Graph showing wave and wind components of the resultant drift of an isolated floe](image)

**Fig. 1.** Wave and wind components of the resultant drift of an isolated floe ($L = 100$ m, $B = 50$ m, $H = 2$ m, $\delta = 0.75$, $\rho_i = 0.92$ kg/m$^3$, $\rho_w = 1.03$ kg/m$^3$) as a function of wave height $h$ of 3% likelihood.

1--wind drift velocity
2--wave drift velocity
3--displacement by wave current

2. Motion due to the hydrodynamic impact of the waves: the wave proper.

The wave current displaces the ice floe because the wave orbits are open and, unlike the drift, this displacement constitutes absolute motion relative to the bottom. The floe thus drifts relative to the water and is transported by the current relative to the bottom. This factor is invariably ignored in the literature, hence our interest in the effect of the hydrodynamic impact of waves on the drift of an isolated floe. Our treatment is based on A. N. Krylov's hypothesis concerning the permeability of a floating object to wave energy. In the light of this hypothesis, the wave drift of a floe should be considered as the outcome of the interaction of the kinetic energy of the floe and the wave energy.

The equation of energy balance for a particle of unit mass is

$$
e = \frac{d}{dt} \left( \frac{V^2}{2} + gZ + jc_T \right) - \frac{1}{\rho} \cdot \frac{\partial \rho}{\partial t},$$

(2)

where $\frac{1}{\rho} \frac{\partial \rho}{\partial t}$ is the change in energy on compression, $j$ is the mechanical
equivalent of heat, \( c_p \) is the specific heat at constant pressure, \( T \) is the temperature, \( Z \) is the vertical displacement of the center of gravity of the particle, and \( g \) is the gravitational acceleration.

We introduce a number of assumptions:

1) \( \frac{1}{\rho} \frac{\partial \rho}{\partial t} = 0 \) --i.e., the liquid is essentially incompressible;

2) \( j \sigma_p T = 0 \) --i.e., the change in the temperature of the water and the ice as a result of internal friction and friction against the surface of the floe is neglected;

3) \( gZ \ll \frac{V^2}{2} \) --i.e., the greater part of the total flux of wave energy is expended in horizontal motion of the floe.

\( \frac{V^2}{2} \) is the energy imparted to unit mass of the ice floe by the hydrodynamic impact of waves.

According to our assumptions, we have the following relation for a drifting floe:

\[
\frac{1}{2} m V_w^2 = A, \tag{3}
\]

where \( A \) is the flux of wave energy relative to the floe, \( m \) is the floe mass, and \( V_w \) is the velocity of the floe produced by wave impact. In what follows, \( V_w \) will be called the wave drift velocity.

The total mean energy of a three-dimensional wave according to Yu. M. Krylov (the statistical theory of sea waves) [3] is given by

\[
E = \frac{3\pi}{128} g \rho h^2, \tag{4}
\]

where \( g \) is the gravitational acceleration and \( \rho \) is the density of sea water.

For the energy flux we have

\[
\overline{A} = \frac{3\pi}{128} g \rho h^2 \frac{C}{2} \cdot \frac{V_E^2}{1 - \Gamma \left( \frac{5}{4} \right)} \approx 0.96 \rho V_E E, \tag{5}
\]

where \( C \) is the phase velocity of a wind-driven wave, \( V_E \) is the rate of wave energy transport relative to the bottom, and \( \Gamma \) is the gamma function.
Relation (4) expresses the energy of the wavy sea surface within the layer of water churned by waves. The floe interacts with the wave energy confined to a layer which extends from the surface approximately to the depth of submergence $H$ of the floe.

Relation (4) now can be written in the form

$$E = \frac{3\pi}{128} \frac{g}{\rho} \bar{h}^2 \left[ 1 - \exp\left( -\frac{-2\pi \bar{H}}{\lambda} \right) \right].$$  \hspace{1cm} (6)

The total energy interacting with the floe is thus

$$E = \frac{3\pi}{128} \frac{g}{\rho} \bar{h}^2 \left[ 1 - \exp\left( -\frac{-2\pi \bar{H}}{\lambda} \right) \right] S,$$  \hspace{1cm} (7)

where $S$ is the mean area of the floe section interaction with the wave energy

$$S = \delta \bar{L} \bar{H}'.$$  \hspace{1cm} (8)

Here $\bar{H}'$ is the floe thickness, $\bar{L}$ is the floe length, $\delta$ is the area configuration coefficient of the floe, namely

$$\delta = \frac{\bar{L} \bar{H}'}{\bar{L}_{\text{max}} \bar{H}_{\text{max}}},$$  \hspace{1cm} (9)

where $\bar{L}, \bar{H}'$ are the mean dimensions of the floe, and $\bar{L}_{\text{max}}, \bar{H}_{\text{max}}$ are the maximum dimensions of the floe.

The rate of energy transport relative to the ice floe is expressed by

$$\bar{V}_E = \frac{1}{2} \bar{C} - \bar{V}_i,$$ \hspace{1cm} (10)

where $\bar{C}$ is the phase velocity of the wave, and $\bar{V}_i$ is the velocity of displacement of the floe on the sea surface due to the combined action of wind, waves, and currents.

Under real conditions, $C/2 \gg V_i$, so that the second term in (10) can be omitted.

Thus the general expression for the mean wave energy flux relative to the ice floe is

$$\bar{A}' = \frac{0.06 \cdot 3\pi}{128} \frac{g}{\rho} \bar{h}^2 \delta \bar{L} \bar{H}' \frac{C}{2} \left[ 1 - \exp\left( -\frac{2\pi \bar{H}}{\lambda} \right) \right].$$  \hspace{1cm} (11)
Making use of (11) to solve (3), we obtain for the wave drift velocity

\[ V_w = 0.035 \frac{\rho \lambda^2 H^2}{m} \sqrt{\frac{2\lambda}{2\pi} \left[ 1 - \exp \left( - \frac{2\pi H}{\lambda} \right) \right]} / \delta. \]  

(12)

The floe mass \( m \) is obtained from the relation

\[ m = \delta' \rho_1 LBH', \]  

(13)

where \( B \) is the width of the floe, \( \rho_1 \) is the density of sea ice, and \( \delta' \) is the volume configuration coefficient of the floe,

\[ \delta' = \frac{LBH'}{L_{\text{max}} B_{\text{max}}^2}. \]  

(14)

Here \( \bar{B} \) is the mean width of the floe, and \( B_{\text{max}} \) is the maximum width of the floe.

Relation (12) contains the mean wave height with 97\% likelihood and the modal wavelength \( \lambda_m \) corresponding to \( \bar{H} \).

Fixing the floe dimensions and the wave parameters, we can compute the contribution of the wave energy to the net displacement of the floe. The calculations were carried out using the average wave heights obtained by N. N. Davidan for the North Atlantic (see below):

<table>
<thead>
<tr>
<th>( h_{3%} ) (meters)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{h} ) (meters)</td>
<td>0.47</td>
<td>0.95</td>
<td>1.42</td>
<td>2.37</td>
<td>3.32</td>
<td>4.73</td>
</tr>
<tr>
<td>( \lambda_m ) (meters)</td>
<td>9.6</td>
<td>19.0</td>
<td>29.0</td>
<td>48.0</td>
<td>68.0</td>
<td>95.0</td>
</tr>
</tbody>
</table>

The results of these computations are shown in Figure 1.

Another important factor is the relationship between the wave and wind drift velocities, on the one hand, and the wave height and wind speed, on the other. The relationship between wave height and wind speed is borrowed from Krylov [4]. A nomogram has been used to find the mean wave height at a point with a known period \( \tau \) and wind speed \( \bar{W} \). The period \( \tau \) for a given wind speed was taken equal to its mean value. The wind drift coefficient was taken as 0.03 in these computations.
Figure 1 shows that the wave drift velocity $V_w$ (curve 2) is comparable with $V_{wd}$ (curve 1). As the wave height increases, the total contribution of waves $V_w + V_{wc}$ to floe motion increases and becomes approximately equal to the wind drift $V_{wd}$ for waves 7-10 meters in height.

In reality, the effect of wind on ice drift diminishes with increasing wave height because of the screening action of the wave, so that the function $V_{wd} = f(W)$ should be less steep than in Figure 1.

We recall that in our derivation of (12) the wave energy flux was assumed to be completely expended in horizontal displacement of the floe. In fact, this phenomenon is much more complex. A certain fraction of the energy (apparently a relatively minor one) is expended in excitation of vertical motion, roll and pitch, etc. The mass of the floe is another important factor affecting the results of the interaction between the waves and the floe. Small-volume floes and massive floes do not interact with waves in the same way. To allow for the above factors, a dimensionless coefficient $k = 0.7-0.9$ has to be introduced into (12). This correction does not necessarily mean that the results from (12) are wrong; but no experimental data are available which would enable us to assess the validity of (12). The computations of Rudyaev [5] dealing with the loss of speed of sea-going vessels under irregular wave conditions appear to justify our formula. Rudyaev's computations are also based on Krylov's hypothesis. The loss of speed computed from Rudyaev's formula shows adequate agreement with experimental results.

Previous studies of the drift of isolated floes have ignored the wave component of drift. Yet our results indicate that this component must not be neglected. Indeed, we see from Figure 1 that the wave drift velocity is comparable to the wind drift velocity, so that for waves traveling upwind (a rare phenomenon) the net displacement of a floe is zero. When $V_w$ and $V_{wd}$ point in the same direction, the floe velocity is doubled.

The above considerations fully apply to sailing ice near the clear water edge. The propagation velocity of waves in the ice-covered basin is lower than for the free surface, and the wavelength is shorter [7].

Given the wave elements near the clear water edge of ice, we can compute the effect of waves on the closeness of drifting floes and their
displacement. As a result of these effects, close ice is observed near the clear-water edge, whereas farther off the compaction of ice decreases. This pattern is generally observed on a relatively small scale, since the wave energy is a rapidly damping function of distance. The scale of the phenomenon is clearly sensitive to the compaction of the drifting ice. A direct proportionality is observed: the higher the ice compaction, the faster the wave energy damping.

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