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A NEW ARCTIC JOURNAL

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THE PHYSICAL MEANING OF TWO-DIMENSIONAL STRESSES
IN A FLOATING ICE COVER

by

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Introduction

This note tries to clarify a conceptual difficulty presented by two-dimensional models of sea-ice cover. The real ice cover is three-dimensional and the question is: What is the proper relationship of a two-dimensional model discussed, for example, by Campbell [1965], Rothrock [1973], and Glen [1970] to a three-dimensional one such as the ridging model of Parmerter and Coon [1972]? In particular, how should one relate the two-dimensional stresses in the models to the real three-dimensional stresses? This question needs to be answered before one can assess the physical plausibility of any proposed two-dimensional rheology.

Stress analysis

Let \( \sigma_{ij} \) denote stress components in three dimensions (tensile stresses positive). The \( z \) coordinate is vertical. Consider an ice cover of uniform density \( \rho_i \) floating on water of uniform density \( \rho_w \). The thickness of the ice is \( h(x,y) \) and the depth below the waterline is \( h'(x,y) \), where both \( h \) and \( h' \) are slowly varying functions of position (\( \partial h / \partial x \ll 1, \partial h / \partial y \ll 1, \partial h' / \partial x \ll 1, \partial h' / \partial y \ll 1 \)).

If no shear tractions act across vertical planes within the ice,

\[
\rho_i h = \rho_w h'.
\]
Equation (1) is an approximation that may be expected to hold at each point 
\((x,y)\) if the external forces do not change too rapidly and if the spatial 
variation of \(h\) and \(h'\) is sufficiently slow. It does not, of course, mean 
that the stress in the ice is hydrostatic.

The vertical integration of the momentum equation is conveniently 
done by considering the forces acting on a section of the ice bounded by a 
pair of parallel vertical planes perpendicular to \(Ox\) and a similar pair 
perpendicular to \(Oy\). From the balance of vertical forces, and again assuming 
no shear tractions across vertical planes, we have

\[
\bar{\sigma}_z = -\frac{1}{2} \rho_i gh,
\]

where bars will denote averages through the thickness. From the balance of 
horizontal forces (neglecting accelerations)

\[
\begin{align*}
\frac{\partial}{\partial x} (h\bar{\sigma}_x) + \frac{\partial}{\partial y} (h\bar{\tau}_{xy}) + \rho_w gh', \frac{\partial h'}{\partial x} + \tau_x &= 0, \quad (3) \\
\frac{\partial}{\partial x} (h\bar{\tau}_{xy}) + \frac{\partial}{\partial y} (h\bar{\sigma}_y) + \rho_w gh', \frac{\partial h'}{\partial y} + \tau_y &= 0, \quad (4)
\end{align*}
\]

where \(\tau_x, \tau_y\) are the components of the resultant horizontal forces per unit 
area, from wind and water, acting on the top and bottom surfaces of the ice. 
(We neglect the Coriolis force, but it can readily be added if necessary.) The terms in \(\partial h'/\partial x\) and \(\partial h'/\partial y\) are the main concern of this note. They 
arise from the pressure of the water acting upwards on the inclined under-
surface of the ice. If the thickness were uniform they would vanish. With 
the approximation (1) they become

\[
\rho_i gh \cdot \frac{\rho_i \partial h}{\rho_w \partial x} \quad \text{and} \quad \rho_i gh \cdot \frac{\rho_i \partial h}{\rho_w \partial y} .\quad (5)
\]

If the ice is modeled as a two-dimensional continuum the momentum 
equations, under the same assumptions as above, would be written (perhaps 
wrongly, as we shall see) as

\[
\frac{\partial \bar{\sigma}_x}{\partial x} + \frac{\partial \bar{\tau}_{xy}}{\partial y} + \tau_x = 0,
\]

\( (6) \)
\[ \frac{\partial T_{xy}}{\partial x} + \frac{\partial S_y}{\partial y} + \tau_y = 0, \tag{7} \]

where \( S_x, S_y, T_{xy} \) are two-dimensional stress components (forces per unit length). It is not immediately clear how one should relate the two-dimensional \( S \)'s to the three-dimensional \( \sigma \)'s. For example, the relation

\[
\begin{align*}
S_x &= h \bar{\sigma}_x, \\
S_y &= h \bar{\sigma}_y, \\
T_{xy} &= h \bar{\tau}_{xy},
\end{align*}
\]

would be inconsistent because it would not deal with the terms in \( \partial h/\partial x \) and \( \partial h/\partial y \).

To help clarify the question, let us first consider a model in which the (three-dimensional) ice behaves like a homogeneous incompressible viscous continuum of viscosity \( \eta \). Then

\[
\begin{align*}
\bar{\sigma}_x &= \sigma + \bar{\sigma}_x' = \sigma + 2\eta \frac{\partial u}{\partial x}, \\
\bar{\sigma}_y &= \sigma + \bar{\sigma}_y' = \sigma + 2\eta \frac{\partial v}{\partial y}, \\
\bar{\sigma}_z &= \sigma + \bar{\sigma}_z' = \sigma + 2\eta \frac{\partial w}{\partial z}, \\
\bar{\tau}_{xy} &= \eta \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right),
\end{align*}
\tag{8}
\]

and \( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \),

where \( u = u(x,y) \), \( v = v(x,y) \), \( w = w(z) \) are velocity components; \( \bar{\sigma}_x', \bar{\sigma}_y', \bar{\sigma}_z' \) are the mean values of stress deviators through the thickness; and

\[
\bar{\sigma} = \frac{1}{3} (\sigma_x + \sigma_y + \sigma_z).
\]

By differentiation, equation (2) may be written

\[
\frac{\partial}{\partial x} (h\bar{\sigma}_x) + \rho_i g \frac{\partial h}{\partial x} = 0, \tag{9}
\]
and
\[ \frac{\partial}{\partial y} (h \bar{\sigma}_y) + \rho g \frac{\partial h}{\partial y} = 0. \] (10)

Now subtract (9) and (10) from (3) and (4), using (5), to give
\[ \begin{align*}
\frac{\partial}{\partial x} \{ h \overline{\sigma}_x - \bar{\sigma}_x \} + \frac{\partial}{\partial y} \{ h \overline{\tau}_{xy} \} & - \frac{\partial}{\partial x} (h \sigma_y) + \tau_x = 0, \\
\frac{\partial}{\partial x} (h \overline{\tau}_{xy}) + \frac{\partial}{\partial y} \{ h(\overline{\sigma}_y - \bar{\sigma}_y) \} & - \frac{\partial}{\partial y} (h \sigma_y) + \tau_y = 0,
\end{align*} \] (11)

where we have written
\[ \sigma_q = \frac{1}{2} \rho g \left(1 - \frac{\rho g}{\rho_w} \right) gh. \] (13)

The use of (8) gives for the first term in (11)
\[ \frac{\partial}{\partial x} \{ 2h (2u \frac{\partial u}{\partial x} + \eta \frac{\partial v}{\partial y}) \}. \] (14)

Notice that since we have no equation for \( \frac{\partial}{\partial x} (h \bar{\sigma}_y) \) we are compelled to subtract \( \frac{\partial}{\partial x} (h \overline{\tau}_{xy}) \) from \( \frac{\partial}{\partial x} (h \overline{\sigma}_y) \) if \( \bar{\sigma} \) is to be eliminated. This is an essential point.

For a two-dimensional medium having a viscosity \( \eta_1 \) for area increase and \( \eta_2 \) for shear,
\[ S_x = S + S'_x = \eta_1 (\dot{E}_x + \dot{E}_y) + 2\eta_2 \dot{E}'_x, \]
where \( S = \frac{1}{2} (S_x + S_y); \dot{E}_x, \dot{E}_y \) are strain-rates and primes denote deviators.

That is,
\[ S_x = \eta_1 \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + \eta_2 \left( \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right), \]
or
\[ S_x = (\eta_1 + \eta_2) \frac{\partial u}{\partial x} + (\eta_1 - \eta_2) \frac{\partial v}{\partial y}. \] (15)

Comparison with (14) suggests that we write
\[\begin{align*}
\eta_1 + \eta_2 &= 4\eta, \\
\eta_1 - \eta_2 &= 2\eta,
\end{align*}\]

that is,
\[\begin{align*}
\eta_1 &= 3\eta, \\
\eta_2 &= \eta.
\end{align*}\]

It then follows that
\[\begin{align*}
\frac{\partial \xi_x}{\partial x} + \frac{\partial \xi_y}{\partial y} - \frac{\partial}{\partial x} (\hbar \sigma_x) + \tau_x &= 0, \\
\frac{\partial \xi_y}{\partial x} + \frac{\partial \xi_x}{\partial y} - \frac{\partial}{\partial y} (\hbar \sigma_y) + \tau_y &= 0,
\end{align*}\]

and
\[\begin{align*}
\xi_x &= \hbar (\sigma_x - \sigma_z), \\
\xi_y &= \hbar (\sigma_y - \sigma_z), \\
\tau_{xy} &= \hbar \tau_{xy}.
\end{align*}\]

We conclude that, if a three-dimensional incompressible viscous continuum floating on water were looked at as a two-dimensional continuum, (a) the two-dimensional medium would appear to have a bulk viscosity equal to three times its shear viscosity; (b) the \(\xi\)’s would have to be related to the \(\sigma\)’s by (17); and (c) the momentum equations (6) and (7) would not be valid since they omit the derivatives of \(\hbar \sigma_0\). The omission in equations (6) and (7) arises because they ignore one aspect of the interaction between the two-dimensional sheet and the water on which it floats.

It might be thought possible to avoid the extra term in \(\partial (\hbar \sigma_0)/\partial x\) by absorbing it in \(\xi_x\) and then suitably redefining \(\xi_x\). But it is not possible to redefine \(\xi_x\) in this way because it must remain related to the strain-rate components by (15).
However, it may well be that we do not want to model the ice cover as a three-dimensional incompressible viscous continuum. If we relax this requirement, it leaves us free to absorb the terms in $h\sigma_q$. Thus we now define

$$
S_x = h(\sigma_x - \sigma_z - \sigma_q), \\
S_y = h(\sigma_y - \sigma_z - \sigma_q), \\
T_{xy} = h\tau_{xy},
$$

and so write (11) and (12) as

$$
\begin{align*}
\frac{\partial S_x}{\partial x} + \frac{\partial T_{xy}}{\partial y} + \tau_x &= 0, \\
\frac{\partial T_{xy}}{\partial x} + \frac{\partial S_y}{\partial y} + \tau_y &= 0.
\end{align*}
$$

The term subtracted from $h\sigma_x$ to produce $S_x$ is in this case

$$
h(\sigma_z - \sigma_q) = -\frac{1}{2}\rho_w gh'^2
$$

in view of (1), (2) and (13); that is, simply the horizontal hydrostatic force of water acting on the submerged depth $h'$. But in order to continue in this fashion we have to use the two-dimensional stresses thus defined in two-dimensional rheological equations. We note that the first of equations (18) may be alternatively written

$$
S_x = h(\sigma_x - \frac{\rho_l}{\rho_w} \sigma_z).
$$

It is not clear to me that rheological equations in which the stress variable is (20) (leading to momentum equations [19]) are preferable to ones in which it is (17) (leading to momentum equations [16]). Rather the reverse. Consider, for example, thin lead ice that is being crunched between thicker ice floes, as in an early stage of the Parmerter-Coon
FOREWORD

In summer 1972, financial circumstances forced the decision to postpone the Main Experiment to 1975-76. What first seemed a costly hiatus has turned into a period of productive activity, especially in the areas of technical development and theory.

In the technical field, the AIDJEX Office is charged with the responsibility for acquiring navigation systems, meteorological sensors, data logging systems and, in collaboration with NOAA's National Data Buoy Office, ice-going data buoys with a central data recording system. In forthcoming Bulletins we hope to show that the delay of the Main Experiment has been turned to our advantage and will result in more reliable and efficient procedures for data acquisition and handling.

Some recent theoretical work is reported in this Bulletin. The problem of sea ice lies somewhere between plasticity, geophysical fluid dynamics, heat transfer, and rigid body mechanics, and its solution requires a group effort in which no single viewpoint should prevail. Inspired in part by the reduction in size of the Main Experiment (see AIDJEX Memorandum 73-1, dated 14 May 1973), various concepts pertaining to small-scale ice mechanics were given increased attention and have yielded approaches to realistic flow laws and to a method of coupling the dynamics and thermodynamics of the ice by means of the thickness distribution.

Since the inception of AIDJEX, investigators, advisors, and administrators alike have held a residual feeling of concern about the risk attached to a project that endeavors to apply untried scientific concepts to a trouble-prone experiment in a difficult environment. Some of the risk, particularly that related to field work, is objective and will remain with us until the project is finished. But we believe that the papers published in this and other recent Bulletins clearly indicate that progress is being made in our central task of dealing with sea ice as a geophysical material.

N. Untersteiner
AIDJEX Coordinator
ridging process. It seems more natural to relate the rate of deformation of this rubble to the stress difference $\sigma_x - \sigma_z$ in the rubble, rather than to $\sigma_x - (\rho_\text{i}/\rho_\text{w})\sigma_z$, and in this case one would use the momentum equations (16) rather than (19).

To summarize, one cannot say that equations (16) or equations (19) are right or wrong until one has defined the meaning of the $S$'s; but the $S$'s, not being directly observable, may be defined in various ways. On the other hand, the two-dimensional strains or strain-rates are observable and clearly defined. They will be connected by rheological equations with certain combinations of three-dimensional stresses that may be used to define $S_{i,j}$. The rheological equations thus define the $S_{i,j}$, and only then can it be asked whether the correct momentum equations are (16) or (19) or some other form.

Order of magnitude

To see whether the term $h\sigma_0$ is numerically significant we can compare it in magnitude with the two-dimensional stresses of $10^5$ to $10^7$ dyne cm$^{-1}$ thought to be needed to cause ridging. If $\rho_\text{i} = 0.9$ g cm$^{-3}$, $\rho_\text{w} = 1.0$ g cm$^{-3}$, $g = 10^3$ cm s$^{-2}$ and $h = 2$ m, $h\sigma_0 = 2 \times 10^6$ dyne cm$^{-1}$, which is in the middle of the range.

Alternatively, we can compare $\partial(h\sigma_0)/\partial x$ with $\tau_x$; that is to say, $\rho_\text{i} \{1 - (\rho_\text{i}/\rho_\text{w})\}gh \frac{dh}{dx}$ with $\tau_x$. If these terms are equal we find, taking $\tau_x = 1$ dyne cm$^{-2}$ and $\rho_\text{i} = 0.9$ g cm$^{-3}$, that $\partial h/\partial x = 5 \times 10^{-5}$. Thus a thickness change of 1 m in 20 km is sufficient to produce a term equal in size to the wind and water stress. It seems that the term should not be ignored.

Energy argument

The following argument helps to make clear the physical meaning of the two-dimensional stress $\sigma_y h$. Suppose the ice were a floating inviscid slab of liquid of thickness $h$. The stresses in it would be
\[
\sigma_x = \sigma_y = \sigma_z = -\frac{1}{2} \rho \dot{\gamma} h.
\]

An external force \( F_0 \) per unit length would be needed around the edge of the slab to keep it in equilibrium. From the balance of forces
\[
F_0 = \frac{1}{2} \gamma (\rho \dot{\gamma} h^2 - \rho \dot{\gamma} h^2) = \sigma_0 h.
\]

If now the thickness increases so that the perimeter moves in by \( \Delta s \), the work done is \( \sigma_0 h \Delta s \) per unit length of boundary. Since the system is conservative, this is also the increase in potential energy.

Let the ice now have any rheology, but such that under a force \( F \) per unit length applied at the boundary the thickness increases uniformly. If the boundary moves in by \( \Delta s \) the increase in potential energy is as calculated above, namely, \( \sigma_0 h \Delta s \). The total work done, \( F \Delta s \), is therefore made up of two parts: \( \sigma_0 h \Delta s \) is used to increase the potential energy and the remainder \( (F - \sigma_0 h) \Delta s \) is dissipated as work of deformation. We can say that, of the applied force \( F \), a part \( \sigma_0 h \) is used to increase the potential energy and the other part \( (F - \sigma_0 h) \) is used to deform the material. (It would be wrong, even in this case of uniform deformation, to think that the whole force was being used to deform the material, and this has to be borne in mind when assessing a rheological model.)

If the thickness does not increase uniformly (as, for example, in ridging) and we use the same force \( F \) and the same \( \Delta s \), correspondingly more work will be done in increasing the potential energy, and therefore less work will be consumed in deforming the material.

References


THE MEANING OF TWO-DIMENSIONAL STRAIN-RATE
IN A FLOATING ICE COVER

by

J. F. Nye

The Problem

The AIDJEX main experiment includes the measurement of strain in the ice pack on certain scales, and it is proposed to model the ice as a two-dimensional continuum. The strains are to be measured by observing the change in position of points fixed in the ice, or by measuring the change in separation of such points. On the other hand, the existence of leads, pressure ridges, and other features shows that the deformation of the ice pack is inhomogeneous or discontinuous on the scale of these features; so the result of a measurement may depend somewhat on the precise relation between a measurement point and some local feature. One speaks sometimes of a "sampling error." Hibler et al. [1973], in analyzing their mesoscale strain measurements, refer to "inhomogeneity variations." The strain will also vary spatially on the scale of the large-scale weather variations.

In these circumstances how should strain be defined, and what precisely do we, or should we, mean by "strain on a scale of \(x\) km"? To discuss such matters in a precise way a conceptual framework is needed, and the purpose of this note is to suggest a basis on which such a framework could be constructed. We first set up an appropriate definition of strain. Then we ask how what is actually measured relates to the defined quantity.

Definitions

Take axes \(ox, oy\) in the horizontal plane (we ignore the curvature of the Earth) and, at fixed time, consider the horizontal component of the velocity of the ice at the point \((x,y)\). (If the velocity varies through the
thickness of the ice it is to be understood that "velocity" means velocity averaged through the thickness.)

We start, for simplicity, by considering one dimension only. Figure 1 shows how the \( x \) component of velocity \( v_x \) might vary along a line in the \( x \) direction. The \( v_x : x \) curve may have finite discontinuities, as for example at a crack which is just on the point of opening. It will also have sections missing where there is open water, and we arbitrarily fill in these parts, shown dotted in Figure 1, by linear interpolation between the end points. If this curve, which for practical purposes represents the actual physical situation, were differentiated to obtain the strain-rate, any finite discontinuities would give points of infinite strain-rate.

![Fig. 1. Hypothetical distribution of ice velocity \( v_x(x) \) along a line. \( A \) and \( B \) are smoothing functions.](image)

The following conceptual procedure is suggested. Construct a running mean \( v_{x,\ell}(x) \) of the \( v_x(x) \) curve by averaging over a distance \( \ell \) (the broken line in Figure 1). That is, the value of \( v_{x,\ell} \) at a given point \( x \) is to be a weighted average of the values of \( v_x \) at points in the neighborhood of \( x \), the width of the neighborhood being approximately \( \ell \). This could be a simple
centered running mean, as indicated by curve A, or a more smoothly varying weighting function such as B. The resulting curve \( v_{x,L}(x) \) has no discontinuities; at worst it contains discontinuities of gradient if \( v_{x}(x) \) has finite discontinuities and if the top-hat weighting function A is used. The gradient of the \( v_{x,L}(x) \) curve at \( x \) is now defined as the "strain-rate on a scale \( l \) at the point \( x \)" and is denoted by \( \dot{\varepsilon}_L(x) \). The exact values of \( \dot{\varepsilon}_L(x) \) will depend slightly on the particular weighting function that is used to construct the \( v_{x,L}(x) \) curve; but once this is decided, the values of \( \dot{\varepsilon}_L(x) \) at all points along the line (excluding regions of length approximately \( \frac{1}{2}l \) at each end) are precisely defined. There is no question of there being any error in this strain-rate distribution; there will only be errors in our estimates of it when we try to measure it. The strain-rate on a scale \( l \) is a precise quantity, by definition.

Having thus carried out some spatial smoothing, we may ask whether some smoothing over an interval \( \Delta t \) in time might also be appropriate. For the purposes of modeling the response of the floating ice to air and water stresses it would be reasonable when considering the velocities of points to exclude those due to elastic wave propagation through the pack and certain other slightly longer-term variations. An appropriate meaning for velocity in the present context might then be velocity smoothed over time intervals of a few hours. Strictly speaking, then, the time-smoothing should be specified in the definition of strain-rate and we could write the strain-rate as \( \dot{\varepsilon}_{L,\Delta t} \). For many purposes, however, we can take the time-smoothing for granted and concentrate on the more difficult problem of the spatial smoothing. It is more difficult for at least three reasons:

1. While we already have a good deal of information about how the velocity of a point varies in time, we have much less information about how the velocity of points (at given time) varies in space.

2. While there is a fairly clear separation of time scales (as suggested above) there is no evidence at present that any such clear separation of scales exists spatially; that is to say, the velocity may vary spatially on virtually all scales; we shall return to this point later.

3. While the time variations are one-dimensional, the velocity field varies spatially in two dimensions.
The definition of strain-rate on a scale \( \ell \) given above for one dimension is readily generalized to two dimensions. We take the actual vector velocity field \( \mathbf{v}(x,y) \) and then smooth it by a weighting function with diameter approximately \( \ell \). The derivatives \( \partial v_{i,j}/\partial x_j \) \((i,j = 1, 2)\) of this smoothed velocity field are then used to define the components of the "strain-rate tensor on a scale \( \ell \)" and also the "vorticity on a scale \( \ell \)" by the usual formulae (strain-rate being the symmetrical part of the tensor \( \partial v_{i,j}/\partial x_j \), and vorticity being the antisymmetrical part, or its single independent component).

**Measurements**

Let us now consider how strain-rate is measured in practice and how what is measured relates to what we have defined. Suppose, for example, in Figure 1 that two stations separated by a distance \( \ell \) are established on the ice which give the points \( P \) and \( R \) on the curve of \( v_x(x) \). The measured strain-rate over the gauge length \( \ell \) is then the slope of the chord \( PR \), and this is taken as an estimate of the strain-rate at the point halfway between the two stations, measured on a scale \( \ell \). By our definition the true value of the strain-rate is the slope of the smoothed curve \( v_{x,\ell}(x) \) at \( M \) (\( M \) refers to the midpoint of the interval). The question is, what is the relation between the slope of the chord \( PR \) of \( v_x(x) \) and the slope of the tangent to \( v_{x,\ell}(x) \) at \( M \)? It is easily shown that, if the curve \( v_{x,\ell}(x) \) is generated by the top-hat weighting curve \( A \), the two slopes are identical. Formally (omitting the subscript \( x \)),

\[
v_{\ell}(x) = \frac{1}{\ell} \int_{x - \ell/2}^{x + \ell/2} v(x') dx'
\]

and hence,

\[
\frac{dv_{\ell}(x)}{dx} = \frac{1}{\ell} \frac{d}{dx} \int_{x - \ell/2}^{x + \ell/2} v(x') dx' = \frac{1}{\ell} \left( v(x + \ell/2) - v(x - \ell/2) \right)
\]
Thus, in this case, if there are no errors in the velocity measurements, the strain-rate deduced from the slope of the chord is an exact measurement of the strain-rate on a scale \( l \) (as defined). If, on the other hand, some different weighting function were used to generate the smoothed velocity curve this convenient result would no longer hold true. This might constitute an argument for using, in the definition, the simple top-hat curve \( A \), a curve which one would otherwise probably wish to reject in favor of a smoother weighting function like \( B \). But unfortunately this simple result no longer holds true in two dimensions. To see this, note that the strain of a single gauge line crossing an area can be far from the average of all the strain values that might exist along neighboring parallel gauge lines. In two dimensions there seems therefore to be no reason for preferring a top-hat smoothing curve.

In one dimension unless top-hat smoothing is used, and in two dimensions generally, the strain-rate measured between two stations of separation \( l \), even with no measurement errors, will not be exactly equal to the strain-rate on a scale \( l \). It will be an estimate of this strain-rate. How good is the estimate? In one dimension, if we measure the velocity of an intermediate station \( Q \) and note the difference in slope of the chords \( PQ \) and \( QR \), it will not necessarily indicate the error in the assessment. For example, in one dimension with top-hat smoothing the slopes of \( PQ \) and \( QR \) could be considerably different (as in Figure 1), while the error in the estimate of strain-rate on a scale \( l \) would be zero. Again, suppose the true situation were as in Figure 2. The slope of \( PR \) could be a very good estimate of the slope of \( \nu_{x,l}(x) \) at \( M \), that is, of the strain-rate on a scale \( l \), in spite of the observed nonlinearity of \( PQR \).

In two dimensions, on the other hand, further measurements of velocity may help in deciding how good the estimate is. For example, in Figure 3, where the diameter of the circle is \( l \), one might measure the strain-rate (stretch-rate) along gauge line \( BB' \) and take it as an estimate of the strain-rate on a scale \( l \) at the point \( M \). If in addition the strain-rates along the parallel gauge lines \( AA' \) and \( CC' \) were measured, their departures from the value measured along \( BB' \) might be a measure of the error in the estimate.
Fig. 2. Hypothetical distribution of ice velocity $v_x(x)$ along a line.

Fig. 3. Gauge lines $AA'$, $BB'$, $CC'$ are used to estimate the strain-rate on a scale $\lambda$ at point $M$ and the error in the estimate. The circle has diameter $\lambda$. 
One has to say "might." At one extreme the strain-rate on a scale $\lambda$ might be essentially uniform over the area and the differences between measurements on $AA'$, $BB'$, $CC'$ might arise from rapid point-to-point fluctuations in velocity (rather as in Figure 1). Then the differences between the measurements would indicate the error in the estimate. At another extreme the strain-rate even on the smallest scale might vary linearly over the area (rather as in Figure 2, the velocity varying parabolically), and then the difference in the measurements would merely show this linear variation while the estimate from the measurement on $BB'$ would contain no error at all.

Of course, whether the actual spatial variations in velocity are regarded as "significant" or not depends on one's point of view. For example, on a large scale they may be determined primarily by the large-scale pattern of weather and ocean currents. On a smaller scale they may be determined primarily by the spatial variations in thickness of the ice or by pre-existing cracks. If these spatial variations in thickness or pre-existing cracks were known, the resulting velocity variations would be considered as significant information. If they were not, the velocity variations might be regarded as unwanted "noise."

Suppose then that an array of stations is used in two dimensions with the gauge lines having lengths of order $\lambda$ and that a strain-rate tensor and a vorticity are fitted to the measurements by least squares. Our conclusion is that the residuals may or may not help to decide how good is the estimate of the strain-rate tensor and vorticity on a scale $\lambda$ at the center of the array. Whether they do or do not depends on the nature of the underlying velocity field. If one knew in advance the statistical nature of the underlying velocity field, one could then decide the question. At present we do not know it, and that is the difficulty. It may be that a comparison of measured mesoscale and macroscale strain-rates, combined with study of air photographs, will provide the missing information.

We may note that, if techniques of photointerpretation should improve to the point that very full spatial information on velocity were available at a small number of definite times, the problems connected with spatial averaging could be treated rather exactly, but the problem would then be how to deal with the inadequate temporal information.
An implicit assumption is sometimes made that there is a rather clear separation of spatial scales: that there is a small-scale variation of strain-rate due to inhomogeneities in the ice, superimposed on a large-scale variation due to large-scale weather patterns. (The current AIDJEX Scientific Plan [AIDJEX Bulletin No. 15, 1972, p. 30] stipulates that the array of manned stations should be large compared with the ice features, such as leads, ridges, and individual floes, and small enough so that variations of strain within the array are approximately linear.) The photographic evidence that leads form on a very wide variety of scales, together with the mesoscale strain results of Hibler et al. [1973], suggests that the assumption of such a clear separation of scales may not be true. It seems better, at present, to assume that velocity variations exist on virtually all scales (just as height variations in a landscape, whether they are called topography or roughness, exist on virtually all scales).

If, nevertheless, against expectation, there does exist such a clear separation of scales, it would show itself in strain-rate measurements as follows. The strain-rate on a certain scale would be found to be the same as the strain-rate on a larger scale. More generally, the strain-rate distribution would be found to be independent of scale over a certain range of scales, say \( l_1 \) to \( l_2 \). Thus the running mean of velocity would be the same over a range of smoothing lengths. This would imply no variations of velocity on wavelengths between order \( l_1 \) and order \( l_2 \) (that is, the power spectrum would be zero over a corresponding range of spatial frequencies). The variations due to inhomogeneity in the ice would be found by reducing the measurement scale below \( l_1 \), while the variations due to large-scale weather patterns would be on a scale greater than \( l_2 \). However, as we have said, a clear separation of this sort seems unlikely. Its absence may make mathematical modeling of the mechanics of the ice cover more difficult—but at least the strain-rate on a given scale can still be perfectly well defined. Using the landscape analogy, although there are height variations on all scales the geographer can still plot contour maps and a meaning can still be attached to "slope"—after specifying the scale.

In conclusion, we can summarize by saying that a definition of "strain-rate and vorticity on a scale \( l \)" must logically come first. Then
one can discuss how proposed measurement schemes are related to the quantities so defined. And finally one can ask whether the error in the estimate of the defined strain-rate and vorticity on a scale R can be found from the measurements. This note has not succeeded in doing all these things. It has tried to suggest how one might begin.

Acknowledgment

The argument given in this note was developed during discussions with Alan Thorndike; I should like to thank him for his help.

References


IS THERE ANY PHYSICAL BASIS FORassinLING LINEAR VI8COUS BEHAVIOR FOR SEA ICE?

by

J. F. Nye

A number of models for the large-scale behavior of the arctic sea ice use a linear viscous law, or a slightly more elaborate version of it, for the interaction of the ice with itself. The results from the models are then compared with the observed behavior of the ice pack. It is therefore relevant to ask whether there is any sound evidence, either from the degree of success of these models or from independent sources, that the ice does indeed behave in a viscous manner. I suspect the answer is no. Of course, the water drag can produce a retarding force of a viscous type—but the question is, if the water drag were ignored and the ice rested on a frictionless base, with no air stress, would a steady two-dimensional stress in the ice produce a corresponding steady two-dimensional strain-rate? This is what is implied by a viscous constitutive law. I have not heard of any physical basis for such a law, and I am inclined to doubt whether the predictions of the large-scale models are yet sensitive enough to the constitutive law for the broad general agreement they give with the facts to be taken as evidence in support of the constitutive law they use.

If the ice is not viscous, how then does it behave? I do not know, but if required to guess I would support the idea of brittle behavior under tension, combined with strain-hardening behavior under compression. Strain-hardening under compression, by the successive crunching of thicker ice, would be consistent with the Parmerter-Coon ridging model.

There does, however, seem to be a possibility that although the ice, modeled as a continuum, is brittle or plastic on time scales of days or weeks, it may nevertheless appear to be viscous on longer time scales. This could come about in the following way.
As a highly simplified model, to make the point, suppose the driving stress from the wind consists of a steady (say, mean-annual) component with short-term storms superposed. The large-scale two-dimensional stress in the ice will have a similar time dependence. Suppose that the steady component of stress produces no effect at all by itself when acting on the ice in its "normal" compacted state. Now suppose a storm breaks up the ice so that some limited rearrangement is possible. Many rearrangements might take place; in the absence of the steady biasing component of stress the rearrangements would constitute strains in random orientations. The presence of the steady component, however, means that those rearrangements are favored which contribute to a strain which allows the steady stress component to do work. After the storm is over there is thus found to be a small strain in the "direction" of the applied steady stress. It is reasonable to suppose that the amount of this strain is related to the magnitude of the steady stress. Put crudely, large random stresses from the storm jiggled the ice, and the small steady stress took advantage of the favorable opportunities so presented. In this simple picture nothing more happens until the next storm, when the process is repeated. Looked at on an annual time scale, the deformation rate would be found to be related (perhaps proportional) to the stress, that is, one would see viscous behavior. The "viscosity" would be inversely related to the frequency and intensity of the storms, rather than being any intrinsic property of the ice. It is rather like the thermal molecular mechanism of viscosity in liquids, with the storms playing the part of thermal agitation.

This model is no doubt excessively crude. Even if the details (such as there are) are wrong, it does nevertheless suggest that a "property" (viscosity) of the ice exhibited on an annual time scale may be sought in vain, because it does not exist, on a shorter time scale.
A NOTE ON THE POWER SPECTRA OF SEA-ICE PROFILES

by

J. F. Nye

The following comment is prompted by a paper on the power spectra of sea-ice profiles by Hibler and LeSchack [1972]. The emphasis in the paper is on looking for significant spectral peaks corresponding to spatial periodicities in the ice, which may be related, for example, to fractures and ridges. However, we wish to remark that the general form of the power spectra, irrespective of individual peaks, is also informative.

The paper gives the theory for a time series of measurements of a quantity \( \eta \). If \( \eta \) represents height measurements, the dimensions of \( C(f) \) (called the power spectrum in the paper, and the power spectral density by other authors) are [length]\(^2\)[time]. The series under examination are (one-dimensional) spatial rather than time series and so the physical dimensions are [length]\(^3\), and we may denote the power spectral density by \( C(k) \), where \( k \) is a wave number (dimensions [length]\(^{-1}\)). Consider the following hypothetical form for \( C(k) \):

\[
C(k) = ak^{-n},
\]

where \( a \) is a constant. If \( n = 3 \) the spectrum has a special property because \( a \) is dimensionless. For other values of \( n \) the constant \( a \) must contain a length scale, but \( n = 3 \) is the unique case in which the spectrum, and therefore the property of the profile it represents, has no length scale associated with it. A profile having this property would "look the same" on all scales. Natural landscapes tend to be of this character [Nye, 1970, pp. 385-389] over the wavelength range of about 1 km to 1 cm. In all real cases a low-frequency cut-off would have to be applied to equation (1) to prevent \( C(k) \) from becoming infinite at \( k = 0 \), and there would also obviously have to be some high-frequency limit. Putting in numbers [e.g., Nye, 1970, p. 392] shows that
profiles with \( n = 2 \) or \( n = 4 \) would look to the eye very different from a profile with \( n = 3 \). Power spectra of several very different types of terrain (such as a grass airport runway and a lava flow) are shown in Fig. 6 of Jaeger and Schuring [1966]; they all correspond remarkably well to \( n = 3 \), even though the constant \( \alpha \) can be quite different in each case.

It is interesting that the power spectra of Hibler and LeSchack [1972] often approximate to equation (1) with \( n = 3 \) over a certain frequency range. For example, in their Fig. 2a, b, and c the "relative amplitude" falls by about \( 10^3 \) as the wave number increases from 10 to 100 units; this would be represented by \( n = 3 \). On the other hand, the rise in "relative amplitude" as the wave number falls from 10 units to 1 unit is much less marked; in this range the wavelength is 100 m or more and we are no doubt seeing the fact that sea-ice, unlike most natural landscapes, is flat on the large scale.

References


AN APPROACH TO COUPLING THE DYNAMICS AND THERMODYNAMICS OF ARCTIC SEA ICE

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ABSTRACT

Thermodynamics influences the dynamic behavior of sea ice primarily through the formation of new ice on areas of open water, and through ablation and accretion on the horizontal boundaries of existing ice. These mass changes enter into the momentum balance equations and affect the material properties of the ice pack. To satisfy the needs of proposed dynamic models, it will be necessary to describe changes in the total mass, the area of open water, and the area of thin ice within specified regions. Thermodynamic models can predict growth rates as a function of ice thickness and season from climatological data; however, the needs of advanced dynamic models cannot be met without some knowledge of the distribution of ice thickness. The approach adopted here is to partition the continuous range of ice thicknesses into a number of discrete categories. Equations are then developed which describe how the area covered by each category changes as a result of ice growth, advection, and divergence. The method is independent of any particular dynamic model since the only dynamic input required is the velocity field of the ice.

Phase changes on the surface of the Arctic Ocean are of profound consequence; indeed, the striking feature of the Arctic Ocean is that it is covered with ice, formed in thermodynamic response to the climatic conditions which prevail at high latitudes. Sea ice moves and, since the motion is not uniform, deforms, resulting in a rich surface morphology. The sea ice observed in the field is therefore the product of the continuous and simultaneous action of

thermodynamic and dynamic processes. It is vastly different from the ice which would exist if either of these processes were somehow suppressed. In the absence of dynamics, for instance, all the ice would approach a uniform thickness of thermodynamic equilibrium. If, on the other hand, the thermodynamics were suddenly turned off so that there could be no freezing or melting, then in time all the ice thinner than some critical value would be heaved up into pressure ridges, and the ice pack would settle into a dynamic steady state consisting of deformed ice and open water. These extremes emphasize the critical difference in character between the thermodynamics and the dynamics: on a year-long average, the thermodynamics strives for a single equilibrium thickness by net accretion to the thin ice and net ablation from the thick; by contrast, the dynamics creates regions of thick pressured ice and regions of no ice at all. The thermodynamics seeks the mean and the dynamics the extremes. Because of this disparity, a distribution of ice thickness is maintained whose properties are of the first importance in understanding the behavior of sea ice.

On a geophysical scale, sea ice is an aggregate composed of elements which because of their different thicknesses respond differently to similar thermal and mechanical forcing. The bulk properties of the material must therefore be a function of how abundant the various elements are. Consider, for example, the strength of the ice in compression. When open water or very thin ice is present, the pack offers virtually no resistance to compression; but if all the ice exceeds a meter in thickness, the pack becomes extremely strong. Because winter ice growth is very rapid in areas of thin ice and open water, the constitutive behavior of the ice pack may turn out to be quite sensitive to the thermodynamics.

The amount of ice in the Arctic Basin undergoes a seasonal variation on the order of 20-30% as a result of thermodynamic processes. Regional mass changes affect the movement of the ice pack by altering the acceleration, and Coriolis terms in the momentum balance. The importance of the thermodynamic part of the system is a function of the time scales involved; predictions of ice drift on the order of a few days can effectively ignore ice growth, but realistic long-term predictions cannot be made without accounting for ice growth.
Since the large-scale energy fluxes which control the regional rates of ice growth are not greatly modified by ice movement, the rate of ice growth can be taken as given input for purposes of dynamic modeling. Growth rates in the Arctic Basin are a function of position, ice thickness, and season, i.e.,

\[
\frac{dh}{dt} = f(x,y,h,t).
\]

Time-dependent numerical models have been developed which predict growth rates on the basis of specified energy fluxes at the horizontal boundaries of an ice slab. With suitable assumptions regarding the spatial variations of these fluxes, it will be possible to obtain \( f(h,t) \) for each climatic region in the Arctic Basin. Figure 1 illustrates idealized growth curves for summer and winter conditions in the Central Arctic. Note that the winter value of \( f \) changes by two orders of magnitude between 0 and 3 meters, indicating that estimates of total mass changes in a given region cannot be made without some knowledge of the frequency distribution of ice thickness; an average ice thickness is not sufficient. Despite additional difficulties in treating the growth of very thin ice and pressure ice, predictions of ice growth are limited primarily by the quality of available energy flux data.

Having partitioned the ice into a number of thickness categories, we now propose a method to calculate how the area of each category changes as a result of dynamic and thermodynamic forcing. This method, in principle, will supply the information needed by any dynamic model. The number of thickness categories which must be treated will be dictated by (i) the sensitivity of the assumed constitutive law to the ice thickness distribution, \( g \), and (ii) the shape of realistic \( f \) and \( g \) functions.

Assume that we are given the function \( f \) and the velocity field, \( V(x,y,t) \), of the ice pack. Consider some large region \( R \), corresponding, for example, to an Eulerian grid element in a dynamic model. Then there exists some distribution of ice thickness within \( R \) which can be described by a probability density function, \( g(h,t) \). If we define \( A_i(t) \) to be the percent
of area covered by ice in the thickness band between partition points \( h_i \) and \( h_{i+1} \) (Figure 2), then

\[
A_i(t) = \int_{h_i}^{h_{i+1}} g(h,t) \, dh .
\]

For a given thickness band, \( A_i \) can change as a result of thermodynamic ice growth, advection, or mechanical deformation, i.e.,

\[
\frac{\partial A_i}{\partial t} = \phi_i(f, g) + \nabla \cdot \nabla A_i + \psi_i(V) \tag{1}
\]

where \( \phi_i \) is the area change due to thermodynamics and \( \psi_i \) is the area change due to the dynamics. In the simplest case which neglects the effects of shear, \( \psi_i \) will be assumed to depend on the divergence

\[
\psi_i(V) = w_i \nabla \cdot V .
\tag{2}
\]

\( w_i \) is a weighting function which depends on the sign of \( \nabla \cdot V \) and describes how ice is redistributed as a result of convergence. \( w_i \) must satisfy two conditions. From conservation of area, it follows directly that

\[
\sum_i w_i = 0
\]

and if we define \( \bar{h}_i \) to be the average thickness in the \( i \)-th category, and \( \bar{h} \) the average thickness in \( R \), conservation of the volume of ice requires that

\[
\sum_i \bar{h}_i w_i = \bar{h} .
\]

The thermodynamic term, \( \phi_i \), can be obtained by noting that

\[
\frac{\partial g}{\partial t} = - \frac{\partial}{\partial h} \left( fg \right) . \tag{3}
\]
Fig. 1. Dependence of growth rate, $f$, on ice thickness, $h$, in the Central Arctic.

Fig. 2. Postulated probability density function of ice thickness in the Central Arctic.
From the definition of $A_i$ and equation (3) we see that

\[ \phi_i = \left. \frac{dA_i}{dt} \right|_{\psi=0} = -fg \left\{ \begin{array}{c} h_{i+1} \\ h_i \end{array} \right. \]

which indicates that thermodynamic area changes in a given thickness category are dependent only upon the values of $f$ and $g$ at the boundaries of the category. Combining equations (1), (2), and (4), we obtain

\[ \frac{\partial A_i}{\partial t} = fg(h_i) - fg(h_{i+1}) + V \cdot \nabla A_i + w_i V \cdot V . \]

Equation (4) holds only in the continuous sense for an infinitesimal $\delta t$. In actual applications where we must consider a finite $\Delta t$, the derivatives of $f$ and $g$ become important. If we do a Taylor series expansion around $A_i(t=0)$ and neglect terms of order higher than two, we can write down this dependence explicitly:

\[ \phi_i = \left. \left[ \begin{array}{c} -fg \left( 1 - \frac{\Delta t}{2} \frac{\partial f}{\partial h} \right) + \frac{\Delta t}{2} \left( f^2 \frac{\partial g}{\partial h} - g \frac{\partial f}{\partial t} \right) \end{array} \right] \right|_{h_i} \]

The $\partial f/\partial t$ term will always be at least two orders of magnitude smaller than the other terms and can be neglected; however, the other three terms are all significant. The dominant term is $-fg$, but in a number of situations the $\partial g/\partial h$ term can exceed $fg$. For the growth of very thin ice during the winter, the $\partial f/\partial h$ term can approach $fg$ in magnitude.

In summary, we have formulated the interaction between the velocity field, $V$, the thermodynamics, $f$, and the ice thickness distribution function, $g$, or its discrete representation in terms of the $A_i$'s. In Figure 3, more structure is added to show the operation of a combined dynamic/thermodynamic model of sea ice. The thickness distribution is determined by the velocity field and growth rates; $g$ then feeds back into a determination of the bulk
constitutive properties of the ice pack. At the same time, since $g$ defines the total mass of ice in the region, it feeds directly into the dynamic model for use in the momentum balance equation. Although many problems remain in the exact details of how to treat $g$, this approach appears to meet the demands of all proposed dynamic models, yet is independent of any particular dynamic model since it requires as input only the velocity field of the ice.

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ON THE THICKNESS DISTRIBUTION OF SEA ICE

by

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ABSTRACT

A region on the surface of the Arctic Ocean may contain ice of many different thicknesses, the relative proportions of which are described by an ice thickness distribution function, $G$. A theory is developed to describe how $G$ changes in response to thermodynamic and dynamic forcing.

INTRODUCTION

Two phenomena act to alter the thickness, $h$, of floating sea ice: thermodynamics controls the accretion and ablation of mass in a vertical column of ice, while dynamic processes result in the formation of leads and ridged ice. The combined result of these phenomena is to maintain a range of ice thicknesses from $h = 0$ to some $h_{\text{max}}$. In previous work [1973] we have used an ice thickness distribution to couple the thermodynamic and dynamic processes occurring within the ice pack. In this paper we tidy up the previous treatment and offer a fuller theory for how the thickness distribution responds to the thermodynamic and dynamic forcing.

DEFINITION OF THE ICE THICKNESS DISTRIBUTION

Let us partition the surface of the Arctic Ocean into $N$ simply connected Lagrangian regions which are fixed with respect to the ice. After some time
the regions may have deformed (due to the velocity of the ice), but there will still be \( N \) regions and they will still cover the entire ocean.

Selecting one of these regions, \( R \), we may observe that it encompasses ice of various thicknesses and perhaps some open water. Let \( A(h, t) \) be the area of ice within \( R \) of thickness strictly less than \( h \) at time \( t \). We define \( R(t) \) to be \( A(h_{\text{max}}, t) \), the total area of the region under consideration.

The ice thickness distribution function for the region is denoted by the symbol \( G \) and is defined as

\[
G(h, t) \equiv \frac{A(h, t)}{R(t)}.
\]  

(1)

\( G \) is thus the fractional area of ice in \( R \) which is thinner than \( h \); it is a dimensionless quantity. \( G \) is like a probability distribution function; it is zero for \( h \) less than some \( h_0 \) and unity for \( h \) greater than \( h_{\text{max}} \). Furthermore, \( G \) must be monotonic. A sample \( G \) vs. \( h \) for fixed time is drawn in Figure 1. The step in \( G \) at \( h = h_1 \) implies that there is a finite area of ice of thickness exactly equal to \( h_1 \).

![Fig. 1. A possible ice thickness distribution function.](image)

Such a step is generated at \( h = 0 \) following the end of each summer when all of the open water begins to freeze at the same time; as the winter progresses, continued freezing moves the step to the right. When such a discontinuity exists, the value of \( G \) at \( h_1 \) is given by
\[ G(h_1) = \lim_{\varepsilon \to 0} G(h_1 - |\varepsilon|) \]

i.e., the limit as \( h \) approaches \( h_1 \) from the left. This follows from the strict inequality in the definition of \( A \). Thus, if there is a finite area of ice of thickness exactly equal to \( h_1 \), it is not included in \( A(h_1,t) \) but is included in \( A(h_1 + \varepsilon, t) \) for any \( \varepsilon > 0 \).

Under this definition \( G(0,t) = 0 \), since there is never any ice of thickness strictly less than zero. In the event that there actually is some open water, \( G \) has a step at zero, i.e., the limit of \( G \) as \( h \) approaches zero from the right is not zero. A point on the \( h \) axis of particular importance is the largest \( h \) for which \( G \) is zero, which we call \( h_0(t) \).

\[ G(h_0(t), t) = 0 \quad (2) \]

That such a point exists is again a consequence of the strict inequality in the definition of \( A \).

\( G \) is related to the density function \( g \), discussed in the previous paper, by

\[ G(h,t) = \int_{0}^{h} g(h', t) \, dh'. \]

THE THERMODYNAMICS

The rate of ice growth, \( f \), is determined by the magnitude of the atmospheric and oceanic energy fluxes at the horizontal boundaries of the ice and by the thermal history of the ice. Thus \( f = f(x, y, h, t) \) is a function of position, ice thickness, and season and has the dimensions of length per unit time. Figures 2a and 2b illustrate the dependence of \( f \) on \( h \) and \( t \) in the Central Arctic. \( f \) is independent of ice motions and can therefore be prescribed on the basis of field observations or from theoretical calculations.

To determine how ice growth influences the thickness distribution, consider a region \( R \) which has a certain thickness distribution \( G(h, t) \). We assume that \( f \) is uniform on \( R \) and ignore the velocity field. Further, we
Fig. 2. The thermodynamic growth a) as a function of ice thickness for summer and winter conditions, and b) as a function of time of year for thick and thin ice.

require that \( f \) be smooth in \( h \) and \( t \). After some time \( \delta t \), ice will grow (or melt) from thickness \( h \) to thickness \( h + f(h,t)\delta t \). Then it follows that all the ice which is thinner than \( h \) at time \( t \) will be thinner than \( h + f\delta t \) at time \( t + \delta t \), provided only that \( f \) is smooth and \( \delta t \) is sufficiently small.

Thus

\[
G(h,t) = G(h + f\delta t, t + \delta t) \\
= G(h, t + \delta t) + f \frac{\partial G}{\partial h} \delta t + \text{terms in } \delta t^2.
\]

After rearranging and dividing by \( \delta t \), we obtain

\[
\frac{1}{\delta t} \left( G(h, t + \delta t) - G(h, t) \right) = -f \frac{\partial G}{\partial h} + \text{terms in } \delta t.
\]

Then, in the limit where \( \delta t \) vanishes,

\[
\frac{\partial G}{\partial h} \bigg|_{\text{no dynamics}} = -f \frac{\partial G}{\partial h}.
\]
THE DYNAMICS

Consider the behavior of the Lagrangian element $R$ in response to the velocity field $U(x, y, t)$ of the ice. In the absence of thermodynamics, the change in the area, $R$, with time is

$$\frac{dR}{dt} = \oint_{\text{boundary}} U \cdot n \, d\sigma$$

where $n$ is the outward unit normal to the boundary of the region. Using the divergence theorem and dividing by $R$, we obtain

$$\frac{1}{R} \frac{dR}{dt} = \iint \nabla \cdot U \, dx \, dy$$

and, with some loss of rigor, we call this quantity $\text{div} \, U$, the average value over the region of the divergence of the velocity field $U$. As the ice diverges ($\text{div} \, U > 0$) or converges ($\text{div} \, U < 0$), ice within $R$ is redistributed to form open water and/or "pressure ice." This dynamic redistribution of ice is described by a function $W(h, t)$ such that

$$W \, \text{div} \, U = \frac{1}{R} \frac{\partial A}{\partial t}.$$  \hspace{1cm} (4)

$R \, W \, \text{div} \, U$ is the rate of change of the area of ice in $R$ of thickness less than $h$, due to dynamic processes alone. Before examining the form of $W$, we proceed formally to derive the dynamic equation

$$\frac{\partial G}{\partial t} = \frac{\partial}{\partial t} \left( \frac{A}{R} \right) = \frac{1}{R} \frac{\partial A}{\partial t} - \frac{A}{R^2} \frac{dR}{dt}$$

$$= W \, \text{div} \, U - \frac{A}{R} \, \text{div} \, U$$

or

$$\left(\frac{\partial G}{\partial t}\right)_{\text{no thermodynamics}} = (W - G)\text{div} \, U. \hspace{1cm} (5)$$

Since the rate of change of $A(h, t)$ is $R \, W \, \text{div} \, U$, we observe that

$$W = \begin{cases} 0, & h \leq h_0 \\ 1, & h \geq h_{\text{max}} \end{cases}$$  \hspace{1cm} (6)
To express the total volume of ice in $R$, we note that $dA = \frac{\partial A}{\partial h} dh$ is the area of ice in $R$ in the thickness band $h$ to $h + dh$. Multiplying by the thickness $h$ and integrating over all $h$, we find the total volume

$$V = \int_0^{h_{\text{max}}} h \frac{\partial A}{\partial h} dh.$$  

Since $W$ merely redistributes existing ice, it cannot alter the total volume of ice in $R$. This is a consequence of choosing a Lagrangian framework where no new ice is brought into $R$ through dynamic processes. Therefore,

$$\frac{dV}{dt} = 0,$$

and we obtain the following integral constraint:

$$\frac{dV}{dt} = \int_0^{h_{\text{max}}} h \frac{\partial A}{\partial h} \frac{\partial W}{\partial t} dh = \int_0^{h_{\text{max}}} h \frac{\partial A}{\partial h} (R \cdot W \cdot \text{div} \ U) dh,$$

which implies

$$\int_0^{h_{\text{max}}} h \frac{\partial W}{\partial h} dh = 0.$$  \hspace{1cm} (7)

Examining special cases helps to clarify the definition of $W$ and the constraints on it.

A. **Divergence**

During an episode of divergence, physical intuition suggests that open water is created, but otherwise the ice in $R$ is unaltered by the dynamics. Thus, in a time interval $\delta t$, the amount of open water which forms in $R$ is equal to $R \cdot \text{div} \ U \cdot \delta t$; and consequently, for all $h > 0$, the area of ice thinner than $h$ is increased by $R \cdot \text{div} \ U \cdot \delta t$. Equation (4) then implies that

$$W = \begin{cases} 0, & h \leq 0, \\ 1, & h > 0. \end{cases}$$
In the simplest case of $R(0) = 1$, $A(0^+, 0) = 0.5$, $\text{div } U > 0$, we obtain the solutions $R(t) = e^{\text{div}Ut}$, $A(0^+, t) = e^{\text{div}Ut} - 0.5$, $G(0^+, t) = 1 - 0.5e^{-\text{div}Ut}$, sketched in Figure 3.

![Figure 3](image)

**Fig. 3.** Time-dependent solutions for the constant divergence situation. The area of the region $R(t)$ and the area of open water within $R$ both grow without bound. The fractional area of open water approaches unity for large $t$.

**B. Convergence with rafting**

Consider the situation in which $R$ is entirely covered with ice of 10 cm thickness, and $\text{div } U$ is negative. Assume a simple rafting model in which the area of 10 cm ice must decrease and the area of 20 cm ice increase. $\mathcal{W}(h, t)$ has the following form:

$$
\mathcal{W} = \begin{cases} 
0, & h \leq 10 \text{ cm}, \\
C, & 10 < h \leq 20 \text{ cm}, \\
1, & h > 20 \text{ cm}.
\end{cases}
$$

From the sketch in Figure 4 we see that $\mathcal{W}$ has a step at $h = 10$ cm and at $h = 20$ cm. For the purpose of applying equation (7), the integral constraint, we interpret the derivative of $\mathcal{W}$ in terms of delta functions:

$$
\int_0^{h_{\text{max}}} h \frac{\partial \mathcal{W}}{\partial h} \, dh = \int_0^{h_{\text{max}}} h \left[ C \delta(h - 10) + (1 - C)\delta(h - 20) \right] dh = 0.
$$
Thus $10C + 20(1 - C) = 0$, which implies that $C = 2$. The result is that the area of 10 cm ice, $A(10,t)$, is consumed at a rate greater than the rate of convergence. From equation (5) we see that the normalized area of 10 cm ice, $G(10,t)$, initially changes at a rate $\text{div } U$ but approaches a rate of $2(\text{div } U)$ as $G(10,t)$ approaches zero. Letting $R = 1$ initially, we have sketched the solutions $R(t) = e^{\text{div } Ut}$, $A(10,t) = 2e^{\text{div } Ut} - 1$, and $G(10,t) = 2 - e^{-\text{div } Ut}$, in Figure 5.

Fig. 5. Time-dependent solutions to the constant convergence with rafting situation. When the original area is decreased by one half, $R(t_1) = 0.5$, all the 10 cm ice has been consumed in rafting, $A(10,t_1) = G(10,t_1) = 0$. At this time all of the ice is 20 cm thick; $A(20,t_1) = 0.5$, $G(20,t_1) = 1$. 

Fig. 4. The ice redistribution function, $\tilde{w}$, for the simple rafting case.
C. General convergence

We are free to choose any form of $\dot{W}$ consistent with the previously mentioned constraints; however, field data and physical intuition further restrict the choice. Observations indicate that ridging in multiyear ice is infrequent, and we therefore consider the area change due to such ridging to be negligible. Observations also show that rafting and ridging can result in the creation of pressure ice of almost any thickness up to some $h_{\text{max}}$. A reasonable model therefore appears to be one which crushes the thinnest ice first and then redistributes it in some specified manner over the entire range of ice thickness. For the present discussion, we assume that the area of ice destroyed by crushing is redistributed uniformly over the range $h_0$ to $h_{\text{max}}$. With this assumption and using equation (7), we find that $\dot{W}$ has the form

$$\dot{W} = \begin{cases} 
0, & h \leq h_0 , \\
\frac{h_{\text{max}} + h_0}{h_{\text{max}} - h_0} - \frac{2h_0}{(h_{\text{max}} - h_0)^2} (h - h_0), & h_0 < h < h_{\text{max}} , \\
1, & h \geq h_{\text{max}} .
\end{cases} \quad (8)$$

A sketch of this $\dot{W}$ is shown in Figure 6.

![Figure 6. A proposed form of $\dot{W}$. This redistributor crushes only the thinnest ice available ($h = h_0$) and, conserving volume, creates areas of deformed ice smeared uniformly over thicknesses from $h_0$ to $h_{\text{max}}$.](image)

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Several comments are in order. First, although area is distributed uniformly with our particular choice of \( \mathcal{W} \), the volume is not, and the net effect is that proportionally more of the volume of crushed ice goes into creating thick ridges than goes into creating thin ridges. Second, given that the thinnest ice gets crushed first, it is true in general that the rate of crushing exceeds the rate of convergence. Finally, the simple form of \( \mathcal{W} \) discussed in the diverging case (Eq. 6) is seen to be a special case of equation (8) in which \( h_0 = 0 \). As long as \( h_0 \) remains at zero, the area being consumed is covered by open water (ice of zero thickness) and the redistribution function is trivial.

**THE GOVERNING EQUATION FOR THE ICE THICKNESS DISTRIBUTION FUNCTION**

The thermodynamic and dynamic processes act independently in the sense that it has been possible to describe each process without reference to the other. Nevertheless, the two processes do interact because they both depend on \( G \) and they both alter \( G \). Having developed the response of \( G \) to the thermodynamics and to the dynamics acting separately, we now write that the net change in \( G \) is the sum of the dynamic and thermodynamic changes given in equations (3) and (5).

\[
\frac{\partial G}{\partial t} + f \frac{\partial G}{\partial h} = (\mathcal{W} - G) \text{div} U
\]  

(9)

This equation describes the behavior of \( G(h,t) \) defined on a region \( R \) for \( 0 < h < h_{\text{max}} \) and for all time. The characteristics of the equation are defined by \( \frac{dh}{dt} = f \), and are sketched as solid lines in Figure 7. The dashed lines represent the zero crossings for \( f \). Along the characteristics, equation (9) reduces to the ordinary differential equation

\[
\frac{DG}{Dt} = (\mathcal{W} - G) \text{div} U .
\]  

(10)
Fig. 7. Characteristics in \((h,t)\) space for equation (9). The slope of the characteristic passing through a point \((h,t)\) is given by \(\frac{dh}{dt} = f(h,t)\). The characteristics slope to the left in the summer and, except for the very thickest ice, to the right in the winter. There is a special characteristic (the broken line) which repeats itself exactly from year to year. Since all other characteristics approach this curve, it is called the thermodynamic equilibrium characteristic.

A. Boundary Conditions

The integration of equation (10) can proceed in either direction along a characteristic, provided some value of \(G\) on that characteristic is specified. This applies to characteristics which cross the time axis, just as it does to characteristics crossing the \(h\) axis. Seeking solutions to the open rectangular region in Figure 7, we take the following as initial and boundary conditions:
1. \( G(h,0) \) is some prescribed initial condition which must be monotonic, but not necessarily smooth.

2. \( G(h_{\text{max}}, t) = 1 \) for all \( t \). Note that \( h_{\text{max}} \) is prescribed, but is not necessarily the smallest \( h \) such that \( G(h, t) = 1 \).

3. \( G(0^+, t) = \begin{cases} 0, & f(0^+, t) > 0, \\ \text{unspecified}, & f(0^+, t) \leq 0. \end{cases} \)

The last condition requires some comment. For \( f(0^+, t) > 0 \), there can be no open water. If, during divergence, open water is created, it is instantaneously acted upon by the thermodynamics to create very thin ice. Therefore, \( G \) is continuous at zero and \( G(0^+, t) = 0 \). However, when \( f(0, t) \leq 0 \), values on the characteristics crossing the \( h = 0^+ \) boundary (see Figure 7) have been specified previously, and it would overdetermine the problem to specify them again.

B. An Example

A simple example serves to illustrate the basic physical ideas and how they are incorporated into the governing equation. Starting with ice of some thickness \( h_1 \) covering the entire region, consider what happens if the ice is converging and if the thermodynamic function \( f(h, t) \) is simply a positive constant \( f \). To proceed with the simplest mathematics, we adopt, for this example, the pure rafting model discussed earlier. The effect of the thermodynamics is to move the step, initially at \( h = h_1 \), to the right with a velocity \( f \). By contrast, the effect of the dynamics is to reduce the area of ice of thickness \( h_0 \) and to build, by rafting, ice of thickness \( 2h_0 \). Figure 8 compares the effects of the dynamics and the thermodynamics. Initially, \( h_0 \) moves at the rate \( f \), i.e., it follows a characteristic. To see this formally, we use the fact that \( G(h_0(t), t) = 0 \). Then by differentiating

\[
0 = \frac{dG}{dt} = \frac{\partial G}{\partial h} \frac{dh_0}{dt} + \frac{\partial G}{\partial t},
\]

and using equation (9),

\[
\frac{dh_0}{dt} = \left( f - \frac{\dot{W}}{\partial G/\partial h} \right)_{h=h_0}.
\]
In the present example $G$ is initially a step at $h = h_0$, so that $\partial G/\partial h$ is a delta function. Consequently, $\partial h_0/\partial t = f$, as expected. After some time, $\tau$, however, the dynamics will have eliminated the step at $h_0$, $\partial G/\partial h$ will be finite, and the motion of $h_0$ will involve both terms on the right-hand side of equation (11). Figure 9 shows the characteristics and the lines $h_0(t)$ and $2h_0(t)$. The characteristics in this case are straight lines with slope $f$. Following a characteristic in $(h,t)$ space, we have

$$\frac{DG}{Dt} = \begin{cases} 0 & h < h_0(t), \\ (2 - G)\text{div } U & h_0(t) \leq h < 2h_0(t), \\ 0 & h_0(t) \leq h. \end{cases}$$

$G$ has exactly the same shape following any characteristic (Fig. 10). If some particular characteristic, $AA'$ say, intersects the $2h_0(t)$ line at time $t_1$, then

$$G = \begin{cases} 1 & t < t_1, \\ 2 - e^{-\text{div } U(t - t_1)} & t_1 < t < t_2, \\ 0 & t_2 < t. \end{cases}$$
Fig. 9. The solution region for the case of constant $f$ and constant convergence with simple rafting. The lines $h_0(t)$ and $2h_0(t)$ divide the region into three parts. Changes in $G$ occur only between the two lines.

The time interval $t_2 - t_1$ is a constant equal to $-\frac{1}{\text{Div} \ U}$ ln2. A profile of $G$ versus $h$ for fixed $t$ crosses characteristics (see the line $BB'$ in Figure 9) and discontinuities are possible; such a situation is drawn in Figure 11. Physically this discontinuity is a residue from the initial step assumed in $G(h,0)$.

To summarize, the importance of this example has been to illustrate the following:

1. How the thermodynamics determine the characteristics.
2. How the dynamics change the value of $G$ following a characteristic.
3. That $h_0(t)$ is determined by both the thermodynamics and the dynamics.
4. That the solution surface is well behaved along, but may be discontinuous across, the characteristics.
5. That discontinuities are damped out by the dynamics.
Fig. 10. The shape of $G$ following the characteristic $A A'$ in Figure 9. Along any characteristic $G$ obeys the ordinary differential equation (10).

Fig. 11. $G$ versus thickness for fixed time along the line $B B'$ in Figure 9. The step in $G$ at $h_0$ will decrease because of the dynamics until it disappears at time $T$. After $t = T$, the line $h = h_0(t)$ moves to the right faster than the characteristics.

DISCUSSION

The ice thickness distribution, $G$, has ramifications in all areas of sea ice research. For example, $G$ regulates the heat exchange between the atmosphere and the ocean, and it determines the navigability of ice-covered waters. Of foremost importance, from our point of view, is that $G$ determines the rheological properties of the ice pack. It is for this reason that Coon and others (1973, unpublished notes) have correctly identified $G$ as a state variable, or, more precisely, a state function. Other authors have tried to use the compactness to characterize the state of the ice, but in doing so
have failed to allow for the disparity in properties between thick sea ice and thin.

The theory advanced here explains how $G$ varies in time. This treatment is based on the following assumptions:

A. There is a given function $f(h,t)$ which describes the growth and melting of sea ice.

B. The change in area of a region of sea ice results in a redistribution of ice within the region. In divergence, the ice breaks apart; rather than stretching and becoming thinner, it fails, creating open water. In convergence, the thinnest ice is crushed first and piled in some arbitrary way to create thicker, ridged ice.

Weaknesses in the present formulation are that shearing is ignored and that $W$ is arbitrary. Nevertheless, the equations developed from these assumptions do appear to contain the basic physics. In future work, we intend to explore the consequences of these equations. First, we will seek a stable periodic solution using realistic forcing. Second, we plan to examine the sensitivity of the periodic solution to assumptions imbedded in $W$, $f$, and $\text{div } U$. Finally, we intend to determine the response of $G$ to transients in the forcing functions. The theory will be tested by comparing these results with any available field observations, notably those of Wittmann and Schule [1966].

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REFERENCES


THE STEADY DRIFT OF AN INCOMPRESSIBLE ARCTIC ICE COVER

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ABSTRACT

The steady drift of sea ice in an idealized Arctic Basin has been calculated assuming that the ice is incompressible and inviscid. The momentum and continuity equations for the ice are solved for the velocity and the ice pressure. The divergence of velocity is assumed to be $0.33 \times 10^{-8}$ sec$^{-1}$. The boundary conditions require that no ice flow across coastal boundaries, but that ice flow out of the basin into the Greenland Sea and into the basin from the Kara Sea.

The patterns of calculated velocities and vorticities are realistic, but their magnitudes are too high. When the wind stress is reduced to one-third the strength first assumed, realistic speeds and vorticities are obtained. The maximum calculated ice pressure of about $10^8$ dyne cm$^{-1}$ (pressure integrated through the ice thickness) is marginally able to ridge thick ice, according to the ridging model of Parmerter and Coon [1973]. These maximum values occur near Greenland where Wittmann and Schule [1966] report intense ridging. Coastal shear zones on the order of 100 km wide might be represented by the added effect of a shear viscosity of about $6 \times 10^{12}$ gm sec$^{-1}$.

INTRODUCTION

Over the past few decades, as research stations and a few beset ships have drifted over the Arctic Ocean, a general picture of the mean drift pattern of arctic sea ice has evolved (Fig. 1). The pattern consists of an anticyclonic gyre in the Beaufort Sea, the transpolar drift stream running from the New Siberian Islands to the Greenland Sea, and a cyclonic gyre east of Severnaya Zemlya. Drift velocities (calculated from net rather than total distance traveled) are known to be about 3 cm sec$^{-1}$ over most of the
basin. Ice leaves the basin, between Greenland and Spitsbergen, with roughly three to four times this velocity.

Considering that there have seldom been more than three points on the arctic ice cover simultaneously tracked, one could hardly claim that the field of motion of the ice is well documented. Indeed, it is not even known to what degree the large-scale deformation of the ice cover is spatially continuous. Throughout this paper, velocity and its spatial gradients are assumed to be defined and spatially continuous. In view of the sparse data, any more complex assumption would be unjustified and premature.

Theories for the motion of pack ice have been proposed at many levels of complexity. They all agree that wind and currents are the dominant driving forces. In fact, the free drift (free of the influence of internal stresses) is not a bad first approximation to the observed drift, as a comparison of Figure 2 with Figure 1 shows. But some details of the free drift
are unrealistic. For instance, the transpolar drift stream is directed toward Greenland and Ellesmere Island rather than out to the Greenland Sea, and the gyre in the Beaufort Sea appears to be centered at least 200 km too far to the east. In addition, the free drift does not generally conform to the boundaries of the Arctic Basin.

The concept of internal ice stress seems to have been used first by Sverdrup [1968]. This quantity is assumed to account for lateral transfer of momentum in pack ice through the poorly understood mechanisms by which neighboring pieces of ice scrape, push, bump, and grind against one another. Much of the difference between various theories of ice dynamics lies in the variety of hypotheses about the constitutive equation relating this stress to other variables. Three major classes of hypotheses have been made by various researchers in actual calculations: (1) that the ice possesses

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Fig. 2. The calculated drift of sea ice with no internal stress, similar to that calculated by Fel'zenbaum [1958]. A velocity vector one grid space long represents 10 cm sec$^{-1}$. 

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a shear viscosity; (2) that the ice behaves like a cavitating fluid; and (3) that ice is incompressible and cannot sustain shear stresses.

Laikhtman [1958] and Ruzin [1959] introduced the idea that ice can be described as viscous in shear; that is, that the internal ice stress equals twice the shear viscosity multiplied by the strain rate deviator. This law is unsatisfactory, because it in no way restricts the divergence of velocity (hereafter, simply divergence). Consequently, the theory predicts an unrealistically large convergence in the Beaufort Sea, as Campbell [1965] has shown and discussed. Campbell and Rasmussen [1972] have demonstrated, in one of their cases, that the addition of a bulk viscosity (a stress term proportional to the divergence) reduces this convergence.

In a second hypothesis, introduced by Nikiforov et al. [1967], the ice behaves essentially like an ideal, cavitating fluid. This means that it has two phases. In the compact phase, it is incompressible and, in the absence of sources, nondivergent. Pressures are positive in this phase. Rather than support negative pressures (tension), the material opens up, or cavitates, and enters a second phase in which compactness decreases to below unity. The motion in this phase is identical to the free drift described above. Doronin [1970] has combined the cavitating behavior with a shear viscosity which increases linearly with compactness. This is the most complete model yet reported. It includes thermal source terms in prognostic equations for both compactness and ice thickness, as well as a momentum equation. An unappealing aspect of the method used by Nikiforov et al. and by Doronin is that the pressure is not explicitly retained in the compact phase. Instead, the nondivergence at compact points is maintained by an artifice.

The hypothesized behavior studied in this paper and previously by Rothrock [1972] and by Witting and Piacsck [1972] is a simplification of the behavior just discussed. It is assumed that only the compact phase exists, so that the ice is everywhere incompressible (and inviscid). As in the ideal, cavitating fluid, shearing motion develops no stress. Any driving forces tending to cause compression or dilatation are balanced by the gradient of the isotropic, two-dimensional internal ice stress, called pressure. As defined here, pressure is a true stress integrated through the thickness of the ice and has dimensions of force divided by length.
Incompressibility requires only that the divergence be unrelated to the pressure, not that it be zero.

The problem is formulated in terms of the unknown ice velocity and pressure. These quantities are determined from the continuity equation, in which the divergence is specified, and the momentum equation. The velocities, vorticities, and shear rates are compared to observed quantities. The pressure field is interpreted in the light of known characteristics of sea ice distribution and theoretical work relating pressure and ridge formation.

Only the steady case is considered in this paper. High-frequency motions of the ice in response to storms, for example, are assumed to be superimposed on the steady motion. They cause repeated cycles of opening of the ice cover, production of young ice on this open water, and closing accompanied by some ridging. The net effect of these cycles is to cause the long-term mean divergence in the present formulation.

This steady divergence is assumed to have a uniform spatial distribution. Because of the small magnitude of the nondimensional divergence, this assumption is not so restrictive as it might seem. This point will be amplified later. A value for the average divergence (in space and time) can be obtained by using Gauss's theorem, which relates the average divergence within an area to the net export from the area. Two drifting stations, NP-1 and Arlis II, have exited the basin between Greenland and Spitsbergen, and it is known that velocities there are about 10 cm sec\(^{-1}\) over a width of about 400 km. This gives an area export of \(4 \times 10^8\) cm\(^2\) sec\(^{-1}\), which is near the median of the values reviewed by Vowinkel and Orvig [1961]. Drift from the Kara Sea between Franz Josef Land and Severnaya Zemlya is also poorly documented, but the drift of the St. Anna [Lappo, 1958] suggests that a northward velocity of about 3 cm sec\(^{-1}\) exists there over a width of about 600 km, giving an import of \(1.8 \times 10^8\) cm\(^2\) sec\(^{-1}\). Because no documentation has been found for them, exports between Spitsbergen and Franz Josef Land and through the Canadian Archipelago and the Bering Sea are assumed to be negligible. The net export, then, divided by the idealized basin area of \(6.24 \times 10^6\) km\(^2\) implies an average divergence of \(+0.35 \times 10^{-8}\) sec\(^{-1}\).
There is no \textit{a priori} reason that the average divergence must be positive. If the driving forces were in the right direction, there might be a long-term mean import into the Arctic Basin of ice formed in, say, the Barents and Kara seas. The ice might be ridged into thicknesses sufficiently great that ablation exceeds accretion. However, the scanty data referred to above imply a positive divergence, and results to be described here show that a negative divergence causes unrealistic drift patterns.

**THE GOVERNING EQUATIONS**

The momentum equation and the continuity equation for the steady drift of an incompressible ice cover have been solved in an idealized Arctic Ocean. The ocean is represented (as in Figure 1) by the interior of a rectangle 2000 km wide and 3200 km long with two rectangular deletions corresponding to land masses. Boundary conditions applied to various boundary segments will be described later. The origin of the coordinate system is taken at the corner near Spitsbergen (78.72° N, 6.34° E). The \(y\)-axis lies along the Canadian Archipelago. The \(x\)-axis, which makes an angle of 52.3° with the Greenwich meridian, runs north of the Barents Sea and through the Kara Sea. All vectors and vector operators lie in the \((x,y)\)-plane with the exception of the unit vector \(\mathbf{k}\) perpendicular to this plane. In vector products, \(\mathbf{k}\) serves to rotate other vectors through an angle of \(\pi/2\) in the plane.

Terms representing the stresses applied to the upper and lower surface of the ice by the air and water will appear in the momentum equation. These stresses are defined by a modified Ekman layer theory, in which stress is proportional to the velocity of the geostrophic flow (just outside the frictional layer) relative to the ice. The angle of turning from this relative geostrophic velocity to the stress vector is less than the 45° of the classical Ekman theory.

Using this theory,

\[
\tau_a = \kappa_a f^{-3/2} (\cos \theta_a \mathbf{a} \cdot \mathbf{v} - \sin \theta_a \mathbf{V}_P)
\]

where \(\kappa_a\) is the eddy viscosity in air, \(f\) is the Coriolis parameter, and \(\mathbf{V}_P\)
The angle $\theta$ is the counterclockwise angle from the surface geostrophic flow vector (in the direction $\overrightarrow{U}$) to the stress vector. The ice velocity has been neglected, with an error of a few percent, in comparison to the geostrophic wind. Observations of the atmospheric boundary layer [Fiedler and Panofsky, 1972] show angles of turning of $15^\circ$ to $30^\circ$ under the stable conditions of the arctic winter and less than $15^\circ$ in neutral conditions. In addition, such reduced angles are predicted by several boundary layer theories, including a secondary flow theory [Brown, 1970] of the steady boundary layer and solutions of the Navier-Stokes equations [Deardorff, 1972]. In this paper, $\theta$ is taken to be $20^\circ$. The value of $3 \times 10^8$ cm$^2$ sec$^{-1}$, which is assumed for $\kappa$, is given by Faller and Kaylor [1966] for a neutral boundary layer with a surface roughness appropriate for arctic sea ice.

The relative motion of the ice cannot be ignored in the calculation for water stress. The resultant expression is

$$\overrightarrow{\tau}_w = C[\cos\theta_w (\overrightarrow{U}_w - \overrightarrow{u}) + \sin\theta_w \overrightarrow{kx}(\overrightarrow{U}_w - \overrightarrow{u})]$$

where the drag coefficient $C$ is $\rho_w \frac{1}{2}$, $\rho_w$ being the density of sea water and $\kappa$ the eddy viscosity. $\overrightarrow{u}$ is the ice velocity, $\theta_w$ is the angle of turning, and $\overrightarrow{U}_w$ is the geostrophic flow beneath the modified Ekman layer and is equal to $gf^{-1/2} \overrightarrow{kx}H$, where $g$ is the gravitational acceleration. Coachman's [1962] long-term values have been used for the sea surface height $H$. For convenience, the stress $\overrightarrow{\tau}_w$ is redefined in two parts. The first part,

$$\overrightarrow{\tau}_{w1} = -C(\cos\theta_w \overrightarrow{u} + \sin\theta_w \overrightarrow{kx}\overrightarrow{u})$$

is linear in the unknown velocity $\overrightarrow{u}$, and is the stress caused by dragging the ice over a stagnant ocean. The second part,

$$\overrightarrow{\tau}_{w2} = Cgf^{-1}(\cos\theta_w \overrightarrow{kx}H - \sin\theta_w \overrightarrow{VH}),$$

is the stress imposed on the ice by the movement of currents. It is independent of the velocity $\overrightarrow{U}$. Values for $\kappa$ and $\theta_w$ of $24$ cm$^2$ sec$^{-1}$ and $20^\circ$
have been adopted, based on the observations of Hunkins [1966]. It should be emphasized that estimates of both the air and water stress are subject to errors of a factor of two or three.

The first governing equation is the momentum equation which involves these forces per unit area: the Coriolis force \(-mfk\sin\omega\) (where \(m\) is the mass of ice per unit area), the air and water stresses just defined, the gravitational force pulling the ice down the slope of the sea surface \(-mg\nabla H\), and the negative gradient of ice pressure \(-\nabla p\). The prescribed stresses are summed in one term

\[
\bar{\tau} = \bar{\tau}_\alpha + \bar{\tau}_\omega - mg\nabla H
\]

which is dominated by the wind stress \(\bar{\tau}_\alpha\). (At a typical point in the tranpolar drift stream, the magnitudes of the three vectors on the right-hand side of (1) are, respectively, 0.67, 0.09, and 0.06 dyne cm\(^{-2}\).) Then the momentum equation can be written in form

\[
-A\bar{\omega} - Bf\omega - \nabla p + \bar{\tau} = \bar{\mu}\bar{u} \cdot \bar{\omega} \approx 0
\]

where \(A\) is \((C\cos\theta_\omega)\), and \(B\) is \((mf + C\sin\theta_\omega)\).

The advective acceleration \(\bar{\mu}\bar{u} \cdot \nabla \bar{u}\) has been neglected. It is three orders of magnitude smaller than the dominant terms in equation 2 as the following scaling argument demonstrates. Since the velocities forced by winds and currents can be seen from (2) to be of order \(|\bar{\tau}|/A\), the ratio \(|\bar{\mu}\bar{u} \cdot \nabla \bar{u}|/|\bar{\tau}|\) is roughly \(m|\bar{\tau}|/(LA^2)\) where \(L\) is the shortest significant length scale. Inserting reasonable values, the ratio equals

\[
\frac{(300 \text{ gm cm}^{-2}) \cdot 1 \text{ dyne cm}^{-2}}{(500 \text{ km})(0.06 \text{ gm cm}^{-2} \text{ sec}^{-1})^2} \approx 1.6 \times 10^{-3},
\]

which is indeed small.

The other governing equation is a simplified form of the continuity equation

\[
\frac{\partial m}{\partial t} = - \text{div}(\bar{\mu}\bar{u}) + m\Phi
\]

(3)
in which $\Phi$ represents sources of ice by thermal processes. The left-hand side is zero in the steady problem, and $m$ is assumed to be uniform in space. Then the equation becomes

$$\text{div } \mathbf{u} = \Phi$$  \hspace{1cm} (4)

As discussed earlier, the divergence and, therefore, the area source are assumed to be spatially uniform.

To solve equations 2 and 4 for $\mathbf{u}$ and $p$, a stream function $\psi$ and a velocity potential $\phi$ are introduced, so that the velocity can be written as

$$\mathbf{u} = \nabla \psi (\mathbf{p}^\perp) + \nabla \phi$$  \hspace{1cm} (5)

The velocity potential must satisfy $\nabla^2 \phi = \Phi$, obtained by substituting (5) into (4). It is taken to be $(L^2 \tau^2 - \omega L)\phi$, where $L$ is the basin width (2000 km). The curl of equation 5 is the Poisson equation

$$\nabla^2 \psi = -\omega$$  \hspace{1cm} (6)

where $\omega$ is the vorticity (curl $\mathbf{u}$) and is found to be

$$\omega = A^{-1} \text{curl } \mathbf{r} - BA^{-1} \phi$$  \hspace{1cm} (7)

by combining the curl of equation 2 and equation 4. By eliminating the vorticity between the curl and the divergence of equation 2, a Poisson equation for $p$,

$$\nabla^2 p = m_f A^{-1} \text{curl } \mathbf{r}_a - (A^2 + B^2) A^{-1} \text{div } \mathbf{u},$$  \hspace{1cm} (8)

is obtained, which must satisfy the same boundary conditions applied to equation 6 but expressed in terms of $p$.

One of the following boundary conditions is assumed on each segment of the boundary. Particular cases will be discussed below. For convenience, a coordinate system is adopted in which $n$ is the distance along the outward normal to the boundary and $s$ is the distance in the tangent direction to the left of the outward normal. Superscripts denote vector components.
Known flux: \[ u^n = U(s) = \text{known} \]
\[ \Rightarrow \frac{\partial \psi}{\partial s} = - \frac{\partial \phi}{\partial n} + U(s) \]
\[ \Rightarrow \frac{\partial p}{\partial n} + BA^{-1} \frac{\partial p}{\partial s} = \tau^n + BA^{-1} \tau^s - (A^2 + B^2)A^{-1}U(s) \]

No slip: \[ u^s = 0 \]
\[ \Rightarrow \frac{\partial \psi}{\partial n} = \frac{\partial \phi}{\partial s} \]
\[ \Rightarrow \frac{\partial p}{\partial s} - BA^{-1} \frac{\partial p}{\partial n} = \tau^s - BA^{-1} \tau^n \]

Constant pressure: \[ p = \text{constant} \]
\[ \Rightarrow u^s + BA^{-1} u^n = A^{-1} \tau^s \]
\[ \Rightarrow \frac{\partial \psi}{\partial n} - BA^{-1} \frac{\partial \psi}{\partial s} = BA^{-1} \frac{\partial \phi}{\partial n} + \frac{\partial \phi}{\partial s} - A^{-1} \tau^s \]

(The no-slip condition does not restrict the velocity component normal to the boundary.)

Equation 7 and the two elliptic equations 6 and 8 are solved for \( \omega \), \( \psi \), and \( p \), finally obtaining \( \bar{u} \) from (5). Of course, if equations 5, 6, and 7 have been solved for the velocity, then the pressure can be found from equation 2. Conversely, if the pressure is first obtained from equation 8, the velocity can be obtained without integration from the expression

\[ \bar{u} = \frac{[A(\bar{\tau} - \bar{V}_p) - B\bar{k}x(\bar{\tau} - \bar{V}_p)]/(A^2 + B^2)} \]

which follows directly from equation 2. The procedure followed here is to find the velocity and ice pressure independently, and then to confirm that they satisfy equation 2.

Each of the Poisson equations (6 and 8) is solved by successive over-relaxation, the operator \( \nabla^2 \) having been replaced by the standard five-point difference operator. A square mesh of 200 km is used. At points where the boundary condition specifies the unknown, the value is held fixed during the relaxation. The treatment of boundary conditions involving derivatives depends on the nature of the boundary point. At regular points, the first-order
derivatives in the boundary condition are expressed as centered differences involving an image point outside the boundary, as used by Fox [1950]. The boundary conditions and the difference Poisson equation applied at the boundary point provide two equations to determine the values of the unknown at the boundary point and its image point. Singular points on the boundary (corners or points separating boundary segments on which different conditions apply) have no associated image point. There, the boundary condition of one of the adjacent segments is applied, expressed in first-order differences.

RESULTS

Equations 5-8 have been solved for several cases. In the standard case, the parameters have these values:

\[ m = 270 \text{ gm cm}^{-2} \]
\[ f = 1.458 \times 10^{-4} \text{ sec}^{-1} \]
\[ \kappa_\alpha = 3.0 \times 10^4 \text{ cm}^2 \text{ sec}^{-1} \]
\[ \theta_\alpha = 20^\circ \]
\[ \rho_\omega = 1.030 \text{ gm cm}^{-3} \]
\[ \kappa_\omega = 24 \text{ cm}^2 \text{ sec}^{-1} \]
\[ \theta_\omega = 20^\circ \]
\[ C \equiv \rho_\omega f^{1/2} \kappa_\omega^{1/2} = 6.18 \times 10^{-2} \text{ gm cm}^{-2} \text{ sec}^{-1} \]
\[ g = 983.217 \text{ cm sec}^{-2} \]
\[ A \equiv C \cos \theta_\omega = 5.70 \times 10^{-2} \text{ gm cm}^{-2} \text{ sec}^{-1} \]
\[ B \equiv mf + C \sin \theta_\omega = 6.01 \times 10^{-2} \text{ gm cm}^{-2} \text{ sec}^{-1} \]
\[ \phi = (1/3) \times 10^{-8} \text{ sec}^{-1} \]

The standard boundary conditions are that there be no components of velocity normal to the boundary anywhere, with the following exceptions. Along the two grid intervals on the Greenland Sea, the outward normal component of velocity is specified to be 9.46 cm sec\(^{-1}\). An inward normal component of 2.84 cm sec\(^{-1}\) is specified on three grid intervals in the Kara Sea.
The flow pattern for the standard case, shown in Figure 3, is in good agreement with the observed pattern. All the major features are present, including the cyclonic circulation east of Severnaya Zemlya. The transpolar drift stream heads in a quite realistic direction, unlike the stress-free case, and unlike the completely compressible viscous cases investigated by Campbell [1965]. There is a cyclonic circulation, although it does not appear in the figure, which occupies the corner cell in the Chukchi Sea.

Fig. 3. The calculated velocities and isobars for the standard case in which the ice is incompressible. A velocity vector one grid space long represents 10 cm sec\(^{-1}\). The isobars are labeled in units of 10\(^7\) dyne cm\(^{-1}\). Grid points are 200 km apart.

The velocity magnitudes, however, agree less satisfactorily with observations. In the transpolar drift stream, calculated velocities are twice the observed mean of 2.8 cm sec\(^{-1}\). Near the coasts bordering the Beaufort Sea, calculated velocities are about four times the observed values.
of 3.5 cm sec\(^{-1}\). The tendency for faster than average flow just off Prince Patrick Island is in agreement with observations, but it is greatly overestimated by this calculation.

The vorticity of sea ice can be estimated both from analyses of the positions of arrays of drifting stations and from the azimuth and position records of single drifting stations. Because the vorticity (\(\text{curl} \vec{u}\)) of a continuum is twice the average local rate of rotation, the vorticity of a drifting station is defined here as twice its rate of rotation. Although these two methods of estimating vorticity give similar values, no thorough attempt has been made to compare them. The station azimuths of T-3 [Bushnell, 1959; Cabaniss, 1962; and Hunkins, Kutschale, and Hall, 1969] and of stations Alpha and Charlie [Cabaniss, 1962] imply typical vorticities in the Beaufort Sea between \(-0.4 \times 10^{-7}\) and \(-2.4 \times 10^{-7}\) sec\(^{-1}\). These values, listed in Table 1, exclude the anomalously high summer rotations of T-3. They are compatible with array data (A. S. Thorndike, unpublished analysis, 1971) and the estimate by Bushuyev et al. [1967].

**TABLE 1**

**OBSERVED LONG-TERM RATES OF ROTATION FOR DRIFTING STATIONS**

<table>
<thead>
<tr>
<th>Label in Figure 11</th>
<th>Drifting Station</th>
<th>Period</th>
<th>Vorticity = twice rate of rotation, in units of (10^{-7}) sec(^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>T-3</td>
<td>Jan 1958 - Jan 1959</td>
<td>-0.48</td>
</tr>
<tr>
<td>b.</td>
<td>T-3</td>
<td>Sep 1964 - Aug 1965</td>
<td>-0.80</td>
</tr>
<tr>
<td>c.</td>
<td>T-3</td>
<td>Sep 1965 - Sep 1966</td>
<td>-2.4</td>
</tr>
<tr>
<td>d.</td>
<td>Charlie</td>
<td>Jun 1959 - Dec 1959</td>
<td>-1.2</td>
</tr>
<tr>
<td>e.</td>
<td>Alpha</td>
<td>Jul 1957 - Oct 1958</td>
<td>-0.44</td>
</tr>
<tr>
<td>f.</td>
<td>NP-1</td>
<td>Jul 1937 - Jan 1938</td>
<td>-2.0</td>
</tr>
<tr>
<td>g.</td>
<td>Sedov</td>
<td>Jul 1939 - Jan 1940</td>
<td>+0.27</td>
</tr>
</tbody>
</table>

In the standard case, the calculated vorticity is typically \(-2 \times 10^{-7}\) sec\(^{-1}\), several times larger than observed values. The vorticity for a case with reduced wind stress agrees more closely with these data and will be presented later in this paper.
The pressure varies on the scale of the whole basin. It allows the divergence to be maintained at its specified value and the flow to be restricted to obey the boundary conditions. The pressure is known only to within an additive constant, unless the boundary conditions fix its value at some point. In all cases discussed here, the zero of pressure is chosen arbitrarily to be the minimum calculated value in the idealized basin. Relatively low pressure is interpreted as being associated with open water, and relatively high pressure as suggestive of ridging. Of course, neither ridging nor opening can strictly be related to pressure in the incompressible approximation. In a more exact model the incompressible approximation must be modified to permit precisely these responses of opening (cavitation) and ridging at the extremes of pressure.

The pressure field shown in Figure 3 exhibits two regions of low pressure in Amundsen Gulf and near the New Siberian Islands. Both these regions have open water earlier in the spring and later in the autumn, as shown in Figure 4, than other regions. This phenomenon is undoubtedly related to continental run-off. However, regardless of the thermodynamic processes, substantial opening can occur only where the pressure is low. This comparison is justified since spring and autumn mean monthly surface winds do not differ qualitatively from the annual winds [Namias, 1958].

The high pressure in the Chukchi Sea coincides with the tendency, exemplified in Figure 4, for ice to remain in contact with the Siberian coast near Wrangel Island when there is coastal separation on either side. There is no pressure maximum in the center of the Beaufort Sea gyre. In fact, all maximums and minimums occur on the boundary. The greatest pressure occurs near Greenland and seems to be associated with the intense ridging just north of Greenland and Ellesmere Island, as shown in Figure 5. Intense ridging may also be related to a high rate of shear. The calculated shear field (similar to that shown in Figure 10) shows high shears in several locations around the periphery of the basin. The region observed to be highly ridged is that which combines high shear with high pressure.

The maximum pressure difference within the basin, for the standard case, is $12 \times 10^7$ dyne cm$^{-1}$. This value may be compared with the pressure necessary to crush a slab of sea ice (horizontally) when no buckling or
bending is allowed. For a failure stress in compression of $2 \times 10^7$ dyne cm$^2$, ice as thin as 30 cm requires a pressure of $60 \times 10^7$ dyne cm$^{-1}$ to be crushed. Even with the fetch of the whole Arctic Basin, such pressures are not available in the steady case.

Parmerter and Coon [1973] have calculated pressures necessary for ridges to form. For ice 3 m thick, their limit height model predicts that a pressure of about $5 \times 10^7$ dyne cm$^{-1}$ is required for a ridge to continue building indefinitely. Their kinematic model, which is more realistic, predicts that the required instantaneous pressure rises as high as $15 \times 10^7$ dyne cm$^{-1}$. Thus, thick ice is marginally capable of ridging in the limited region of high pressures shown in Figure 3. For 1 m ice, the range of pressures given by these two ridge models is an order of magnitude lower. Thinner ice can, therefore, be ridged almost everywhere in the basin.
Fig. 5. The observed ridging index, defined as the number of ridges per 30 nm, after Wittmann and Schule [1966].

The force balance expressed by equation 2 is shown in Figure 6(a). The net driving stress $(\tau - \nabla p)$ acts at an angle $\tan^{-1}(BA^{-1})$ to the left of the velocity $\vec{U}$ and balances the Coriolis force $\vec{C}$ and the ocean drag $\tau_{o1}$. ($\nabla p$ is zero for free drift.) The vectors $\tau$ and $\nabla p$ presumably can point in any direction as long as their sum satisfies the constraint of equation 2.

In the vicinity of the drift of the ice station Alpha, the velocity $\vec{u}$ is calculated to be $37^\circ$ to the right of the stress $\tau$ (which is approximately $\tau_{o1}$), as seen in Figure 6(b). In their analysis of the drift of that station for the year beginning July 1957, Reed and Campbell [1962] found this angle to be $35^\circ$. Unfortunately, $-\nabla p$ is not so significant in this region as in other regions [Figures 6(c) and (d)]. A more definitive test would result from comparison with a similar analysis of long-term mean wind and drift in another region. Figure 6(c) shows the balance typical of the transpolar drift stream, where the flow is more nearly in the direction of $\tau$. 

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Fig. 6. The force balance satisfying equation 2. The velocity (dashed) is also shown. (a) The general case. (b) The average for the area of the drift of ice station Alpha. (c) In the transpolar drift stream. (d) In the region of fast drift between the Canadian Archipelago and the Beaufort Sea gyre.

There are two regions where the negative pressure gradient performs work on the flow ($-\nabla p \cdot u > 0$). These are the areas in which the ice flows into a low-pressure region: near the Canadian Archipelago and near the East Siberian Sea. The balance in Figure 6(d) typifies these regions. Except in these regions, $-\nabla p \cdot u$ is negative. However, the work done by the driving stress $u \cdot \nabla$ is positive almost everywhere, substantiating the concept of a wind-driven flow.

Other solutions have been calculated with different combinations of boundary conditions on the outlet (between Greenland and Spitsbergen) and
on the $z$-axis. In one such case, the condition of no-slip is assumed on the outlet, and the $z$-axis is assumed to be a line of constant pressure. This condition is inspired by the fact that the observed position of the free boundary of the arctic pack (which must be a zero pressure boundary) fluctuates in the vicinity of the $x$-axis, although it is several hundred kilometers to the south during most of the year. In this solution, the area of high pressure moves to a position north of Greenland and Ellesmere Island, so that the isobars resemble quite closely the lines of constant ridging indices in Figure 5.

The corresponding flow field, however, is unsatisfactory due to an increased influx from the Kara Sea. The export through the outlet is about double the amount specified in the standard case, if the divergence is held at the standard value. If the divergence is given some negative value to reduce the export through the outlet, the flow pattern becomes unrealistic. The transpolar drift stream flows more in the direction from the Laptev Sea into the Greenland Sea. When the no-slip condition is applied on both the outlet and the $x$-axis, there is more export to the Barents Sea than to the Greenland Sea.

Other cases of interest are those in which islands (Spitsbergen and Franz Josef Land) occur as segments in the boundary. On either side of such an island boundary segment, either the no-slip or the constant-pressure boundary condition may be applied. (Neither of these conditions explicitly restrains the flux normal to the boundary.) One additional scalar must be specified for each boundary island in order to specify the circulation around that island: either the pressure difference from one end of the segment to the other, or the net flux across one of the adjoining boundary passages. However, there is no sound basis for setting pressure differences; and, if net fluxes are specified, the calculated velocity fields are not very different from those in which boundary fluxes are specified everywhere along the boundary.

In the remainder of this section, solutions will be described in which parameters are varied singly from their values in the standard case. The standard boundary conditions are retained. The velocities in the transpolar
drift stream and the Beaufort Sea gyre are unaffected by either a doubling or a halving of the mass concentration \( m \) to within 1\% in magnitude and 1\% in direction. As \( \theta \) increases from 0\° to 45\° or \( \theta_a \) decreases through this range, speeds typically increase by 25\%, while velocity directions vary by less than 2\° at most points. The variation in speed is due primarily to the change of the projections of \( \vec{T} \) and \( \vec{T}_{\parallel} \) on the velocity.

The pressure field exhibits much greater variability. Figure 7 shows the pressure field for a mass concentration double that of the standard case. The positions of the highs and lows are unaltered (by either \( m \) or \( \theta_b \)). The increased bulge of, say, the \( 6 \times 10^7 \) dyne cm\(^{-1} \) isobar compared with Figure 3 is associated with the accentuated curvature of the surface \( p(x,y) \) described by equation 8 (in which the last term is negligible). Figure 8 illustrates the sensitivity of the positions of highs and lows on the boundaries to \( \theta_a \). As \( \theta_a \) increases from 0\° to 45\°, the low-pressure region near the Laptev Sea

Fig. 7. The pressure field, in units of \( 10^7 \) dyne cm\(^{-1} \), for \( m = 540 \).
Fig. 8. The pressure field, in units of $10^7$ dyne cm$^{-1}$, for $\theta_\alpha = 45^\circ$ moves from the east coast of Severnaya Zemlya to the New Siberian Islands, a distance of about 900 km.

The overestimation of the kinematic quantities in the standard case has been described. These quantities are roughly proportional to the typical magnitude $\tau$ of the imposed stress divided by the water drag coefficient $A$. Because of the uncertainty in both $\tau$ and $A$, a case was calculated with the wind stress reduced by a factor of three. The typical drift speed in the transpolar drift stream, as seen in Figure 9, compares very closely with the observed mean of about 2.8 cm sec$^{-1}$. The speeds of about 5 cm sec$^{-1}$ near Prince Patrick Island are much more realistic than those in the standard case. Figure 10 shows a rate of shear, defined as

$$\frac{1}{2} \left[ \left( \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right]^{1/2}$$

where $(u, v)$ is the velocity, of about $0.2 \times 10^{-7}$ sec$^{-1}$ in the interior of
the basin. This long-term mean is compatible with observed instantaneous values of about $0.6 \times 10^{-7}$ sec$^{-1}$ (A. S. Thorndike, unpublished analysis, 1971). In Figure 11 the rotations of stations listed in Table 1 are compared to the calculated vorticity.

Fig. 9. The calculated velocities and isobars with one-third the wind stress of the standard case. A velocity vector one grid space long represents $5$ cm sec$^{-1}$. The isobars are labeled in units of $10^7$ dyne cm$^{-1}$.

The pressure, whose scale is the product of $\tau$ and the fetch (here the basin width), is also considerably changed, as Figure 9 shows. The point of maximum pressure near Greenland has shifted one mesh pont westward. The high-pressure region is now more isolated from the free boundary which, in reality, is not far south of the origin. The pressure magnitudes are such that thick ice cannot be ridged anywhere, but ice less than 1 m thick can still be ridged almost everywhere in the basin, according to the ridging model of Parmerter and Coon [1973].
A salient feature of the present theory is the neglect of shear stresses. This may be the reason that kinematic quantities tend to be overestimated when standard values are used for the parameters.

The effects of shear viscosity can be considered in some detail. If a term equal to the shear viscosity $\mu$ times twice the strain rate deviator is included in the internal ice stress, then a term $\mu \nabla^2 \tau$ is added to the left-hand side of the momentum equation 2. The importance of this term relative to the water stress is given by the dimensionless ratio $\mu/(L^2 A)$. ($L$ is the basin width, and $A$ the water drag coefficient.)

If $\mu/(L^2 A)$ is small, the velocity field of the incompressible case is affected only in a boundary layer of width $(\mu/A)^{1/2}$ along the coast. The flow
The calculated vorticity, in units of $10^{-7}$ sec$^{-1}$, with one-third the wind stress of the standard case. The straight line segments crudely represent the drift tracks of the stations whose vorticities are listed in Table 1.

parallel to the boundary within the layer is of the form

$$U\{1 - \exp[-(A/\mu)^{1/2} y]\}$$

where $U$ is the tangential velocity at the boundary predicted by the inviscid theory and $y$ is the perpendicular distance from the boundary into the basin.

When considered in the context of the Beaufort Sea, this boundary layer has a particularly noticeable feature: the vorticity, which is generally negative in the interior region, is positive in such a boundary layer. The sign change occurs at a distance of order $(\mu/A)^{1/2}$ from the coast. Furthermore, the magnitude of the vorticity is $U(\mu/A)^{-1/2}$, which is larger than the vorticity in the basin's interior by a factor of $[\mu/(L^2 A)]^{-1/2}$. 
For any value of $\mu/(L^2A)$, large or small, this sign change in vorticity should occur. As $\mu/(L^2A)$ becomes large, the coastal region of vorticity with sense opposite to that of curl $\vec{u}$ grows to extend more than half the distance to the center of the basin.

The drift of T-3, designated by $a$ in Table 1 and in Figure 11, was characterized by a vorticity of the same sign and magnitude as is typical of the rest of the Beaufort Sea. Provided that the rotation of this ice island is representative of the vorticity of the neighboring sea ice, a viscous boundary layer, if it exists at all, must be thinner than 100 km. An upper bound on the shear viscosity $\mu$ is therefore about $(100 \text{ km})^2A$ or $6 \times 10^{12}$ gm sec$^{-1}$.

Another relevant observation of rotation is reported by Lappo [1958]. It concerns the highly transient motions of the Chelyuskin while it was in close pack ice in the southeast Chukchi Sea, several miles off of the coast. According to Lappo’s description, the ship was drifting along the coast to the southeast rotating clockwise with vorticities of about $-1.7$ day$^{-1}$. The sense of rotation changed when the velocity reversed its direction. This description suggests the time-dependent version of the viscous boundary layer described here. It is not clear from Lappo whether the Chelyuskin (and the ice) or the ocean surface current is moving at 0.75 nm hr$^{-1}$ (40 cm sec$^{-1}$). However, with this value, the boundary layer thickness $(\mu/A)^{1/2} \approx U/\omega$ is 20 km, and the viscosity $\mu \approx A U^2/\omega^2$ is $2.4 \times 10^{11}$ gm sec$^{-1}$.

Campbell [1965] concluded that a viscosity of $0.9 \times 10^{15}$ gm sec$^{-1}$ gave the most realistic solution. Such a large value is appropriate only if combined with another assumption which permits slip at the coast. Doronin’s [1970] most realistic solution was obtained with a viscosity of $3 \times 10^{11}$ gm sec$^{-1}$, which is in good agreement with the above estimates.

**CONCLUSIONS**

This study was undertaken to test the assumption that the Arctic ice cover moves essentially as an incompressible fluid in the long-term mean. The results show that, under this assumption, realistic flow patterns which conform to the geometry of the basin can be calculated. The divergence of
of the velocity is restrained to have a more realistic value than that calculated either by neglecting internal ice stress altogether or by including only viscous shear stress. Resistance to compression is an important mechanical property of the Arctic ice cover.

When the uniform specified divergence is negative (with suitable boundary conditions), the transpolar drift stream is directed too much toward Ellesmere Island; all the ice exported to the Greenland Sea is supplied from the Kara Sea and cannot be older than first-year ice. In contrast, flow patterns in which the ice is assumed to diverge are more realistic. It is most likely that some regions tend to be less divergent than others, perhaps even convergent, but the average divergence over the whole basin is positive. Such spatial variations of divergence would not change the calculated flow patterns described above if the divergence had the same basin average and about the same magnitude. The divergence has little effect on the interior flow. This flow is dominated by the stress \( \tau \) as reflected in the small size (0.08) of the nondimensional ratio \( AL \) div \( \tilde{u}/\tau \).

The transmission of the curl of the wind stress through the ice to the ocean surface is important in ocean circulation theory. The curl of the stress applied to the ocean surface

\[
\text{curl} \left( -\vec{w} \right) = -m_f \text{div} \tilde{u} + \text{curl} \vec{\tau}_a \quad (9)
\]

is obtained from the curl of (1) and (2). Only a term involving div \( \tilde{u} \) affects the transmission. Two cases are of concern. In the incompressible case, the term \( m_f \text{div} \tilde{u} \) is less than one percent of curl \( \vec{\tau}_a \) for typical values of all quantities. If shear stresses were present, another term would need to be considered on the right-hand side of (9).

In the case of free drift, div \( \tilde{u} \) can be obtained from (8) (with \( \nabla^2 p = 0 \)). Then the ratio \( -m_f \text{div} \tilde{u}/(\text{curl} \vec{\tau}_a) \) equals \( -(m_f)^2/(A^2 + B^2) \), which is about -0.22 for values adopted here. Thus, the curl of the wind stress is transmitted through the ice with an efficiency of about 78%.

The calculated pressure field correlates well with observed synoptic ice conditions. However, the positive pressure in the corner near the Greenland Sea is not consistent with the presence of a free boundary of zero
pressure in the vicinity. Within the framework of the present theory, this corner pressure is sensitive to the intensity of the wind stress, as has been illustrated, to the local details of the flow which have been somewhat crudely represented here, and to the magnitude of the flux through the outlet. The corner pressure might also be changed by effects neglected here, such as cavitation (occurring anywhere in the basin) or shear stresses, which may permit ice jamming and steep pressure gradients near the outlet.

It could be primarily the high transient surface stresses associated with storms which drive the ridging processes. But internal stresses are proportional to the product of the surface stresses and their fetch, and it has not yet been shown whether this product is greater for transient phenomena or for the case studied here. It does appear, from a comparison of the present calculations and observed ridging intensity, that ridging is related to the long-term mean stress field.

Uncertainty in the magnitudes of $\bar{\tau}_a$ and $\bar{\tau}_w$ has made some conclusions tenuous. In particular, it cannot be stated whether the high velocities and vorticities calculated in the standard case are due to overestimates of $\bar{\tau}_a$ or to underestimates of the water drag coefficient or an unrealistic constitutive equation for the pack ice. The comparison of calculated pressures with those pressures predicted by ridging models is necessarily approximate.

Whether or not the inviscid incompressible approximation is sufficiently realistic, it does not appear that shear viscosity can be invoked to improve the prediction of flow in the interior of the basin. It may be proper to describe the shear zones near coasts as viscous boundary layers, with a shear viscosity of not more than about $6 \times 10$ gm sec$^{-1}$, but these zones might also be explained by other material properties such as plasticity.

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REFERENCES


DIFFERENTIAL SEA ICE DRIFT I:
SPATIAL AND TEMPORAL VARIATIONS IN
MESOSCALE STRAIN IN SEA ICE

by

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ABSTRACT

Measurements of mesoscale strain in sea ice were carried out over a five-week period in spring 1972 at the main AIDJEX camp in the Beaufort Sea. They have been analyzed to determine inhomogeneities in the strain as well as a least squares strain tensor time series. The least squares divergence between Julian day 83 and 112 exhibited five significant strain events consisting of dilatation followed by convergence. Net areal changes were as large as 3%. Data taken every three hours indicated divergence rates up to 0.12% per hour and shear rates as large as 0.10% per hour. In the principal axis coordinate system, the events typically exhibited a much larger compression (or extension) along one axis than along the other. It was found that by using a number of strain lines 8 km or longer the average strain rate tensor could be calculated with rate magnitudes larger than the inhomogeneity variation (variability of the average strain due to inhomogeneities). The inhomogeneity variation was found to scale inversely with the square root of the average length of the strain lines and to be only slightly dependent on frequency for frequencies above one cycle per day. The vorticity is, in most cases, similar to the rotation of the central station of the array and may be adequately estimated by calculating the average rotation of a set of randomly oriented strain lines.

Spectral and cross-spectral studies indicate that sampling intervals of up to 10 hours are generally adequate for resolving low-frequency strain rates without intolerable aliasing, that low-frequency events show significantly greater spatial correlation than do higher-frequency events, and that there is a significant coherence in the divergence rates of the different-sized arrays at approximately two cycles per day (in individual spectra these peaks every 12 hours are largely masked by random ice motion). Longer time series of several months are needed to adequately resolve the low-frequency behavior of the ice.
INTRODUCTION

One of the prime goals of the Arctic Ice Dynamics Joint Experiment (AIDJEX) is an improved understanding of the drift of pack ice. To this end one urgently needs accurate field observations of the deformation of different types of pack ice performed on a variety of time and space scales. To partially satisfy this need, a series of detailed mesoscale strain measurements were made at approximately 3-hour intervals over a 30-day period in the spring of 1972 in the Beaufort Sea. These measurements are particularly useful since in earlier deformational studies—as reviewed, for example, in Hibler et al. [1973]—there were usually large and random time intervals between observations which precluded the computation of accurate time series. Also, and perhaps even more important, those studies included no detailed investigation of the variations in the strain tensor that result from inhomogeneous spatial variations or fluctuations in the deformation of the ice, which are due to the highly heterogeneous nature of the pack ice and the random occurrence of deformation zones (leads and new ridges).

Therefore, our analysis described in this paper was undertaken with two primary goals in mind: first, to provide a detailed time series of the least squares strain rate tensor and vorticity (complete with inhomogeneity variation "error bars") over the 25-day period, Julian day 88-113 (28 March-22 April), 1972; and second, to study the magnitude, scale, and frequency dependence of the inhomogeneity variations.* These results can then be compared with predictions from theoretical drift calculations and with data collected on the remote sensing overflights. In addition, persistent effects in the deformation at different scales were examined, and spectral and cross-spectral studies were made to determine necessary sampling rates as well as coherent modes of deformation in different-sized arrays. Besides providing insight into the nature of pack ice dynamics, such spectral and strain inhomogeneity information is helpful in designing future strain arrays.

*By inhomogeneity variation we mean the statistical variability of the average strain obtained from a given strain array due to inhomogeneities in the velocity field.
SITE LOCATION AND DATA COLLECTION PROCEDURES

The measurements used for this study were made in the vicinity of the main 1972 AIDJEX camp, located approximately at 75°00'N, 148°30'W. The camp and the research programs carried out from it are described in AIDJEX Bulletin No. 14 (July 1972). The strain array was established by erecting a series of targets which consisted of corner cubes mounted on the tops of aluminum poles. The height of the targets varied from 3 to 10 meters above the ice surface depending on the distance and the obstructions between the target and main camp. A diagram of the strain array is shown in Figure 1, together with an overlay of active leads and ridging zones. (The leads shown in this mosaic were obtained from 1500 meter photography. Photos taken at higher altitude do not always resolve all active leads.) The angles of the targets were measured with an average accuracy of better than ±1 minute and were referenced to a fixed stake on the multiyear floe on which the main camp was sited. The line between the laser and the stake was then tied into the true north determinations (sun shots) made by Alan Thorndike and Allan Gill [Thorndike et al., 1972]. Distances were measured to the nearest 0.1 ft. because the large strains that were experienced obviated any greater precision.

This strain measurement system was found to be vastly superior to the use of manned tellurometer sites [Hibler et al., 1973]. With it, a large number of strain lines could be determined easily without manning the remote stations. It was also relatively quick and easy to install and placed a minimal reliance upon "black boxes." However, visibility problems (wind-blown snow, sea smoke from leads) made acquisition of continuous, equally spaced time series difficult (laser measurements were impossible approximately 10 percent of the time, and once high winds caused a gap of almost two days in the strain line time series). In addition, the system required manpower 24 hours a day.
Fig. 1. Schematic diagram of the strain array together with an overlay of active leads and ridging zones. Leads and ridges obtained from a 1500 meter aerial photo mosaic taken on 6 April 1972.
DATA ANALYSIS

Extrapolation and Smoothing of Strain Data

Data taken in the field consisted of distances and angles of targets relative to a fixed reference stake. As a first step in the data reduction, these angles were converted to angles relative to true north as measured on Julian day 81 (21 March). Rotations of the array were not taken into account for strain calculations; the coordinate system used for this study is therefore slightly different from the true north coordinate system (the maximum difference is, however, less than 5 degrees). Rotations of the array were, of course, included in the vorticity calculations. The time scale was converted to GMT by using four GMT calibration times obtained in the field to find a least squares rate for the clock used in the measurements. The data point times were recomputed with this rate and then all data (both angles and distances) were linearly interpolated and resampled every hour. Using this new data set based on the one-hour interval, the time series were smoothed with a low pass filter having a transition band width with periods from 8.0 to 6.15 hr. The smoothed time series was then resampled every third hour. If there was reason to expect that the data contained time intervals greater than 3 hours, a low pass filter having a transition band from 20 to 11.4 hrs was used before resampling. Both filters had less than 0.6% side lobe errors and consisted of 81 symmetric weights designed according to the procedure discussed by Hibler [1972].

This process of interpolation followed by smoothing may be viewed as a consistent way of constructing a smooth curve (with no high-frequency components past a reasonable cutoff dictated by the average sampling rate) through the randomly spaced data points. Alternatively, the curve may be considered an accurate representation of the low-frequency portion of the linearly interpolated curve.

Examples of the smooth curves generated by this process are shown in Figure 2. Curve a, which results from the filter with the higher-frequency cutoff, follows the data quite closely. This indicates that there is little variance associated with periods shorter than 6 hours.
Fig. 2. Typical results of the extrapolation and smoothing process used to generate equispaced values for the strain analysis. Curve a was obtained with a smoothing filter transition band from 8.0 to 6.15 hrs and curve b with a transition band from 20.0 to 11.4 hrs.

Experimental Error Estimation

The primary error in the lengths of the triangulated strain line comes from the measurement error in the target angle. Since angle measurements were generally accurate to ±1 minute and distance errors were small, the distance error in the triangulated strain lines was determined from

\[ \Delta C = \frac{AB}{C} |\sin \theta| |\Delta \theta| \]  . (1)

where \( C \) is the triangulation line joining the measured lines \( A \) and \( B \) with enclosed angle \( \theta \). The angular error, \( \Delta \theta \), was taken to be 2 minutes since \( \theta \) represents the difference between two angles accurate to ±1 minute. The smoothing was always carried out after triangulations were complete. The triangulations followed the linear interpolation of the data. Typical percentage errors \((\Delta C/C)\) varied from 0.05% to 0.10%.

To determine the errors for the percentage rate changes we proceeded as follows. First, any error due to extrapolation was neglected since sufficient smoothing was carried out so that such variations would generally be small. Each smoothed point was then assigned a percentage error, \( \Delta C/C \),
as determined above. The rate, \(d\), was obtained by subtracting adjacent points—call them \(x_1, x_2\)—so that for uncorrelated errors [i.e., \(\text{cov}(x_1, x_2) = 0\)],

\[
\frac{\Delta d}{d} = \sqrt{2} \frac{\Delta C}{C}
\]

The assumption of no correlation was used for data smoothed with the higher pass filter. However, for the lower pass filter much of the high-frequency noise was removed, and we therefore estimated \(\text{cov}(x_1, x_2)\) to be given by \(\text{cov}(x_1) (\sin \pi/2)/(\pi/2)\) and consequently

\[
\frac{\Delta d}{d} = \sqrt{2(1 - 1/\pi)} \frac{\Delta C}{C}
\]

This would be the case for a white-noise signal filtered by a low pass filter with a perfectly sharp cutoff at one-half the Nyquist frequency [Holloway, 1958]. In practice our filter smoothed the data somewhat more, so that the above would give an upper estimate for the error. For lines on which no triangulation was necessary we assumed no experimental error. These experimental errors were input to obtain experimental error bars on the strain rate tensor as discussed in the next section.

### Strain Tensor and Inhomogeneity Variation Computations

Using tensor notation, the strain rate tensor \(\dot{\varepsilon}_{ij}\) is defined by

\[
\dot{\varepsilon}_{ij} = \frac{1}{2} \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right)
\]  \hspace{1cm} (2)

where \(v_i\) is the velocity of the ice pack and \(i, j = 1, 2\) since we are only concerned with the horizontal motion of the ice pack. Considering \(N\) non-collinear strain lines along which linear strains are measured, we denote by \(y_i\) the linear strain rate along the \(i\)th line and by \(\theta_i\) the angle of the \(i\)th line to the \(x\)-axis. The predicted strain rate, \(\tilde{y}_i\), along the \(i\)th line, is given by [Nye, 1957]

\[
\tilde{y}_i = \varepsilon_1 \cos^2 \theta_i + \varepsilon_2 \sin^2 \theta_i + \varepsilon_3 \sin 2\theta_i
\]  \hspace{1cm} (3)

\[\equiv \varepsilon_j X_{ij}\]
where
\[ \varepsilon_1 \equiv \varepsilon_{11}, \quad \varepsilon_2 \equiv \varepsilon_{22}, \quad \varepsilon_3 \equiv \varepsilon_{12} \]  
\[ \text{(4)} \]

The measured strain rate \( y_i \) will differ from the predicted strain rate \( \tilde{y}_i \) by some "error" \( z_i \) which we wish to minimize by the appropriate choice of the \( \varepsilon_i \)'s. The least squares estimates of the \( \varepsilon_i \)'s are obtained by differentiating
\[ \sum_{i=1}^{N} (y_i - \varepsilon_j X_{ij})^2 \]
with respect to \( \varepsilon_k \), which yields the matrix equation for the least squares estimates of \( \varepsilon_i \) (denoted by \( \bar{\varepsilon}_i \)) [Jenkins and Watts, 1969, p. 132]:
\[ \bar{\varepsilon} = M^{-1} \bar{X}' Y \]  
\[ \text{(5)} \]
where \( M = \bar{X}' \bar{X} \) and primes denote transposes.

In our case \( z_i \) consists of two parts:
\[ z_i = z_i^M + z_i^I \]  
\[ \text{(6)} \]
where \( z_i^M \) is the measurement error and \( z_i^I \) is the variation due to the inhomogeneity of the strain over the region of ice samples.

To estimate the variation of the parameters \( \varepsilon_i \) due to the error \( z_i \), it is necessary to calculate the covariance matrix of the \( \varepsilon_i \) which is easily shown to be given by
\[ C = M^{-1} \bar{X}' V \bar{X} (M^{-1})' \]  
\[ \text{(7)} \]
where \( C_{ij} = \text{cov} (\varepsilon_i, \varepsilon_j) \) and \( V_{ij} = \text{cov} (z_i, z_j) \).

Consequently the variation in \( \varepsilon_i \) due to measurement errors only--call it \( C_{ij}^M \)--may be obtained by using equation (7) with \( V_{ij} = \text{cov} (z_i^M, z_j^M) \).

To find confidence limits of some linear combination of the \( \varepsilon_i \) due to the total "error" \( z_i \), we make the usual assumption that the \( z_i \)'s are
uncorrelated with the same mean and variance so that \( \text{cov}(z_i, z_j) = \delta_{ij} \sigma^2 \).

Thus, equation (7) reduces to

\[
C_{ij} = \text{cov}(\varepsilon_i, \varepsilon_j) = (M^{-1})_{ij} \sigma^2
\]

(8)

Given a linear combination of the \( \varepsilon_i \), \( b = a_i \varepsilon_i \), we have

\[
\text{cov}(b) = a_i a_j C_{ij}
\]

(9)

Taking the point estimator of \( \sigma^2 \) to be given by

\[
\hat{s}^2 = \frac{\sum_{i=1}^{N} (y_i - \bar{y}_i)^2}{N - 3}
\]

the deviation of \( b \) from the least squares value \( \bar{b} \) is such that

\[
\frac{(b - \bar{b})/s}{\sqrt{a_i a_j (M^{-1})_{ij}}}
\]

has the \( t \) distribution with \( N - 3 \) degrees of freedom assuming the errors are normally distributed [Bennett and Franklin, 1961, p. 250]. Consequently, confidence limits may be calculated using a \( t \) distribution table. We also note that equation (9) may be used to calculate the contribution of the measurement errors to the error in \( b \) by replacing \( C_{ij} \) by \( C_{ij}^M \). This would also apply to experimental situations in which only three strain lines were used.

In this paper we are concerned principally with the inhomogeneity \( z_i^T \), which strictly speaking is not an error, but represents the variation of the strain tensor over the region sampled. In the cases we have studied, the total variation error (\( \sqrt{\text{cov} b} \)) was generally found to be considerably larger than the measurement error, and thus the confidence limits primarily reflect inhomogeneities. Consequently, as a matter of convenience we will often refer to the computed total variation error as the inhomogeneity variation. This is a good first approximation, although it should be remembered that by inhomogeneity variations we mean not experimental errors.
but the statistical estimate of the variability of the average strain (for a given strain array) due to inhomogeneities.

**Vorticity Computations**

The vorticity is defined by

\[ \Omega = \frac{1}{2} \left[ \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right] \]  

(11)

Using this definition and that of the strain rate tensor \( \varepsilon_{ij} \), the predicted rotation rate \( \dot{\theta} \) of the \( i \)th strain line oriented at angle \( \theta \), to the \( x \)-axis is given by

\[ \dot{\theta} = \Omega + \varepsilon_{12} \cos 2\theta + \frac{1}{2} (\varepsilon_{22} - \varepsilon_{11}) \sin 2\theta \]  

(12)

This equation is obtained easily by transforming the tensor \( \partial v_i / \partial x_j \) into a coordinate system with the \( x \)-axis parallel to the \( i \)th strain line. Thus by measuring the rotation and linear strain along \( N \) lines, we may combine equations (3) and (12) and carry out a combined least squares calculation minimizing

\[ \sum_{i=1}^{N} \left[ (y_{i} - \tilde{y}_{i})^2 + (\dot{\theta}_{i} - \tilde{\theta}_{i})^2 \right] \]

where \( \tilde{\theta}_{i} \) is the observed rotation rate of the \( i \)th line. The resulting least squares matrix equations and error equations are completely analogous to equations (5) through (10).

When these combined equations are used, it should be noted that adding a constant rotation to all angles changes only the vorticity and not the strain. This is easy to see because the equations for \( \dot{\theta} + C \) where \( \Omega \) is replaced by \( \Omega + C \) are the same as for \( \dot{\theta} \) with \( \Omega = \Omega_1 \). By averaging over several angles we see from equation (12) that \( <\dot{\theta}> = \Omega \), and in fact, actual calculations show the average rotation to be very nearly equal to the least squares vorticity for randomly oriented lines.

Clearly the combined equations represent a useful way to calculate both the vorticity and the strain tensor, especially when the distances
and angles to $N$ targets are measured from a central point. However, the strain calculation alone, using only linear strains, is useful for comparing scaling effects due to different lengths of strain lines.

**STRAIN RESULTS**

For comparison, strain-rate tensor time series have been calculated utilizing several different sets of strain lines; Table 1 describes all combinations of strain lines used in the calculations. All strain lines were chosen to be independent in a measurement sense; that is, no line length of any array may be determined from knowledge of the other line lengths in the array. Some care must be taken to insure independence, since it is well known (see for example Flugge [1962], section 26, p. 7) that there can be only $2n - 3$ independent lengths connecting $n$ independent $(x, y)$ points.

**TABLE 1**

**STRAIN LINE COMBINATIONS USED IN THIS PAPER**

<table>
<thead>
<tr>
<th>Array &quot;Name&quot;</th>
<th>Strain Lines in Array Connecting Targets</th>
<th>Range of Strain Line Lengths</th>
</tr>
</thead>
<tbody>
<tr>
<td>16 km array</td>
<td>1-3; 1-5; 1-8; 1-10;</td>
<td>14.8 - 19.3 km</td>
</tr>
<tr>
<td></td>
<td>2-5; 2-8; 2-9; 3-9;</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5-9; 8-10</td>
<td></td>
</tr>
<tr>
<td>8 km - 12 target array</td>
<td>1-2; 2-3; 8-9; 9-1;</td>
<td>7.5 - 12.1 km</td>
</tr>
<tr>
<td></td>
<td>1; 2; 3; 8; 9; 4-11;</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4-13; 11-13</td>
<td></td>
</tr>
<tr>
<td>8 km - 9 target array</td>
<td>1-2; 2-3; 8-9; 9-1;</td>
<td>7.6 - 12.1 km</td>
</tr>
<tr>
<td></td>
<td>1; 2; 3; 8; 9</td>
<td></td>
</tr>
<tr>
<td>5 km array</td>
<td>11-7; 7-4; 4; 7; 11; 13</td>
<td>4.3 - 5.2 km</td>
</tr>
<tr>
<td>16 km triangle</td>
<td>9-2; 2-5; 5-9</td>
<td>14.8 - 17.5 km</td>
</tr>
<tr>
<td>8 km triangle</td>
<td>11-13; 13-4; 4-11</td>
<td>7.9 - 8.4 km</td>
</tr>
<tr>
<td>5 km triangle</td>
<td>11; 7; 11-7</td>
<td>4.5 - 4.6 km</td>
</tr>
</tbody>
</table>
Over the time interval studied in this paper there was only one major gap in the time series, and this portion is blanked out in the plots. However, for spectral studies, root mean square error estimation, and correlation studies, the whole curve was used (data points every 3 hours) including linearly extrapolated data through the gap, with the linear extrapolation being done on each strain line as discussed previously.

Strain Tensor Time Series

The 16 km array consists of 10 long intersecting lines that are reasonably well distributed over the study region. Therefore, the least squares strain tensor computed from this array should be a good representation of the average strain for the study region. The results of the least squares calculations are given in Figures 3, 4, and 5, which present the two invariants of the strain rate tensor both separately and in the form of the net divergence, \( \varepsilon_{ij} \) (sum of the principal axis components), and the net maximum shear \((\varepsilon_1 - \varepsilon_2)/2\) (difference of the principal axis components divided by 2). Also plotted are the divergence rate \( \dot{\varepsilon}_{ij} \) and the maximum shear rate, \((\dot{\varepsilon}_1 - \dot{\varepsilon}_2)/2\). In the net calculations the initial time was 0900 hours (GMT), day 88 with the angles of all the targets taken to be the angles at that initial time. For the strain rates, the angle of the strain lines at the beginning of each 3-hour time interval was used in the least squares calculations. The continuous errors in Figures 3 and 4, which are the \( \Delta \) values shown below each curve, represent the total variation error (calculated using equations 8, 9, and 10) which is due primarily to inhomogeneities. The maximum experimental error was calculated by first calculating the error for each point in the time series according to equations (7) and (9) and then finding the maximum error over the time series.

It is clear from the figure that in general both the divergence rate and the shear rate are of greater magnitude than their respective total variation errors. The results also indicate that the inhomogeneity variation generally increases with increasing strain rate. It is important to remember that the inhomogeneity variations shown are for 10 targets and
Fig. 3. Least squares strain tensor time series using 10 strain lines from the 16 km array with a smoothing filter transition band from 8.0 to 6.15 hrs. The small error bar represents the maximum uncertainty due to measurement error. Divergence rate and net divergence are shown with total variation errors on each.
Fig. 4. Maximum shear rates and net maximum shear with total variation errors using 10 strain lines from the 16 km array. The smoothing filter transition band is from 8.0 to 6.15 hrs.
Fig. 5. Divergence rate and principal axis components of the least squares strain rate tensor obtained using 10 strain lines from the 16 km array.
would be smaller for a larger number of targets. This is analogous to the error on the slope of a simple least squares line which becomes smaller as more points are added even though the standard error of the estimate remains the same. In particular, for the same residual error along each linear strain line, an equilateral strain triangle would have a divergence rate error 1.7 times that shown in Figure 3.

One striking aspect of the deformation, best illustrated in Figure 5, is that in the principal axis coordinate system most of the expansion or contraction is taking place along one axis. Moreover, there is usually contraction along one axis and extension along another. Alternatively, thinking in terms of the strain ellipse for strain occurring over a given 3-hour interval, the ellipse will be very eccentric.

The maximum observed divergence rate is seen to be $0.125\% \pm 0.036\%$ per hour with a maximum convergence rate of $0.106\% \pm 0.035\%$ per hour. The largest maximum shear rate is $0.105\% \pm 0.022\%$ per hour. The net divergence curve indicates a net areal change of about $3\% \pm 0.5\%$, with the general trend being a convergence. The net maximum shear curve, on the other hand, indicates an extremum shear value of $3.2\% \pm 0.5\%$. The deformation appears to be distributed equally between area change and (tensor) shear.

### Dependence of the Strain Tensor on the Scale of the Measurement Array

To investigate the dependence of the strain tensor and the inhomogeneity variation on the length of the strain lines, the 8 km and 5 km arrays were used for similar calculations. Since the targets in the 5 km array were not measured so frequently as those at greater distances, the linearly extrapolated data were smoothed with a low pass filter having a transition band from 20.0 to 11.4 hrs, as discussed in the section on data analysis procedures. For comparison the other two arrays were smoothed in the same manner. Figure 6 presents the net divergence and Figure 7 the divergence rate and the maximum shear rate curves for the 16 km (curve a), the 8 km (curve b) and the 5 km (curve c) arrays. Figure 8 shows the total variation rate errors for the divergence rates. For numerical comparison
Fig. 6. Comparison of least squares net divergences from different arrays using a smoothing filter with a transition band from 20.0 to 11.4 hrs. Curves labelled by (a) represent the 16 km array, (b) the 8 km - 12 target array, and (c) the 5 km array.
Fig. 7. Comparison of least squares divergence rates (1) and maximum shear rates (2) using a smoothing filter with a transition band from 20.0 to 11.4 hrs. Curves labeled by (a) represent the 16 km array, (b) the 8 km - 12 target array, and (c) the 5 km array.
of the total variation errors we have also listed in Table 2 the relevant time-averaged root mean square errors and maximum experimental errors for the various arrays.

The RMS residual error given in Table 2 is particularly relevant since it estimates a quantity that is independent of the number of strain lines. The divergence error, on the other hand, will, for random sampling, decrease as the number of lines increases. In practice, since the target angles do not change significantly, the residual error time series is of the same form as the divergence error time series multiplied by a positive scaling factor.

It can be seen from Table 2 that (1) the residual variation error (and hence the inhomogeneity variation) increases as the average target length decreases; and (2) most of the total variation error is due to inhomogeneities and not to experimental error. A third point of interest is the very large maximum variation error for the net divergence curve. This suggests that it would be difficult to assign an accurate trend (say, generally converging or generally diverging) to the deformation as measured by a short target array. On the other hand, a visual comparison of the net
TABLE 2

SUMMARY OF ERRORS FROM DIFFERENT STRAIN TENSOR TIME SERIES

<table>
<thead>
<tr>
<th>Array Description</th>
<th>16 km</th>
<th>8 km-12 target</th>
<th>5 km</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMS residual variation error</td>
<td>$1.67 \times 10^{-4} \text{hr}^{-1}$</td>
<td>$3.01 \times 10^{-4} \text{hr}^{-1}$</td>
<td>$4.44 \times 10^{-4} \text{hr}^{-1}$</td>
</tr>
<tr>
<td>RMS divergence variation rate error</td>
<td>$1.08 \times 10^{-4} \text{hr}^{-1}$</td>
<td>$1.76 \times 10^{-4} \text{hr}^{-1}$</td>
<td>$3.64 \times 10^{-4} \text{hr}^{-1}$</td>
</tr>
<tr>
<td>Maximum experimental error for div. rate</td>
<td>$2.98 \times 10^{-5} \text{hr}^{-1}$</td>
<td>$4.69 \times 10^{-5} \text{hr}^{-1}$</td>
<td>$4.28 \times 10^{-5} \text{hr}^{-1}$</td>
</tr>
<tr>
<td>Maximum net divergence variation error</td>
<td>0.75%</td>
<td>1.5%</td>
<td>5.0%</td>
</tr>
<tr>
<td>RMS divergence rate</td>
<td>$2.55 \times 10^{-4} \text{hr}^{-1}$</td>
<td>$2.60 \times 10^{-4} \text{hr}^{-1}$</td>
<td>$3.08 \times 10^{-4} \text{hr}^{-1}$</td>
</tr>
</tbody>
</table>

divergence curves indicates that deformational events consisting of divergence followed by convergence are similar in all the arrays. A more quantitative measure of these correlations is given in Table 3, which presents correlation coefficients between the net divergences, divergence rates, and maximum shear rates for the three arrays.

TABLE 3

CORRELATION COEFFICIENTS BETWEEN DIFFERENT STRAIN TENSOR TIME SERIES

<table>
<thead>
<tr>
<th>Arrays</th>
<th>Net Divergence Correlation</th>
<th>Divergence Rate Correlation</th>
<th>Maximum Shear Rate Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>16km-8km</td>
<td>0.98</td>
<td>0.83</td>
<td>0.95</td>
</tr>
<tr>
<td>16km-5km</td>
<td>0.92</td>
<td>0.24</td>
<td>0.63</td>
</tr>
<tr>
<td>8km-5km</td>
<td>0.93</td>
<td>0.57</td>
<td>0.70</td>
</tr>
</tbody>
</table>

Although the correlation coefficients are all significant at the 5% level, there are several basic differences. For one, the correlations between the maximum shear rates are significantly larger than the correlations.
between the divergence rates. For another, the largest correlations occur between the net divergence curves. Since the divergence rate may be viewed as the result of a high pass differential filtering operation on the net divergence, the higher correlation for the net divergence indicates that the low-frequency portions of the net divergence curve are much more coherent than the high-frequency portions. The higher correlations for the maximum shear also suggest that, between the smaller and the larger arrays, the shearing aspect of the deformation is more consistent than the divergence rate.

The erratic nature of the rates for shorter target arrays is also apparent in Figure 7. This would certainly be expected since the shorter lines average motions over fewer leads. The strain line length record thus contains more sharp motions. This effect is somewhat mitigated by the least squares analysis, but it is obvious in plots of individual triangles. In Figure 9, for example, we illustrate net divergences of three overlapping triangles together with the 16 km array least squares results. The more rapid, large-magnitude motion of the smaller triangles is apparent. The curves do illustrate a general correlation of strain events consisting of dilatation followed by convergence. However, the overall correlation is clearly not so good as that obtained between the least squares curves in Figure 6. Figure 9 graphically indicates some of the difficulty in using only one triangle to measure ice deformation.

Statistical Model for Error vs. Strain Line Length

To estimate inhomogeneity variations for different-sized arrays, it is useful to compare our observed residual error variation with the expected functional form obtained by a simple statistical argument. In particular it seems reasonable that the residual error should scale inversely with the square root of the average strain line length.

To derive such a result let us consider the distance change along a strain line of interest to be due to the stretching or contraction of \( N \) randomly selected smaller strain lines (call them sub-strain lines) with \( N \) proportional to \( L \), the length of the long strain line. Assume that the
Fig. 9. Comparison of net divergences of overlapping triangles of different sizes using a smoothing filter with a transition band from 20.0 to 11.4 hrs. Curve (a) represents the 16 km array least squares net divergence; curve (b) the divergence calculated using the 16 km triangle; curve (c) the 8 km triangle; and curve (d) the 5 km triangle.
differential length changes of the sub-strain lines $\Delta L'_i$ are described by some distribution function (not necessarily Gaussian) $P(\Delta L')$ such that $N \Delta L'$ is the "expected" length change in $L$. The observed strain is given by

$$\frac{\Delta L}{L} = \frac{\sum_{i=1}^{N} \Delta L'_i}{L}$$

(13)

The standard deviation $\sigma$ of the strains measured from long lines is related to the standard deviation $\sigma'$ of the strains measured by the shorter lines by

$$\sigma = \frac{\sigma'}{\sqrt{N}}$$

(14)

and since $N$ is proportional to $L$

$$\sigma = \frac{K \sigma'}{\sqrt{L}}$$

where $K\sigma'$ may depend upon time and angle. We would expect the time-averaged (rms) residual error to be a reasonable average of $\sigma$ over time and angle, so that $\sigma \text{ (resid)} = C L^{-\frac{1}{2}}$ with $C$ a constant which presumably varies with season and which can be determined from data such as we have collected.

To determine whether our observations have such a length dependence, we illustrate in Figure 10 a log-log plot of the rms residual error versus the average strain line length for the 16 km, 8 km-9 target, and 5 km array. The 8 and 5 km array errors fit a $L^{-\frac{1}{2}}$ dependence quite well, whereas the 16 km array error is somewhat smaller than predicted. These results are quite reasonable. The calculated 16 km error is probably unrealistically low because the intersecting strain lines resample the same region and are thus not really random. The 8 km-9 target and 5 km arrays, on the other hand, have no intersecting lines and would thus be expected to yield errors representative of the true lateral inhomogeneity variation.

When using Figure 10 for array size design, one should keep in mind that typical least squares computations (see Table 2) will reduce divergence rate confidence limits to about half the residual error, whereas computations using only one triangle, with an assumed constant residual error along each
Fig. 10. Observed (dots) and expected (solid line) residual errors versus average strain line length. For comparison, the average rms divergence rate of the three arrays is $3 \times 10^{-4}$ hrs$^{-1}$.

leg, will yield a divergence rate error larger than the residual error. Thus the figure suggests that a 20 km least squares array would be adequate for strain measurements. In fact such an array would, according to Figure 10, yield a least squares divergence rate error less than that from a 80 km triangle. Figure 10 also indicates that there is little advantage in extending the length of the strain lines from 20 km to 30 km, although there is a significant advantage in extending the lines from 8 km to 20 km.

**Frequency Dependence of Inhomogeneity Variation**

One question of interest with respect to the inhomogeneity variation is whether certain frequencies noted in the individual strain line observations are more coherent with other strain line motions. If this were the case, these frequencies would show small strain fluctuations. One would also expect the spectra of the least squares predicted linear strain rate to be less modified at the frequencies that are coherent.

To determine the frequency dependence of the inhomogeneity variation, we carried out three low pass filtering operations on the 8 km - 9 target array using progressively lower cutoffs. In Figure 11 the resulting
Fig. 11. Effect of low pass filtering with progressively lower cut-offs on the divergence rate and total variation error of the 8 km - 9 target array. The transition bands for curves a, b, and c are respectively 8.0 - 6.15, 20.0 - 11.4, ∞ - 26.6 hrs.

A. Divergence rates.
B. Total variation errors.
divergence rate curves are illustrated together with the total variation rate errors. Table 4 also lists the ratio between the root mean square total variation error for the divergence rate and the variance of the least squares divergence rate curve. For coherent frequencies this ratio would be expected to be small.

**TABLE 4**

RESULTS OF LOW PASS FILTERING OF THE DIVERGENCE RATE TIME SERIES

<table>
<thead>
<tr>
<th>Filter Transition Band(s)</th>
<th>MS Error</th>
<th>Variance of Signal</th>
<th>MS/Var</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.0 - 6.15 hrs</td>
<td>6.69 $10^{-8}$ hrs$^{-2}$</td>
<td>12.06 $10^{-8}$ hrs$^{-2}$</td>
<td>0.55</td>
</tr>
<tr>
<td>20.0 - 11.4 hrs</td>
<td>4.62 $10^{-8}$</td>
<td>8.41 $10^{-8}$</td>
<td>0.55</td>
</tr>
<tr>
<td>∞ - 26.6 hrs</td>
<td>1.32 $10^{-8}$</td>
<td>2.31 $10^{-8}$</td>
<td>0.57</td>
</tr>
</tbody>
</table>

Table 4 generally indicates that in the high-frequency range studied by this operation (essentially periods from 1 day to 6 hours) there are no outstanding coherent frequencies, although there appears to be a slight increase in coherence for the lower frequencies. Overall, these time-averaged results plus the behavior shown in Figure 11 suggest that the inhomogeneity variation is approximately proportional to the strain rate amplitude. We note, however, that because of the length of the time series we cannot adequately resolve lower frequencies with periods of several days.

**Coherence Spectra Between Strain Rates from Different Arrays**

To determine any spectral peaks or frequencies common to arrays of different lengths we calculated the spectra and coherence spectra between the divergence rate curves for the three arrays illustrated in Figure 7. The results are shown in Figure 12. In calculating the spectra and coherence, we used the lag product method as discussed, for example, by Rayner [1971, p. 94] and applied a Hamming spectral window to both the quadrature spectrum and cospectrum. The 10% confidence limit was calculated according to the
Fig. 12. Spectra of and coherence between least squares divergence rates from the 16 km, 8 km - 12 target, and 5 km arrays.

A. Spectra of least squares divergence rates. Curve a represents the 16 km array spectra, curve b the 8 km - 12 target array, and curve c the 5 km array.

B. Coherence between a-1 (the 16 km and 8 km - 12 target arrays), a-2 (the 16 km and 5 km arrays), and a-3 (the 8 km and 5 km arrays).

procedure described by Panofsky and Brier [1968, p. 158]. The smoothing filter applied to the data had a pass band extending to 8.0 hrs and therefore did not affect (within a 0.6% error) the portion of the spectra illustrated in Figure 12.

As can be seen in Figure 12 the individual spectra show a slight peak in the results from the 8 km and 16 km arrays at a period of about 12 hours. The coherence effect is striking; there is a significant peak in the coherence of the large and small arrays at roughly a period of 12 hours. The absence of a strong spectral peak at this wavelength in the individual spectra could well be due to the peak's being masked by the large amount of random noise in the high frequencies. This is especially true in the smaller 5 km array. In any case the coherence does clearly indicate that there is a common coherent mode of deformation between the large and small arrays with a period of 12 hours.

This effect could be caused by variations in water currents due to inertial oscillations [Hunkins, 1967]. Measurements of ocean currents by
Newton and Coachman [1973] during previous AIDJEX experiments have indicated 12-hour cycles in the currents, with the oscillations displaying coherence over distances of about 20 km. Different drag coefficients for different ice floes could couple with these currents to create a differential ice motion.

We emphasize that this coherence study does not adequately resolve lower frequencies such as variations with wavelengths of several days. We would expect a strong coherence here; as noted earlier, the increase of the correlation coefficients between the net divergence and divergence rate indicates that this is the case.

**Necessary Sampling Rate**

Although a high sampling rate is always desirable, it is not always feasible. The spectra in Figure 12 generally suggest that sampling intervals from 3 to 10 hours would yield valid low-frequency information without intolerable aliasing. To determine more directly the effect of a sparse sampling rate, it is useful to compare, as we do in Figure 13, strain calculations performed on data sampled every eight hours, with results sampled every three hours. The comparison at low frequencies is quite close, and there is certainly not an intolerable degree of aliasing. Such a sampling rate would not, of course, resolve the 12-hour oscillations shown in Figure 12, but it would adequately resolve the lower-frequency components required for comparison with synoptic meteorological variations which generally occur on a time scale of several days [Monin, 1972].

**Significance of Particular Strain Events**

The previous comparisons indicate that one of the strongest similarities between the strain tensors from the different arrays is the appearance of similar deformational events spanning several days and consisting of dilatation followed by convergence. This similarity suggests that, although at any given time there may be local fluctuations in the sea ice strain field, the deformation of the ice over a period of several days may be quite similar for the whole mesoscale region. This conclusion may be justified
Fig. 13. Comparison of low-frequency portions of divergence rate time series using different sampling rates. The solid line used data sampled every three hours before smoothing, the dashed line every eight hours before smoothing. The smoothing filter had a transition band from 40 to 16 hrs.

better by methodically determining the significance of particular strain events.

To carry out such a determination we used the net divergence curve of the 8 km - 9 target array and assigned an error bar to each major event in the following manner. First, reasonable before-and-after time limits, say $t_1$ and $t_2$, were picked for each event. Then denoting the time of the peak of the event (maximum dilation) by $t_3$ and the net strain at $t_3$ by $\varepsilon(t_3)$, we calculated the total variation error $\Delta[\varepsilon(t_2) - \varepsilon(t_1)]$ and $\Delta[\varepsilon(t_2) - \varepsilon(t_3)]$ in the usual least squares manner and took the average of these two variation errors as the error bar. The results presented in Figure 14 show that all but one of the deformational events studied are significant. Since we have used the nine target arrays, the errors are in some sense upper limits because there are no overlapping lines.
Fig. 14. Net divergence curve for the 8 km - 9 target array with error bars computed for major deformation events.

COMBINED VORTICITY AND STRAIN RESULTS

To carry out a combined least squares calculation of vorticity and strain rate we used the five targets T1, T2, T3, T8, T9 to obtain five linear strains and five rotation angles for input into equations (3) and (12). This input yielded 6 degrees of freedom for the combined least squares results for the strain rate components and the vorticity. The calculation was carried out in the camp coordinate system so that the camp rotation must be added to obtain the true vorticity. (The camp North coordinate coincided with true North on day 81.) To obtain the camp rotation, a time series \( R = \text{Longitude} - \text{Azimuth} \) was constructed by linearly extrapolating the satellite position and azimuth measurements reported by Thorndike et al. [1972]. This time series was smoothed by the same filter that was applied to the distance and angle data of the targets, and then finite differences were taken using data every 3 hours.

In Figure 15, we illustrate the results of the calculation. All curves were smoothed with a filter having a transition band from \( \infty \) to 26.6 hrs. The dotted lines were based upon measurements of only four targets (T1, T2, T3, T8) made prior to the implementation of the complete array. Because these earlier data have several gaps in them, they should not be taken as highly accurate although the general trends should be valid.
Fig. 15. Divergence rate (a) and vorticity (b and c) time series results using combined least squares equation. Curve b represents the vorticity in the camp coordinate system and curve c the true vorticity obtained by adding the camp rotation to curve b.

We have included the data here for completeness. Part a illustrates the divergence rate, part b is the vorticity in the camp coordinate system, and part c is the true vorticity consisting of part b plus camp rotation. The form of the divergence rate curve shown in Figure 15 is essentially the same as would be obtained from the 16 km array. With respect to the inhomogeneity error, the error curve for the vorticity in the camp coordinate system has essentially the same form as the smoothed divergence rate error curve c-1 in Figure 11 and has an rms time average value of ≈ 0.0001 hrs⁻¹.
Figure 15 shows a negative correlation between the vorticity and the divergence rate. The correlation coefficient for the solid line data is in fact $-0.51 \pm 0.22$, indicating that a convergence is associated with a counterclockwise rotation. This correlation will be discussed in greater detail in relation to linear drift theories and atmospheric pressure variations in a later paper [Hibler, 1973].

With a few exceptions in the earlier data, the true vorticity is essentially equal to the camp rotation, suggesting that the whole mesoscale region generally rotates as an entity. A comparison of the average rotation angles of all the targets with Figure 15 indicates that the average target line rotation is nearly equal to the least squares vorticity (in the camp coordinate system). This is expected, as mentioned earlier, for random strain lines, so that to a good approximation least squares vorticity may be estimated by the average strain line rotation for a set of randomly oriented strain lines.

**SUMMARY**

The results of the least squares analysis indicate that, by using strain lines longer than 8 km, an average strain rate tensor may be calculated with rate magnitudes generally larger than the inhomogeneity variations. The results also show that, although larger errors are encountered, an array consisting of 5 km targets produces least squares results very similar to those obtained from larger arrays, especially at lower frequencies. The inhomogeneity variation was found to scale inversely with the square root of the average strain line length and to have little frequency dependence for frequencies above one cycle per day.

The calculations also show that in most cases the vorticity is similar to the camp rotation. For a better estimate, the average rotation of a set of randomly oriented strain lines may be used; this result differs insignificantly from the least squares results.

One of the most persistent modes of deformation present in strain networks of all sizes was a dilatation followed by a compression, with most
of the expansion (or compression) taking place along one direction. In particular, four such events were observed in the time series analyzed: at Julian days 88-92, 94-96, 99-103, and 111-113 (an earlier event from days 82-84 was also observed before the array was completely installed). These events were present in all the array sizes that were studied and would be one feature that could be obtained from a small array consisting of only a few strain lines.

Cross-spectral studies between the divergence rates of different-sized arrays indicated a significant coherence at approximately two cycles per day between the large and small arrays. Individual spectra yielded only weak peaks at this frequency, suggesting that this mode of deformation is largely masked by the random motions present in individual time series. Comparison of correlation coefficients between divergence rates and net divergences of different arrays indicates that lower frequencies have a significantly greater correlation than the higher ones and points out the need for long time series of several months to adequately compare this low-frequency behavior. The spectrum also indicates that sampling intervals up to ten hours are generally adequate for resolving low-frequency strain rates without intolerable aliasing.

CONCLUSION

This study shows that mesoscale strain, with certain important limitations, can be used as a valid measure of the differential drift of the arctic ice pack. The basic qualification is that the mesoscale strain record must be ergodically averaged in some way. Thus the higher-frequency components (hours) of the strain rate record represent more of a local phenomenon, whereas the lower-frequency components (days) are more spatially coherent and therefore representative of the whole mesoscale region and of the overall pack ice drift. These conclusions are also supported by the general agreement between expected differential sea ice drift using linear drift theories and the observed strain results [Hibler, 1973].
This type of ergodic validity of mesoscale strain measurements is physically reasonable; although the mesoscale region contains a small number of leads, over a period of several days it would be expected to diverge if the pack is generally diverging and converge if the pack is converging. This behavior is perhaps best illustrated by the presence of the same significant strain events in different arrays, consisting of dilatation followed by convergence occurring over a period of several days.

It should also be noted that these conclusions apply to 8 km strain lines; less ergodic averaging should be necessary for longer strain lines. In particular the results indicate that the error scales inversely with the square root of the average strain line length.

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DIFFERENTIAL SEA ICE DRIFT II:
COMPARISON OF MESOSCALE STRAIN MEASUREMENTS
WITH LINEAR DRIFT THEORY PREDICTIONS

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ABSTRACT

A comparison of mesoscale strain measurements with the atmospheric pressure field and the wind velocity field indicates that the ice divergence rate and vorticity follow the local pressure and wind divergence: for low atmospheric pressures and converging winds, the divergence rate is negative and the vorticity counterclockwise; for high pressures and diverging winds, the divergence rate is positive and the vorticity clockwise. This behavior agrees with predictions based upon the infinite boundary solution of a linearized drift theory in the absence of gradient current effects and using the constitutive law proposed by Glen [1970] for pack ice. The best least squares values of the constitutive law parameters $\eta$ and $\zeta$ are given by $\sim 10^{15}$ g/sec. Using typical divergence rates these values yield compressive stresses of the magnitude of $10^6$ dyne/cm which are similar to values suggested by the Parmerter and Coon [1972] ridge model. The infinite boundary solution of the linear drift equation indicates that in a low-pressure region the ice converges for high compactness (winter) and diverges for low compactness (summer).

Calculations were also carried out using a more general linear constitutive law which allows the viscosities to vary with temporal frequency and which includes as special cases a generalized Hooke's law as well as the Glen law. A best fit of this more general calculation with strain measurements agrees better with viscous behavior than with elastic behavior, with the frequency dependence of the estimated viscosities approximating closely the Glen law behavior.
LIST OF SYMBOLS

\( \nu \)  ice vorticity
\( \Delta \)  ice divergence rate
\( m \)  ice mass per unit area
\( \eta \)  ice velocity
\( \psi \)  wind velocity
\( \mathbf{f} \)  Coriolis vector
\( \mathbf{f} \)  Coriolis parameter equal to magnitude of the Coriolis vector times the sine of the latitude
\( \lambda \)  \( \text{mf} \)
\( \mathcal{E} \)  internal ice stress
\( \mathcal{I}_w \)  water stress on ice
\( \mathcal{I}_a \)  air stress on ice
\( \phi \)  Ekman angle in air
\( \theta \)  Ekman angle in water
\( B \)  wind stress constant \( = \rho (\mathbf{f} K_a / 2)^{1/2} \)
\( D \)  water stress constant \( = \rho_w (\mathbf{f} K_w / 2)^{1/2} \)
\( \rho \)  air density
\( \rho_w \)  water density
\( K_a \)  eddy viscosity of air
\( K_w \)  eddy viscosity of water
\( U_g \)  \( x \) component of geostrophic wind
\( V_g \)  \( y \) component of geostrophic wind
\( P \)  atmospheric pressure
\( \eta \)  shear viscosity of ice
\( \zeta \)  bulk viscosity of ice
\( H \)  divergence rate response function
\( G \)  vorticity response function
INTRODUCTION

One important use of mesoscale strain measurements is in the comparison of the differential drift—i.e., strain results—with sea-ice drift theories, both to test the theories and to determine certain unknown parameters. These comparisons provide a more critical measure of certain constitutive law parameters than do comparisons involving the drift of only a single point. To make such a comparison in this paper, we will utilize a linearized drift theory similar to that used by Egorov [1970, 1971], Rothrock [1972], and Witting [1972], which, although not so exact as other calculations [Campbell, 1965, 1973], does suffice for quantitative estimates of the dominant drift effects.

Recent calculations using linear drift theories have generally been of two types. In the first, as carried out by Rothrock and Witting independently, the average yearly circulation of the arctic ice cover has been calculated assuming the ice is incompressible. Such calculations, although interesting, cannot be compared to mesoscale strain measurements. The second approach, as carried out by Egorov, obtains an approximate infinite boundary solution for a linear drift theory that uses a shear viscosity to explain the rheological behavior of the ice and neglects gradient current effects. This approach yields results which compare more directly with strain measurements.

We will use a linear drift equation similar to Egorov's. The rheological behavior of the ice is taken into account by using the constitutive law proposed by Glen [1970], which includes bulk viscosity as well as a shear viscosity. (Later in the paper, calculations will be carried out using a more general constitutive law that allows the viscosities to vary with frequency and includes a generalized Hookes law as well as the Glen law as special cases.) Like Egorov, we will neglect gradient current effects.

However, unlike Egorov, we will formulate the infinite boundary solution without approximation in terms of a linear response function. The resulting real space solution consists of a straightforward integral operator which may be applied to the pressure field to obtain expected differential ice drift. This response function form of the solution clearly
illustrates ice drift behavior expected in winter as opposed to summer. In addition the solution allows a rapid determination of the scales of variation in the atmospheric pressure field that are important for given bulk and shear viscosity values. Furthermore, the comparison of the strain measurements to pressure data allows a least squares estimation of the bulk and shear viscosity parameters.

**LINEAR DRIFT EQUATIONS**

Following Egorov's example, we consider a steady state equilibrium drift equation for the case in which the gradient current term varies so slowly in space and time that it may be neglected. In this case the equilibrium equation takes the form

\[-m\overline{\omega} + \tau_w + \tau_a + \mathcal{F} = 0\]  

where \(u\) is the ice velocity, \(f\) the Coriolis vector, \(m\) the ice mass per unit area, \(\mathcal{F}\) the internal ice stress, and \(\tau_w\) and \(\tau_a\) the water and air stresses respectively. The components of water and air stresses are given by

\[
\begin{align*}
\tau_{ax} &= B(\cos \phi U_g - \sin \phi V_g) \\
\tau_{ay} &= B(\cos \phi V_g + \sin \phi U_g) \\
\tau_{wx} &= D(-\cos \theta u_x + \sin \theta u_y) \\
\tau_{wy} &= -D(\sin \theta u_x + \cos \theta u_y)
\end{align*}
\]

where \(\phi\) and \(\theta\) are the Ekman angles in the air and water, respectively. The parameters \(B\) and \(D\) are proportionality constants related to the turbulence coefficients for the atmosphere and the ocean. For the classical Ekman layer solution [Lettau, 1967] \(B\) and \(D\) are given by \(\rho \sqrt{f(K/2)}\) where \(\rho\) and \(K\) are the density and eddy viscosity, respectively, of either air or water. \(U_g\) and \(V_g\) are the geostrophic wind components given by
\[ U_g = -\frac{1}{\rho_f} \frac{\partial P}{\partial y} \]
\[ V_g = \frac{1}{\rho_f} \frac{\partial P}{\partial x} \]  

where \( \rho \) is the air density and \( P \) is the atmospheric pressure. In equation (2) it is assumed implicitly that the ice velocity is small compared to the wind velocity and may be neglected. For the internal ice stress \( \tilde{F} \) we use the constitutive law proposed by Glen [1970]:

\[ \tilde{F} = \eta \nabla^2 \mathbf{u} + \zeta \nabla (\nabla \mathbf{u}) \]  

where \( \zeta \) and \( \eta \) are bulk and shear viscosity constants that can vary with ice compactness and therefore season. Calculations using a somewhat more general constitutive law are discussed later.

**ICE DRIFT SOLUTIONS**

In our case we are interested primarily in the solutions of the linear drift equations for the ice divergence rate, \( \Delta (\Delta \equiv \partial u_x / \partial x + \partial u_y / \partial y) \) and ice vorticity \( \omega (\omega \equiv \frac{1}{2} (\partial u_y / \partial x - \partial u_x / \partial y)) \). By taking the divergence and curl of equation (1) we obtain the two linear equations for \( \Delta \) and \( \omega \):

\[ [(\eta + \zeta) \nabla^2 - D \cos \theta] \Delta + [\lambda + D \sin \theta] 2\omega = \frac{B \sin \phi}{\rho_f} \nabla^2 P \]  

\[-[\lambda + D \sin \theta] \Delta + [\eta \nabla^2 - D \cos \theta] 2\omega = \frac{-B \cos \phi}{\rho_f} \nabla^2 P \]

where \( \lambda \equiv m_f \). These equations represent a linear system with the input being the pressure field \( P \) and the output being \( \Delta \) and \( \omega \). Such systems [Jenkins and Watts, 1968] may be described by response functions in wavenumber space \( \tilde{H}_1(k), \tilde{H}_2(k) \) so that \( \tilde{\Delta}(k) = \tilde{H}_1(k) \tilde{P}(k) \) and \( \tilde{\omega}(k) = \tilde{H}_2(k) \tilde{P}(k) \) where we denote wavenumber space functions with a tilde and \( k \equiv |k| \). The response functions may be obtained straightforwardly by Fourier transforming equations (6) and (7) yielding wavenumber space equations

\[ \tilde{\Delta}(k) = \frac{B}{\rho_f} \frac{\tilde{P}(k)}{\eta + \zeta} \left[ 1 - \tilde{H}(k) \right] \]  

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\[ \ddot{\omega}(k) = \frac{-B}{\rho_f} \frac{\ddot{P}(k)}{2\eta} [1 - \ddot{G}(k)] \]  

(9)

where

\[ 1 - \ddot{H}(k) = \frac{k^2[(\eta k^2 + D \cos \theta)\sin \phi - \cos \phi(\lambda + D \sin \theta)](\eta + \zeta)}{[\lambda^2 + D^2 + 2D \sin \theta \lambda + (\eta + \zeta)\eta k^4 + D \cos \theta(2\eta + \zeta)k^2]} \]  

(10)

\[ 1 - \ddot{G}(k) = \frac{k^2[((\eta + \zeta)k^2 + D \cos \theta)\cos \phi + \sin \phi(\lambda + D \sin \theta)]\eta}{[\lambda^2 + D^2 + 2D \sin \theta \lambda + (\eta + \zeta)\eta k^4 + D \cos \theta(2\eta + \zeta)k^2]} \]  

(11)

By the convolution theorem these equations yield simple integral equations in real space. For example, for \( \Delta(x) \)

\[ \Delta(x) = \frac{B}{\rho_f(\eta + \zeta)} [P(x) - P'(x)] \]  

(12)

where (using polar coordinates)

\[ P'(x) = \int_0^\infty H(r - r') \int_0^{2\pi} P(r', \theta)r'd\theta dr' \]  

(13)

and

\[ H(r) = \frac{1}{2\pi} \int_0^{\infty} \ddot{H}(k) \frac{k}{k} J_0(kr) dk \]  

(14)

It should be noted that these equations apply exactly only to an ice cover and pressure field of infinite extent and, in fact, represent solutions using boundary conditions \( P(x), \Delta(x), \omega(x) \) finite at \( x, y \to \pm \infty \). However, in practice they may be applied to a finite case with the necessary extent of the ice cover and pressure field determined by the spatial extent of a finite filter \( H(r) \) that approximates the wavenumber response \( \ddot{H}(k) \) in equation (10).

The wavenumber space form of the response function \( \ddot{H}(k) \) and \( \ddot{G}(k) \) contain considerable information. In the case of the divergence rate for \( \eta, \zeta \) large, \( 1 - \ddot{H}(k) \) is generally positive for large \( k \) and negative or zero for small \( k \). Thus the divergence rate is essentially the result of a high pass filtering operation on the pressure field with the high wavenumbers contributing positively to the divergence rate and the low wavenumbers negatively with a smaller amplitude. For the vorticity, the response
function is also a high pass filter, but there is no change in the sign of the contribution from different wavenumber components of the pressure field. The wavenumber of the filter cutoff decreases as $\eta$ and $\zeta$ increase, so that different types of behavior are expected for $\eta, \zeta$ small as compared to $\eta, \zeta$ large. These different types of behavior may be characterized by examining the two limiting cases $\eta, \zeta \to 0$ and $\eta, \zeta \to \infty$.

**Limiting Cases**

For the first case nothing that $(1 - \hat{H}(k))/(\eta + \zeta)k^2$ and $(1 - \hat{G}(k))/\eta k^2$ are finite as $\eta, \zeta \to 0$ we have

$$\lim_{\eta, \zeta \to 0} \Delta(x) = \frac{-B}{\rho f} \nabla^2 P(x) \frac{[D(\cos \theta \sin \phi - \sin \theta \cos \phi) - \cos \lambda]}{\lambda^2 + D^2 + 2D \sin \theta \lambda}$$

$$\lim_{\eta, \zeta \to 0} \omega(x) = \frac{B}{2\rho f} \nabla^2 P(x) \frac{[D(\cos \theta \cos \phi + \sin \theta \sin \phi) + \lambda \sin \phi]}{\lambda^2 + D^2 + 2D \sin \theta \lambda}$$

Thus we see that for equal Ekman angles and small $\eta, \zeta$ the ice diverges in a low ($\nabla^2 P > 0$) and converges in a high, and the vorticity is positive (counterclockwise) in a low and negative (clockwise) in a high.

For the second limiting case, we note that for $\eta, \zeta$ very large $\hat{H}(k)$ and $\hat{G}(k)$ pass only the very long spatial wavelengths with the pass band frequency cutoff scaling as $1/\eta$ and/or $1/\zeta$. Consequently the real space response functions $H(x)$ and $G(x)$ approach constants (with integrated areas of unity) for very large $\eta, \zeta$. Therefore, $\int H(x - x')P(x')dx'$ and $\int G(x - x')P(x')dx'$ approach the average pressure for large $\eta$ and $\zeta$; as a result, the large $\eta, \zeta$ limiting equations are

$$\lim_{\eta, \zeta \to \infty} \Delta(x) = \frac{B}{(\eta + \zeta)f} (P(x) - \overline{P}) \sin \phi$$

$$\lim_{\eta, \zeta \to \infty} \omega(x) = \frac{-B}{2\eta \rho f} (P(x) - \overline{P}) \cos \phi$$
where \( \bar{P} \) is the mean pressure over the infinite \( x, y \) space which would be approximately constant in time. For \( \eta, \zeta \) large but finite, \( \bar{P} \) would be replaced by the very low wavenumber components of the pressure field, which would be expected to be reasonably constant in time if the cutoff wavelength is longer than the synoptic variation scale of the pressure field.

As can be seen from equations 17 and 18, in the large \( \eta, \zeta \) limiting case the divergence rate and vorticity are proportional to the local pressure deviation from the overall mean pressure, with a low pressure indicating a convergence and a positive vorticity. Note that there is no dependence on the water stress in this limiting case. In fact the large \( \eta, \zeta \) case is equivalent to neglecting all stresses except the internal ice stress and wind stress. An alternative derivation, for example, would be to delete the water stress and Coriolis terms from equations 6 and 7 and solve a boundary value problem with \((\Delta - P)\) and \((\omega + P)\) finite at \( x, y + \pm \infty \). It is also important to note that the large \( \eta, \zeta \) solution includes lateral transfer of stress through the pack up to infinite distances via the \( \bar{P} \) term. However, this term becomes a constant because the lateral stress averages out and thus \( \Delta \) and \( \omega \) follow the local pressure.

To the extent that \( \eta, \zeta \) may be considered very large in the winter and small in the summer, the two limiting cases would suggest that sea ice (far from coastal boundaries) converges in a low in winter and diverges in a low in summer, with vorticity always positive in a low. Such predicted behavior agrees with earlier mesoscale strain measurements [Hibler et al., 1973], with the more extensive results reported in this paper, and with Russian observations [Volkov et al., 1971]. It is also what one would expect intuitively: that in winter the ice is tightly held and cannot move rapidly, so that the water and Coriolis forces would be smaller than they are in the summer.

Finally, we note that if we used a series solution for the drift as given by Egorov [1970] it would be impossible to draw the above conclusions, because the series diverges for frequencies higher than the high pass cutoff frequency in \( 1 - \tilde{H}(k) \).
General Case: Wavelength Dependence

Clearly, the "cutoff wavelengths" for $1 - \tilde{H}(k)$ and $1 - \tilde{G}(k)$ are critical. To illustrate typical forms of $\tilde{H}(k)$ and $\tilde{G}(k)$ we used the following numerical values: $f = 1.46 \cdot 10^{-4}$ sec$^{-1}$, $m = 300$ g/cm$^2$, $\theta, \phi = 30^\circ$, $\rho_{air} = 1.3 \cdot 10^{-3}$ g/cm$^3$, $K_{air} = 1.5 \cdot 10^5$ cm$^2$/sec, $K_{water} = 200$ cm$^2$/sec. The results for different values of $\eta$ and $\zeta$ are illustrated in Figure 1. For the $\tilde{H}(k)$ curve the key wavelength is the transition from positive to negative response. The results generally indicate that for $\eta, \zeta \sim 10^{15}$, the "high wavenumber" pressure variations are those with wavelengths shorter than $\sim 3000$ km. Since the synoptic scale for the Arctic is of the order of 500 km [Egorov, 1971], we would expect the large $\eta, \zeta$ limiting case to be reasonably valid for $\eta, \zeta \sim 10^{15}$. For $\eta, \zeta \sim 10^{12}$, on the other hand, the high wavenumber cutoff is such that one might expect the small $\eta, \zeta$ case to

![Frequency space response functions for the divergence rate and vorticity of the ice pack for different values of $\eta$ and $\zeta$. The response functions operate on the atmospheric pressure field.](image)

**Fig. 1.** Frequency space response functions for the divergence rate and vorticity of the ice pack for different values of $\eta$ and $\zeta$. The response functions operate on the atmospheric pressure field.
be more applicable. This wavelength dependence explains why most calculations by Campbell [1965] and Campbell and Rasmussen [1973] have indicated diverging ice in a low pressure region. This is especially true of the yearly average drift where the mean yearly pressure field contains few high wavenumber spatial variations.

COMPARISON OF THEORY WITH MESOSCALE MEASUREMENTS

To determine how well the limiting forms of the predicted $\Delta$ and $\omega$ values for large $\eta$ and $\zeta$ infinite boundaries compare with observations reported in Hibler et al. [1973], we have made a comparison between the local pressure at the main AIDJEX 1972 camp, the measured divergence rate, and the measured vorticity. The resulting time series are illustrated in Figure 2. In addition to these three time series, Figure 2 shows the calculated divergence of the wind velocity field and the fluctuations of the local pressure from the average pressure over a region approximately 600 km in diameter. The average pressure was estimated by taking the average of the camp pressure, four remote data buoy pressures located around the camp about 300 km away, and the Point Barrow pressure. For calculation of divergence of the wind velocity field we used local wind speed and direction measurements at each of the three manned stations. The distances and relative angles between the stations were taken as constant and estimated from position data for 19 March as reported by Thorndike et al. [1972]. The basic computational equations are similar to those used in the strain calculations [Hibler et al., 1973]. All of the time series shown in Figure 2 were smoothed with the same low pass filter having a transition band from $0 - 3/80$ cycles/hr [Hibler, 1972].

Correlation coefficients were calculated between all five time series; the results are listed in matrix form in Table 1. The standard error is based upon a number of degrees of freedom equal to the number of points correlated times the fraction of the spectrum passed by the filter.

As can be seen from Figure 2 and Table 1, there is a positive correlation between the local pressure and the divergence rate and a negative correlation between pressure and vorticity as predicted by the limiting case
Fig. 2. Comparison of experimental time series calculated from AIDJEX 1972 data. All curves were smoothed with a low pass filter having a transition band from 0 to 3/80 cycles per hour.

of the linear drift theory. The results also indicate that the spatial pressure fluctuation time series is quite similar to the camp pressure time series and has a similar correlation to the divergence rate and vorticity. This generally indicates that the pressure field varies considerably at wavelengths shorter than 600 km and justifies to a limited extent the use of the infinite boundary solution for comparison.
The correlation between vorticity and divergence rate shows the expected negative value, with the magnitude of the vorticity being generally larger than the divergence rate. In particular the ratio of the vorticity variance to the divergence rate variance is 4.5. The correlations between the wind divergence and the local pressure and pressure fluctuations are also positive, indicating the expected wind convergence in a low pressure region and divergence in a high.

There are indications that at higher temporal frequencies little linear correlation exists between the pressure and the divergence rate at periods shorter than 24 hours. This is reasonable since the atmospheric pressure variation is very nearly band limited. This is illustrated by the spectra of the pressure and divergence rate time series given in Figure 3. Clearly the pressure time series has comparatively little variance at periods shorter than 24 hours, which agrees with typical expected synoptic variation scales [Monin, 1972, p. 9]. The divergence rate spectra on the other hand are relatively flat, although they do fall off by about a factor of 4 at 24-hour periods. These curves indicate that although the meteorological driving forces on the ice are relatively smooth, the response of the ice is more complex and erratic in time, probably due to random bumping of floes and opening and closing of leads.
Estimates of Constitutive Law Parameters $\eta$ and $\zeta$

To estimate $\eta$ and $\zeta$ we utilized the slopes of the regression lines of $\Delta$ and $\omega$ upon $P$ (using the curves in Figure 2) and, also for comparison, the regression lines of $\Delta$ and $\omega$ upon $P - \bar{P}$. Equating these regression line slopes to the predicted slopes in equations 17 and 18 and inserting numerical values for $B$, $\phi$ and $f$ (as previously listed), we obtained the results shown in Table 2.

**TABLE 2**

<table>
<thead>
<tr>
<th>Regression line used</th>
<th>$\eta$</th>
<th>$\eta + \zeta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta, \omega$ upon $P$</td>
<td>$(1.09 \pm 0.32) \cdot 10^{15}$ g/sec</td>
<td>$(2.9 \pm 1.0) \cdot 10^{15}$ g/sec</td>
</tr>
<tr>
<td>$\Delta, \omega$ upon $P - \bar{P}$</td>
<td>$(0.38 \pm 0.07) \cdot 10^{15}$ g/sec</td>
<td>$(1.97 \pm 1.18) \cdot 10^{15}$ g/sec</td>
</tr>
</tbody>
</table>

Fig. 3. Spectra of atmospheric pressure and mesoscale ice divergence rate at the main AIDJEX 1972 camp.
According to the estimates in Table 2, \( \eta \) and \( \zeta \) are of the order of \( 10^{15} \) g/sec, with \( \zeta \), the compressive viscosity, being somewhat larger than \( \eta \), the shear viscosity. This result agrees with intuitive expectations that the ice offers greater resistance to pure compression than to pure shear.

The table also shows that the estimates of \( \eta \) and \( \zeta \) from different regression lines are quite similar. This illustrates that much of the observed correlation between the pressure field and the differential ice motion is due to higher wavenumber (spatial) variations in the pressure field. Such behavior explains why the infinite boundary solution works reasonably well, since high wavenumber variations may be extracted by a real space response function that is well limited in space. Response functions to extract lower wavenumber variations, on the other hand, extend much farther spatially; boundary effects would consequently be expected to be more critical at lower wavenumbers.

As a final point, referring to Figure 1, we see that values of \( \eta, \zeta \sim 10^{15} \) g/sec indicate a wavenumber cutoff of about 2000 km. This cutoff wavelength is relatively large compared to expected synoptic variations in the pressure field, so that the use of the large \( \eta, \zeta \) limiting case appears to be justified for the data analyzed in this paper. This limiting case may also be justified for other boundary conditions (see Appendix).

It is interesting to note that compressive stresses predicted by our estimated values of \( \eta \) and \( \zeta \) are reasonable in terms of stresses predicted by Parmerter and Coon [1972]. For example, maximum values of \( \Delta \) are of the order of 0.0004 hrs\(^{-1} \) which yields for \( \zeta = 10^{15} \) g/sec a compressive stress of \( 1.1 \times 10^8 \) dyne/cm where we have used the Glen constitutive law. This is close to the 0.1 to 0.4 \( \times 10^8 \) dyne/cm needed to cause ridging in 2-meter ice by the bending-failure ridge model of Parmerter and Coon. It is also similar to the maximum pressure difference of \( 2 \times 10^8 \) dyne cm\(^{-1} \) obtained by Rothrock [1972] assuming the ice is incompressible.

To make a numerical comparison of the calculated wind divergence rate with that estimated from the curvature of the pressure field, we take as an estimate of the Laplacian \(-\frac{4}{5} (\nabla^2 P) / a^2 \) where \( a = 300 \) km. Using the regression line of \( \nabla \cdot \nu \) upon \( P - \overline{P} \) we find an observed relation which yields
an Ekman angle of 44°. Certainly these comparisons are only approximate, but they do indicate that the wind divergence estimated from the pressure field using the geostrophic approximation and a constant Ekman angle is the same order of magnitude as the calculated wind divergence rate.

A MORE GENERAL LINEAR CONSTITUTIVE LAW

The previous sections generally indicate that most of the dominant aspects of the observed mesoscale drift behavior may be explained using a simple viscous ice rheology. To see if similar agreement can be obtained using a linear elastic law, we will carry out calculations using a more general constitutive law which allows the viscosities (bulk and shear) to vary with frequency and which can include both elastic and viscous behavior. One such law that is computationally similar to the Glen law is given by

\[
F(t) = \int_{-\infty}^{t} \eta(t - t') \nabla^2 u(t') dt' + \int_{-\infty}^{t} \zeta(t - t') \nabla(\nabla \cdot u(t')) dt' \tag{19}
\]

Taking the temporal Fourier transform of this equation we obtain

(for convenience we simply replace \( t \) by \( \omega \) to denote temporal transforms)

\[
F(\omega) = \eta(\omega) \nabla^2 \mu(\omega) + \zeta(\omega) \nabla(\nabla \cdot u(\omega)) \tag{20}
\]

where \( \eta(\omega) \) and \( \zeta(\omega) \) are analytic in the upper half plane to guarantee causality. Two particular limiting cases of this law are:

A. Glen Viscous law

\[
\begin{align*}
\eta(t) &= \eta \delta(t) \\
\zeta(t) &= \zeta \delta(t)
\end{align*} \tag{21}
\]

or in frequency space

\[
\begin{align*}
\eta(\omega) &= \eta \\
\zeta(\omega) &= \zeta
\end{align*} \tag{22}
\]

where \( \eta \) and \( \zeta \) are constant viscosities.
B. Generalized Hookes law

\[ \eta(t) = \eta \theta(t) \]
\[ \zeta(t) = \zeta \theta(t) \]
where \( \theta(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases} \)

or in frequency space

\[ \eta(\omega) = \frac{i}{\omega + i\delta} \eta \]
\[ \zeta(\omega) = \frac{i}{\omega + i\delta} \zeta \]

with \( \delta \) infinitesimal.

Note that for the Hookes law case \( \eta(\omega) \) and \( \zeta(\omega) \) both decrease with decreasing frequency and have a phase shift. The phase shift is the key indicator of elastic behavior.

Drift Calculations Using Generalized Law

It is clear that by using temporal transforms of all quantities the same formalism used for the simple viscous calculation may be used for the more generalized calculations. In particular, equations 6 through 11 may be formally extended to include the generalize constitutive law by replacing all quantities with temporal Fourier transforms: for example \( \Delta(x) + \Delta(x,\omega)\); \( \tilde{\Delta}(k) + \tilde{\Delta}(k,\omega) \). The argument about limiting cases is also similar, except that the magnitudes of \( \eta(\omega) \) and \( \zeta(\omega) \) are now the determining factors. In particular for the large \( |\eta(\omega)|, |\zeta(\omega)| \) limiting case we have, by analogy to equations 17 and 18, the results:

\[ \lim_{|\eta(\omega)|, |\zeta(\omega)| \to \infty} \Delta(x,\omega) = \frac{B[P(x,\omega) - \bar{P}]\sin \phi}{|\eta(\omega) + \zeta(\omega)| \rho_f} \]  
\[ \lim_{|\eta(\omega)|, |\zeta(\omega)| \to \infty} \omega(x,\omega) = \frac{-B[P(x,\omega) - \bar{P}]\cos \phi}{2|\eta(\omega)\rho_f} \]
Comparison of General Calculations with Observations

To test equations 26 and 27 we need to determine the coherence (and phase lag) at different frequencies between the ice deformation time series and the atmospheric pressure time series. In particular we would like to estimate \( \eta(\omega) \) and \( \zeta(\omega) \). To carry out such an estimation we note that for a linear system the frequency response function may be estimated by a cross-spectral analysis [Jenkins and Watts, 1968, p. 352]. Using the unfiltered time series \( \Delta(t) \), \( \omega(t) \) and \( P(t) \) (the camp atmospheric pressure), a cross-spectral analysis was carried out by the lagged product method. In Figure 4 we show the resulting coherency spectra and phase angles. The phase angle

![Graph showing coherency spectra and phase between vorticity and atmospheric pressure, and divergence rate and atmospheric pressure. The 95% confidence limits for the phase angles vary from ±20° to ±25°.](image)

Fig. 4. Coherency spectra and phase between a) vorticity and atmospheric pressure and b) divergence rate and atmospheric pressure. The 95% confidence limits for the phase angles vary from ±20° to ±25°.
convention is such that a positive phase angle indicates a deformation signal signal lagging behind the atmospheric pressure. Using equations 26 and 27 as a model, a negative phase angle of 90° would occur for a perfect Hooke's law behavior. In Figure 5 we illustrate the resulting amplitudes of $\eta(\omega)$ and $\zeta(\omega)$ obtained from estimates of the amplitude of the response functions of $\Delta$ and $\omega$ upon $P$.

From Figure 5 we see that the bulk viscosity $\zeta(\omega)$ exhibits a general decrease in amplitude with the decrease beginning at about 120-hour wavelengths. The shear viscosity amplitude, on the other hand, is more constant

![Graph showing frequency dependence of the generalized bulk and shear viscosity amplitudes. The amplitudes were obtained using the estimated response function of $\Delta$ and $\omega$ upon $P$.](image)

Fig. 5. Frequency dependence of the generalized bulk and shear viscosity amplitudes. The amplitudes were obtained using the estimated response function of $\Delta$ and $\omega$ upon $P$. 

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with frequency. Referring to Figure 4, the phase angle behavior does show a slight negative tendency indicative of elasticity, especially in the divergence-pressure phase at higher frequencies. However, the overall behavior suggests viscous behavior (0° phase angle) rather than elastic (-90° phase angle).

The decrease in the bulk viscosity amplitude at higher frequencies with the shear viscosity amplitude remaining more constant is plausible on physical grounds. For example, let us imagine forcing a sinusoidal oscillation in the divergence rate of a given region of pack ice. The displacement of the oscillation will scale as 1/ω. Consequently, at very low frequencies the average compressive stress magnitude over one cycle should be larger than at higher frequencies because the larger displacements might cause more thick ice to be crushed. For the shear stress, on the other hand, the stresses due to floes grinding each other might be less dependent upon the amplitude of a shearing motion cycle.

Referring back to Figure 3 we see that most of the atmospheric pressure variance is at wavelengths less than 60 hours. The fact that ζ(ω) and especially η(ω) are relatively flat over these wavelengths (factor of ~ 2 variation) coupled with the small phase shifts at low frequency indicates that the best fit of the generalized calculation agrees closely with the Glen law limiting case. Since a linear law of any kind is probably only a crude approximation of the true ice rheology, the above results suggest that there is little advantage in using a more generalized linear law than that of Glen.

CONCLUSIONS

The most obvious inadequacies of the comparison made here are the neglect of finite boundaries in the drift predictions and the use of a simplified ice rheology. However, the calculations and comparisons in this paper do provide some helpful insight into expected differential sea ice drift for different ice conditions. Specifically this study indicates several conclusions relative to AIDJEX.
1. The general agreement between the infinite boundary linear drift theory predictions and observations indicates that the dominant aspects of the mesoscale differential drift behavior observed in the 1971 and 1972 AIDJEX pilot programs may be explained by using simple boundary conditions and straightforward constitutive laws. Certainly more complete calculations using more realistic constitutive laws are needed to explain detailed drift behavior.

2. The solution of the linear drift calculation indicates the sensitive nature of differential comparisons, in that smaller values of the constitutive law parameters $\eta$ and $\zeta$ would not only change the magnitude of the divergence rate but would completely change its sign.

3. The cross-spectral study between the atmosphere pressure and ice deformation in light of a generalized linear constitutive law indicates that our observed strain results may be explained better by a viscous linear law than by an elastic linear law, with the generalized linear constitutive law yielding only a slight improvement over the Glen law.

4. With respect to spatial scales the infinite boundary linear drift solution indicates that for long wavelength variations in ice deformation the internal ice stress is unimportant, whereas for short wavelength variations the internal ice stress becomes critical.

5. Finally, the fact that differential drift follows the local pressure field reasonably well indicates that the ice velocity field is considerably more nonlinear than has previously been thought. Consequently differential drift estimation using long strain lines may not adequately resolve high wavenumber variations in the ice velocity field. This does suggest, however, that the combined mesoscale and macroscale AIDJEX arrays as planned is an ideal sampling, since the velocity field will be measured in a 25 km grid at enough points to also allow for estimation of the higher wavenumber components of the velocity field.
ACKNOWLEDGMENTS

The author would like to thank D. A. Rothrock for helpful discussions on linear drift theories and continuing constructive criticism of earlier calculations that motivated this research. Discussions with W. F. Weeks and S. Ackley, and comments by J. F. Nye on constitutive laws were also of considerable aid. A. Thorndike and P. Martin kindly provided remote data buoy pressure data. This research was partially funded by the National Science Foundation under NSF Grant AG-344.

APPENDIX

Relative Magnitudes of Differential Drift Forces

A substitution of measured drift parameters into equations 6 and 7 allows for a direct assessment of the relative magnitudes of the wind, water, and Coriolis stress terms independent of the value and functional form of the internal ice stress and independent of boundary conditions. From Figure 2 we see that typical values for $\Delta$, $\omega$, and $\nabla \cdot \nu$ wind are given by $\Delta \sim 0.0002 \text{ hrs}^{-1}$, $\omega \sim 0.0006 \text{ hrs}^{-1}$, $\nabla \cdot \nu \sim 0.14 \text{ hrs}^{-1}$. Using equation 6, this yields the equation

$$(\eta + \zeta)\nabla^2 \Delta - D \cos \theta (5.5 \cdot 10^{-8}) + (m_f + D \sin \theta)(33.3 \cdot 10^{-8})$$

$$= -B (388 \cdot 10^{-8}) \quad (A-1)$$

Using the values of $B$, $D$, $\theta$, $\phi$ and $f$ as mentioned earlier, the equation indicates that the wind stress term is about 10 to 20 times as large as the water and Coriolis stress terms. This certainly shows why water strain and Coriolis terms for compact conditions can be neglected for differential drift, and justifies the use of the large $\eta, \zeta$ drift solution.

It is useful to contrast the differential drift situation with regular drift. For the regular (nondifferential) drift case the wind to ice velocity rate is typically of the order of 50 or less [Reed and Campbell, 1962; Skiles, 1968]. Thus, since $D/B = 50$, water and Coriolis stress terms may
not be neglected for regular drift. For differential drift, the ratio of wind divergence to ice divergence or vorticity is \( \sim 300 \), so that water and Coriolis stresses are relatively small. These scaling factors indicate the critical usefulness of differential drift as compared to regular (nondifferential) drift.

REFERENCES


NUMERICAL INTEGRATION OF QUADRILATERAL ELEMENTS IN THE FINITE ELEMENT METHOD

by

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In the standard formulation of plane problems with isoparametric shape functions, numerical integration is a widely accepted tool for evaluating the elements of the stiffness matrix in various finite element programs [Zienkiewicz et al., 1969; Wilson, 1972]. While the general development of the isoparametric family covers a wide class of elements with curved boundaries [Zienkiewicz et al., 1969], the element with linear boundary (Fig. 1) is most popular and very much in use in everyday practice.

![Fig. 1. An arbitrary quadrilateral element.](image)

A typical term in the stiffness matrix for such elements involves evaluation of integral $I$ as indicated in Przemieniecki [1968]:

$$I = \int_{-1}^{+1} \int_{-1}^{+1} \frac{F(\xi, \eta)}{A_0 + B_0 \xi + C_0 \eta} \, d\xi \, d\eta$$
where

\[ x_{ij} = x_i - x_j \]
\[ y_{ij} = y_i - y_j \]
\[ z_{ij} = z_i \hat{e}_1 + z_j \hat{e}_2 \]

\[ A_0 = (x_{31} - x_{42})(y_{31} + y_{42}) - (x_{31} + x_{42})(y_{31} - y_{42}) \]
\[ = 2(x_{31} y_{42} - x_{42} y_{31}) = 2 \hat{x}_{31} \times \hat{x}_{42} = 4 \text{ Area} \]

\[ B_0 = (x_{34} + x_{12})(y_{34} - y_{12}) - (x_{34} - x_{12})(y_{34} + y_{12}) \]
\[ = 2(x_{12} y_{34} - x_{34} y_{12}) = 2 \hat{x}_{12} \times \hat{x}_{34} \]

\[ C_0 = (x_{32} - x_{41})(y_{32} + y_{41}) - (x_{32} + x_{41})(y_{32} - y_{41}) \]
\[ = 2(x_{32} y_{41} - x_{41} y_{32}) = 2 \hat{x}_{32} \times \hat{x}_{41} \]

The need for numerical integration stems basically from the denominator in the equation being a function of the position variables \( \xi \) and \( \eta \). However, in the case of those elements whose opposite sides are parallel, both \( B_0 \) and \( C_0 \) will be zero since the cross product of parallel vectors is zero. Under such circumstances, analytical evaluation of the integral is possible; observe further that the odd part of \( F(\xi, \eta) \) will contribute nothing to the total because integration limits are doubly symmetric. Such an approach, although elegant, might involve unnecessary branching in a general-purpose algorithm written for numerical integration of arbitrary quadrilaterals. In most formulations \( F(\xi, \eta) \) is a polynomial of degree \( N \), and the Legendre-Gauss quadrature formula [Wilson, 1972] is the one used for the numerical integration scheme. For such cases, an exact evaluation of the integral can be accomplished without any truncation error simply by setting the order of the quadrature formula to \( N/2 + 1 \) inside the program conditioned upon \( B_0 \) and \( C_0 \) being simultaneously zero [Zienkiewicz and Cheung, 1967]. Furthermore, using some criteria on the values of the two skewness parameters \( B_0/A_0 \) and \( C_0/A_0 \) the order of the quadrature formula to be used for the integration may be decided inside a program with a consequent elimination of one input parameter.
REFERENCES


A METHOD FOR CALCULATING BOUNDARY STRESS
IN AN ATMOSPHERIC BOUNDARY LAYER

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INTRODUCTION

A standard way to calculate the boundary stress in a turbulent Ekman layer is to solve the following momentum equations:

\[-fv = -f\nu_g + \frac{d}{dz} (\varepsilon \frac{du}{dz})\]

\[fu = f\nu_g + \frac{d}{dz} (\varepsilon \frac{dv}{dz})\]

assuming that the mean motion of the flow is steady, horizontal, and uniform. Here \(u\) and \(v\) represent the \(x\) and \(y\) components of velocity; \(f\) is the Coriolis parameter; \(\varepsilon\) is the eddy viscosity; and \(z\) the vertical coordinate. \(\nu_g\) and \(\nu_g\) are the \(x\) and \(y\) components of geostrophic wind.

Three further assumptions are usually made:

1) the boundary layer is neutrally stable;
2) the pressure gradient is independent of height; and
3) \(\bar{u} \approx \bar{v}_g\) at \(z \rightarrow \infty\).

If one looks at measurements of atmospheric structure, for example, those of Lettau and Davidson [1957b], one is likely to be convinced that the three assumptions just mentioned are not quite valid. This report describes a preliminary method for the stress calculation in which all three assumptions will be relaxed to some extent.

In order not to get involved with the complexity of the complete atmospheric boundary layer, it seems natural to avoid the integration of the
momentum equations throughout the layer when only trying to find the stress at the boundary. Blackadar and Tennekes [1968] used similarity arguments and obtained a solution of the boundary stress for the neutral atmosphere. But their method could not be easily extended to non-neutral cases [Blackadar and Tennekes, 1968, sec. 7].

Generally speaking, one cannot solve the two momentum equations algebraically because there is not enough information to do so. However, if we are primarily interested in the surface layer where $z = ku^*_s$, then we have two equations with three unknowns, $u$, $v$, and $u^*_s$. Thus an empirical relationship between $u$ and $v$ will supply the third equation. The fact that $u$ and $v$ are related through the angle of turning suggests that the angle of turning in the surface layer should be studied. In actually working out the details, however, it will be shown that the unknowns are $u^*_s$ and $\alpha$, the angle between the surface wind and surface geostrophic wind.

A few sets of data from Lettau and Davidson [1972b] are shown in Figure 1, where the angle of turning is plotted against the vertical distance. It strongly suggests that the angle of turning near the surface is insensitive to what happens farther out. This leads to the possibility of calculating boundary stress by using empirical observations of the angle of turning at the boundary.

**ANALYSIS**

With the $x$ direction set parallel to the surface wind direction, and defining $F = \partial / \partial x$, the momentum equations become

\[-\hat{F} \Omega = -\hat{F} \hat{v}_y + \frac{d}{d\hat{z}} \left( \hat{e} \frac{d\hat{u}}{d\hat{z}} \right)\]

\[\hat{F} \hat{\omega} = \hat{v}_y \hat{a}_y + \frac{d}{d\hat{z}} \left( \hat{e} \frac{d(\hat{F}\hat{a})}{d\hat{z}} \right)\]

where ($\hat{}$) refers to dimensional quantities.

Normalizing on a velocity scale $G_{\lambda}$, the magnitude of the surface geostrophic wind at $z = \lambda$, which is very close to the surface, and a length scale $L$, one gets
Fig. 1. Typical values of angle of turning versus height.

\[-F \frac{fu}{\rho} = -f v_g + \frac{d \tau_x}{dz}\]  
\[(F \frac{du}{dz} + u \frac{dg}{dz}) \frac{de}{dz} + \frac{c}{d \tau_x} \frac{d^2 u}{dz^2} + 2 \frac{dF}{dz} \tau_x + \frac{d^2 F}{dz^2} ue = f(u - u_g)\]  

(1a)  
(2a)

where

\[u_g = \frac{\hat{u}}{G_\lambda} = \frac{-\partial x/\partial y}{\rho \hat{c}_\lambda}\]

\[v_g = \frac{\hat{v}}{G_\lambda} = \frac{\partial p/\partial x}{\rho \hat{c}_\lambda}\]

\[f = \frac{\hat{c}_\lambda}{G_\lambda}\]
\[ u = \frac{\Omega}{G_\lambda} \]
\[ \varepsilon = \frac{\varepsilon}{G_\lambda} \]
\[ \tau_x = \frac{\tau_x}{\rho G_\lambda} = \varepsilon \frac{du}{dz} \]
\[ z = \frac{h}{L} \]

Differentiating (2a) with respect to \( z \) and then substituting (1a) into it one gets (1b)

\[
(Fu' + F'u)\varepsilon'' + 2F'\varepsilon'u' + 2F\varepsilon'u'' + 2F''u\varepsilon' + 3F'''\tau_x
\]
\[ + 2F'(f\nu_g - Ffu) + F''''u\varepsilon = f(u' - u_g') \]  
(1b)

\[
(Fu' + uF')\varepsilon' + F\varepsilon'' + 2F'\tau_x + F''u\varepsilon = f(u - u_g) \]  
(2b)

where primes indicate derivatives of a quantity with respect to \( z \). Thus if one knows something about \( F, \varepsilon, u, \) and \( u_g \) as functions of \( z \) in the surface layer, one can then solve for \( \tau_x \) and \( \alpha \) algebraically. In the surface layer,

\[
\varepsilon = ku_* z \]
\[
\varepsilon' = ku_* \]
\[
\varepsilon'' = 0 \]

\[
u = \frac{u_*}{k} \ln \frac{z}{z_0} \]
\[
u' = \frac{u_*}{k} \frac{1}{z} \]
\[
u'' = -\frac{u_*}{k} \frac{1}{z^2} \]
\[
u''' = \frac{2u_*}{k} \frac{1}{z^3} \]

In the Ekman layer

\[ F = F(\hat{G}, \frac{G_\lambda}{f}, \frac{fz_0}{G}, S) \]
So in the surface layer

\[ F = F \left( \frac{G}{f} \right) \left( \frac{f}{2} \right) \frac{f}{G} \lambda \]  

(5)

where

\[ u_* = \frac{\hat{u}_*}{G} \]

\[ \hat{u}_* = \sqrt{F_x/\rho} \]

\( F \) is a measure of stability

Now \( F \) scales on a large length scale which is of the order of the thickness of the Ekman boundary layer. When we set \( L \) to be that large length scale, \( F', F'', \) and \( F''' \) all become of the same order; furthermore,

\[ F_* = F_0 + F_0'\lambda + \frac{1}{2} F_0''\lambda^2 + \ldots = F^0'\lambda = F^\lambda' \lambda \]  

(6)

where \( \lambda = (S/L) \) is the nearest point from the boundary at which (3) and (4) are still valid.

The surface geostrophic wind component, \( u_z \), is assumed to be independent of \( z \) in the surface layer.

When (2a) was differentiated, a value of negative \( F^\lambda' \) introduces a discrepancy. And yet \( F^\lambda' \) is negative from observations. Therefore, it is concluded that when the \( y \)-momentum equation is differentiated, the inertial terms can no longer be all neglected. In fact, a velocity profile that is not fully developed would produce an upward velocity \( \omega \) (dimensionless). A typical \( \omega \) of 0.01 to 0.02 would make \( [u_\nu'] \) very substantial. Thus the term \( [u_\nu']' \) is added to the differentiated \( y \)-momentum equation as a correction term

\[ (u_\nu')' = (\omega(Fu' + F'u))' \]

\[ = \omega(Fu'' + 2F'u' + F''u) + \omega'(Fu' + F'u) \]

Combining with (4) one gets

\[ (u_\nu')' = F'u'u \]
An assumption of a simple-minded relationship is made:

\[ \frac{F^\prime_\lambda \omega_\lambda}{\mathcal{F}} = - \gamma \]  

(7)

where \( \gamma > 1 \).

Taking the limit of (1b) and (2b) as \( z = \lambda \), and substituting (3), (4), (6), and (7) into (1b) and (2b), together with the approximation \( \tau = u_\ast^2 \), one gets the following equations:

\[ \begin{align*}
0 & \quad + \quad 2(F^\prime_\lambda)(ku_\ast)(\frac{u_\ast}{\lambda}) - 2(F^\prime_\lambda)(ku_\ast)(\frac{u_\ast}{\lambda^2}) + 2F''_\lambda(\frac{u_\ast}{\lambda}) \ln \lambda \frac{L}{\varepsilon_0} \\
- \quad F^\prime(\frac{u_\ast}{\lambda}) + (F^\prime_\lambda)(ku_\ast)(\frac{2u_\ast}{\lambda^3}) + 3F''_\lambda u_\ast^2 \\
+ \quad 2F^\prime_\lambda (\mathcal{F}u_\ast - F^\prime_\lambda \frac{u_\ast}{\lambda} \ln \frac{L}{\varepsilon_0}) + F''_\lambda(\frac{u_\ast}{\lambda} \ln \lambda \frac{L}{\varepsilon_0}) ku_\ast \\
= \quad \mathcal{F}(1 - \gamma) \frac{u_\ast}{\lambda} \\
\end{align*} \]

(1c)

\[ \begin{align*}
\frac{F^\prime_\lambda}{\lambda}(\frac{u_\ast}{\lambda}) (ku_\ast) + (\frac{u_\ast}{\lambda}) \ln \lambda \frac{L}{\varepsilon_0} F^\prime_\lambda \lambda (ku_\ast) - F^\prime_\lambda \lambda (ku_\ast) \frac{u_\ast}{\lambda^2} \\
+ \quad 2F^\prime u_\ast^2 + F''_\lambda (\frac{u_\ast}{\lambda}) \ln \lambda \frac{L}{\varepsilon_0} ku_\ast = \mathcal{F}(\frac{u_\ast}{\lambda} \ln \lambda \frac{L}{\varepsilon_0} - u_\ast) \\
\end{align*} \]

(2c)

Keeping only the terms of order \( 1/\lambda \) in (1c), we get

\[ \frac{F^\prime_\lambda}{\lambda} u_\ast^2 = - \mathcal{F}(\gamma - 1) \frac{u_\ast}{k} \]

\[ \mathcal{F}u_\ast - (E + 2)F^\prime_\lambda u_\ast^2 \]

where \( E = \ln(\frac{\lambda}{\varepsilon_0}) = \ln \lambda (L/\varepsilon_0) \), or

\[ u_\ast = - \frac{\mathcal{F}(\gamma - 1)}{kF^\prime_\lambda} \]

(1d)
\[
\int \cos \alpha = \frac{E' f}{k} u_\ast - (E + 2) F' u_\ast^2
\]  

(2d)

where \( \alpha \) is the angle between the surface wind and the surface geostrophic wind.

Since \( F = \frac{v}{u} = \tan \theta \), where \( \theta \) is the angle between the velocity vector and the surface wind, equations (1d) and (2d) can also be written as

\[
\frac{u_\ast}{G_\lambda} = \frac{(\gamma - 1) \hat{F} L}{k G_\lambda (d\theta/dz)_\lambda}
\]  

(1e)

\[
\cos \alpha = \frac{E + (E + 2)(\gamma - 1) \hat{u}_\ast}{k G_\lambda}
\]  

(2e)

Thus one needs to know only \( f, z_0 \), and \( G_\lambda \) to calculate \( u_\ast/G_\lambda \) and once \( \lambda, L, F_\lambda' \) or \((d\theta/dz)_\lambda \), and \( \gamma \) are determined.

The value of \( \lambda \) should be as small as possible and yet large enough that (3) and (4) hold true. Caldwell et al. [1972] show \( \lambda < 0.006 \). In this document \( \lambda = 0.0017 \) is used. At the time of writing this report, efforts are being made to find a value for \( \lambda \) from a large collection of velocity profile measurements.

Now \( F_\lambda' \) or \((d\theta/dz)_\lambda \) is a function of \( G_\lambda/\hat{f} \) and \( \hat{F} z_0/G_\lambda \). This needs to be established from empirical data of angle of turning. At present, such data are insufficient. However, one may expect the length scale \( L \) to be a simple combination of \( G_\lambda/\hat{f} \) and \( \hat{F} z_0/G_\lambda \) such that \( F_\lambda' \) is a constant. (Note that \( F_\lambda' \) and \( \hat{F} \lambda' \) are different.) Stress calculations show a correlation between \( \hat{u}_\ast/G_\lambda \) and \( \hat{F} z_0/G_\lambda \)^{1/7}, and from (1e) it implies that \( L \) is proportional to \( G_\lambda/\hat{f}(\hat{F} z_0/G_\lambda)^{1/7} \). Thus \( L \) is set to be 0.1 \( G_\lambda/\hat{f}(\hat{F} z_0/G_\lambda)^{1/7} \) which agrees to the same order of magnitude with the boundary layer thicknesses in Caldwell et al. For \( 10^5 < G_\lambda/\hat{f} z_0 < 10^{10} \) this \( L \) works well and should be adequate for calculating the air stress on the arctic ice.

A graph of \( F' \) versus \( L \) is produced using six sets of data for the angle of turning from Lettau and Davidson [1957b]. The \( z_0 \)'s are calculated from the velocity profile as suggested by Lettau and Davidson [1957a], and \( G \)'s are given. (Few sets of data include information about \( G \), and few sets
of angle of turning are smooth; this is why only six sets of data can be used here.) Figure 2 shows this result of $F'_{\lambda}$ versus $L$. It provides a check for the validity of the length scale. It also gives the value of $F'_{\lambda}$, which is about -0.77. For reasons mentioned in the Discussion, the $\alpha_0$ is not considered very accurate, hence Figure 1 must be used with caution. There are about twenty sets of data in Lettau and Davidson [1957b] from a different source that may give smooth angle of turning when processed, and it is hoped that these can be used together with improved values of $\alpha_0$ at a later date to provide a direct calculation of $L$.

Figure 2 shows this result of $F'_{\lambda}$ versus $L$. It provides a check for the validity of the length scale. It also gives the value of $F'_{\lambda}$, which is about -0.77. For reasons mentioned in the Discussion, the $\alpha_0$ is not considered very accurate, hence Figure 1 must be used with caution. There are about twenty sets of data in Lettau and Davidson [1957b] from a different source that may give smooth angle of turning when processed, and it is hoped that these can be used together with improved values of $\alpha_0$ at a later date to provide a direct calculation of $L$.

Equation (1e) is then used with Caldwell's measurements for evaluating $\gamma$. It is found to be equal to 2.0.

Equations (1e) and (2e) can now be written as

$$\frac{\alpha_0}{G_\lambda} = 0.325 \left( \frac{f_{\alpha_0}}{G_\lambda} \right)^{1/7}$$  \hspace{1cm} (1f)

$$\cos \alpha = \frac{2(E + 1)}{0.4} \frac{\alpha_0}{G_\lambda}$$  \hspace{1cm} (2f)
RESULTS AND DISCUSSION

Figure 3 shows a comparison of the measured with the calculated \( u_*/G_\lambda \), while Figure 4 shows the measured and calculated \( \alpha \). The scattering of experimental points is not too bad considering the accuracy with which one obtains \( G \), \( \alpha \), and \( z_0 \) from measurements.

The fact that this method seems to work supports the notion that there is a \( \lambda \) above which the log law applies within the surface layer. In other words, velocity profiles measured below \( z = \lambda \) may not be very good and therefore should not be included in the calculation for \( u_*/G_\lambda \) and \( z_0 \). Efforts are being made to see whether the scattering of \( z_0 \) can be resolved to some extent when the problem of \( \lambda \) is considered. Perhaps if one uses only velocity measurements for \( z > \lambda \), one would get a better value for \( z_0 \), as far as scattering is concerned.
Fig. 4. Comparison of measured and calculated $\alpha$.

**CONCLUSION**

A preliminary method for the calculation of boundary stress in the surface layer of atmosphere has been described. The atmosphere does not have to be neutrally stable; the pressure gradients need not be constant throughout the Ekman layer; and the wind velocity does not have to be equal to the geostrophic wind at infinity. It requires the finding of a characteristic length, the knowledge of a critical distance $\lambda$, and the value of a coefficient $\gamma$ at the surface. When there is enough information, they can be calculated without any arbitrariness.

When more information is collected and processed, quantities such as $\lambda$ and $L$ will be recalculated, and the method will improve. The method may work satisfactorily for both stable and unstable atmospheric conditions. It can also be easily extended to include the effects of stability.
ACKNOWLEDGMENTS

This writer is grateful to Prof. Robert Brown not only for his many helpful comments, but also for the use of the experimental results of his air stress investigations.

REFERENCES


SONAR MAPPING OF THE UNDERSIDE OF PACK ICE

by

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ABSTRACT

An under-ice survey was carried out in a pack ice field near Fletcher's Ice Island (T-3) by means of a 48 kHz Kelvin Hughes transit sonar. The transducer was lowered through a hole, and discrete under-ice scans were made through 360°. Five sets of data have been collected and transformed into polar coordinate mosaics. The echo length, the shadow zone, the arrival time of echoes, and the sounding depth provide the basic information for interpretation. A complete under-ice map has been constructed to show the distribution of the under-ice features. The keel depths of the ridges have been estimated. A surface survey which included a cross-ridge survey was carried out. The top and bottom features have been compared. The average ratio of the peak top elevation to the keel depths is 1/7.6.

INTRODUCTION

Depending upon their age, sea-ice ridges appear on the surface in a variety of configurations ranging from humpy mounds to stacks of broken sheets, and one would expect the underside to be equally complex. This complexity suggests that a great many measurements of the top and the bottom would be needed to establish their statistical relationship, which is a parameter necessary to a theoretical study of sea-ice dynamics.

Methods which have been used to study under-ice features include up-looking sonar mounted on a submarine [Lyon, 1961, 1962, and 1967],
a remotely controlled unmanned vehicle [Francois and Nodland, 1972], narrow-beam sonar [Kovacs, 1972; Breslau et al., 1970], diving [Bright, 1972], and coring. Of these methods, the submarine-mounted sonar allows great mobility in surveying large areas, but it does not relate the surface to the subsurface. The other methods have been used to survey small areas whose dimensions range from a few tens of meters to a kilometer.

On 26, 27, and 28 April 1972, the Geophysical and Polar Research Center of the University of Wisconsin undertook an area survey to map the under-ice features of a typical pack-ice field using a 48 kHz side-looking sonar. The site chosen for the survey lay about 1.6 km off Fletcher's Ice Island T-3 (then at 48°N, 84°W) and just beyond the limit of Colby Bay (Fig. 1). The old pack ice which covers Colby Bay is attached year-round to T-3; the pack ice in our study area undergoes continual deformation. The survey area appears in greater detail in Figure 2, drawn from an oblique photograph taken from a C-130 aircraft. The dashed circle describes the

![Map of Fletcher's Ice Island (T-3)](image)

**Fig. 1.** Map of Fletcher's Ice Island (T-3) showing relative position of T-3 camp, Applied Physics Laboratory (U. Wash.) UARS test site, and our sonar survey area.
driller, (4) a Honda 400-watt generator, and (5) a wooden turntable and clamp to control direction of the transducer and hold the pole firmly in place. (Incidentally, wood is a good material to use in the Arctic because heat transfer from hands to apparatus is slower with wood than with metal.) A nylon line attached to one end of the transducer allowed it to be oriented both vertically and horizontally from the surface, thereby making it possible to insert and remove the transducer through a hole in the ice only 23 cm in diameter.

The sonar transmits 1 ms sound pulse at 48 kHz with 250 watts of acoustical peak power. The active face of the transducer has dimensions of 3.3 x 94 cm, and its radiation pattern has a fan shape with 1.5° of beamwidth in one axis and 51° in the other. The sonar transducer, which was hinged to the terminus of the aluminum pole, was lowered and turned to the 0° direction. Soundings of scattering features were recorded for approximately one-half minute to give about 1 cm of record, and then the direction was changed a few degrees. The increment polar scan was completed when the transducer had been rotated through 360°. The echoes are received by the transducer and amplified with time variable gain (TVG) by the receiver. The TVG is used to compensate the spreading loss of sound transmission in water. TVG is necessary because the graphic recorder has a dynamic range of about 20 db from black to white. The cold and variable temperatures (to -30°C) caused the system gain to vary slowly, and we compensated by using several different gain settings for each direction.

We obtained strong and consistent under-ice echoes (Fig. 3) beneath four major ridges labeled A, B, C, and D on Figure 2. Ridge F was identified on the surface, but we received no corresponding under-ice echoes. The biggest ridge, B, was about 3 m high. Ridges A, C, and D were between 1 m and 2 m high. Area E had complex surface topography.

The transducer shown in Figure 4 can be remotely controlled at the surface to fix it in two orientations, one vertical (Trans V) and one horizontal (Trans H). In Trans V (θ = 0°), the long side axis is vertical--i.e., parallel to the aluminum pole. At this angle, the transducer has a narrow vertical and a wide horizontal beam, and we obtained good vertical
but poor horizontal resolution ($51^\circ$). In the other orientation (Trans H, $\theta = 90^\circ$) the long axis is horizontal—i.e., perpendicular to the pole. At this angle, the transducer has a narrow horizontal and a wide vertical beam, producing records with good horizontal resolution ($1.5^\circ$).

The survey resulted in five sets of data. For each set, we rotated the transducer through $360^\circ$.

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</table>

Surface mapping was carried out with transit and range wheel. The wheel readings were corrected for slippage. Surface elevations and mean ridge elevations were measured with a stadia rod and transit. The surface map and a superposition of sonar map H-12 are given in Figure 3. The elevation of each control point has been corrected to mean sea level, which was about 23 cm below the upper ice surface.

ANALYSIS AND INTERPRETATION

For each set of graphic data, there are either 72 or 36 strips with 1 cm width of information. These records were enlarged photographically and cut into narrow wedges. The wedges were assembled in polar display (Figs. 3, 5, 6, and 7).

First we used H data to determine the positions for each under-ice feature. Because of shadowing, H data taken at different depths give relative depths of some features. Then the V information was added to estimate the depth of the ridges. This information is plotted in Figure 8. These bottom features can then be compared with the surface features shown in Figures 2 and 3.
Fig. 3 (opposite page). Surface survey map superimposed on sonar map H-12. Direction are measured counterclockwise. Ice thickness at S (ice hole) was 3 m.

Fig. 4. Geometry for the case of a tilting transducer. Depth of transducer is $d_0$. A feature is $r$ meters away and $\phi$ degrees right of the direction in which the transducer is facing at tilting angle $\theta$. By turning the transducer counterclockwise, the depth of the feature, $f'$, can be calculated. Depths $d$ and $d_0$ are measured from the bottom of the ice.

Fig. 5 (opposite page). Under-ice sonar map H-29.

Fig. 6 (opposite page). Under-ice sonar map V-12.

Fig. 7 (opposite page). Under-ice sonar map V-23. The transducer was tilted about 10°. Five strong scattering areas indicate deep features. $X$ is the end of ridge C. $X'$ is the echo corresponding to $X$, but was received by the transducer when it was facing 0° azimuth.
Fig. 2. The study site, drawn from an oblique aerial photograph. Labels refer to surface survey. Dashed circle describes sonar range; distance a-b is roughly 280 m. Only those ridges are drawn which were involved in the surface survey. D indicates segments of two ridges. F was found on the surface, but no corresponding under-ice echoes were received.

area, which was 550 m in radius. The baseline is 0° azimuth. The hydro-hut of the group from the University of Washington Applied Physics Laboratory (who were testing their unmanned subsurface vehicle in Colby Bay) was at 42.8° azimuth.

Using the results of this area survey, we intend in this paper to
(1) illustrate the use of sonar in studying under-ice morphology;
(2) determine the relationship between surface and subsurface features; and
(3) estimate an average figure for the ratio of sail heights to keel depths (for our area).

METHOD

The field work of the survey comprised an under-ice sonar survey and a surface survey. Used in the sonar survey were (1) a modified Kelvin Hughes transit sonar, (2) an aluminum pole articulated in 1.9 m sections to support the transducer in the water, (3) a Jiffy gasoline-driven ice auger and
In the two sonar maps (H-12 and H-29), ray tracing calculations were made for transducer depths of 12 m and 29 m. The sound velocity profiles were measured during the same period by the APL scientists and were provided for our use by R. E. Francois.

A comparison of the two H sonar profiles shows the following:

1. The echoes appear on both profiles at the same distance. These echoes correspond to under-ice features which the sonar can detect from both origins;
2. A sequence of echoes appear on the same profile. These echoes represent a spatial sequence of ice blocks which extend away from the transducer;
3. The echoes appear on data H-29 (Fig. 5) but not on H-12 (Fig. 3). These echoes represent features hidden behind obstacles which block the sound rays generated at 12 m but not those originated at 29 m (Fig. 5);
4. The shadow zone between two ridges could be interpreted as being the smooth ice of a frozen lead.

The data in V-12 give a good map (Fig. 6), although the horizontal resolution is poor. The data are used to estimate depths for the ridges as in the following examples. Ridge A had a depth of at least 9.5 m. For 160 m range and transducer depth of 12 m, the vertical illumination range extended from 9.5 m to 14 m (where 4.5 m window corresponds to 1.5° vertical beamwidth). The short echo-length suggests that the sound waves were reflected from a steep wall. The lack of echoes beyond ridge A indicates that either ridge A was as deep as 14 m so that it blocked the 1.5° window completely or the features behind it were too shallow to be detected.

The situation for ridges C and D was different. We notice that both C and D appear on the map (Fig. 6). There are three possible explanations:

1. The two ridges were so situated that C extended slightly into 1.5° of illumination range, and the greater depth of D allowed them both to appear on V-12. Ridge C had a depth of 9.6-11 m, and D was 11 m or deeper.
2. Ridge C had nonuniform depth. Some segments were not deep enough to reach the vertical illumination zone, and D did not have to be deeper than 11 m as suggested above. Due to the wide horizontal beam, both C and D would have appeared on the sonograph in the same way as they did on map V-12.
3. Some of the Trans V data were taken when the transducer was tilted—i.e., when the transducer's long dimension was not exactly vertical. The tilt angle will be referred to as $\theta$.

Data V-23 and V-29 yield results consistent with those of V-12. The geometry of the tilted transducer (Fig. 3) gives the depth of a point in the beam as

$$d = d_0 - r \tan \theta \sin \phi$$

where $d$ and $d_0$ are depths relative to the bottom of the ice. All measurements are reported depths and elevations relative to sea level. The transducer is at depth $z = d + 3$ m, $z_0$, or $d_0$; $\phi$ is the horizontal angle deviation; $r$ is the horizontal distance; and $\theta$ is the tilt angle.

When the transducer was tilted, we found that:

1. The angular shift $\phi$ of the return from a feature is dependent upon both the depth of the feature and the horizontal distance from the transducer.

2. As the fan sweeps by a feature, the feature is first illuminated at $\phi$ and remains illuminated until the trace of the fan on the lower surface of the ice has passed the feature.

3. For each sound there is a series of echoes corresponding to many depths and azimuths. The azimuthal shifts were measured by placing a transparency of Trans H polar scan over the Trans V data. For example, on map V-23, ridge A has a 20° shift and ridge B a shift of less than 10°. Ridge C has nonuniform depth and a 35° shift. Ridge D has a 10° shift (Figs. 6 and 7).

It is a straightforward procedure to determine the tilt angle if one can find a singular feature such as a ridge that ends abruptly in smooth ice. The direction in which scattering from the ridge stops gives the angle for the trace of the sonar beam along the underside of the ice. Since $z$ is 3 m and $z_0$, $r$, and $\phi$ are known, $\theta$ can be estimated. The end of ridge C at 285° (point $X$ on Figures 3, 5, and 8) gave repeated echoes for almost 60°, or six directions of sounding, before it disappeared on data V-23 (from $X$ to $X'$ on Figure 7). Knowing that $z$ is 3 m, $z_0$ is 23 m,
Fig. 8. Map of under-ice features. Dashed lines mark the four UARS tracks, and depths noted on the map were measured by the UARS. Sonar location is indicated by +.

and $\phi$ is $60^\circ$ ($\phi$ can be larger than $25^\circ$, which is half the main beam, because of the side lobes of the sonar beam), we estimated the tilt angle to be $10^\circ$.

There are five strong scattering areas in the study, and they extend to great depth. The ice in areas 3 and 4 of ridge B has a $10^\circ$ shift and a depth of 17 m or 18 m. Area 1 (also ridge B) was about 14 m deep. Areas 2 and 5, each about 23 m deep, do not have angular shift. Ridge A has a
20° shift and has been estimated as 12 m deep. Ridge C had nonuniform depth, with some places shallower than 11 m. The results of data V-29 show the estimated depths as 12 m for ridge A and 16 m for areas 3 and 4.

We can then determine the depth of each feature and construct cross sections (Fig. 9). Figure 8 represents the final work of our interpretations. The subsurface features correlated well with the surface features. A doughnut-shaped feature at 200 m and between 85° and 110° (Fig. 8) appeared on the surface.

The APL subsurface vessel traveled under the Colby Bay area on 9 May 1972, and parts of its four tracks extended into our survey area. Those parts have been superimposed on our under-ice map for comparison (Fig. 8). Their results agree well with ours (Francois, personal communication).

![Diagram](image)

**Fig. 9.** Scattering features and profile of pack ice. The surface elevations were measured along the survey path. Under-ice depths are from sonar data. Starting from left, the ratios of top to bottom thickness are 1.4/10, 1.2/9, 1.5/11, 1.5/10. The dotted portions are in shadow regions for the sonar.
SAIL HEIGHTS AND KEEL DEPTHS

A cross section of the ice and a map view of the under-ice features are shown in Figure 9. The estimated sail heights, keel depths, and their ratios for the four ridges shown in Figure 9 are listed in the caption. The average of the ratios is 1/7.

Sail heights estimated from the other ridges and their corresponding keel depth and ratios have been calculated. The average ratio for ridge A is 1/8.7; ridge B, 1/5.6; ridge C, 1/7.4; ridge D, 1/7.7; and for area E, 1/7.6. The overall average ratio is 1/7.6.

Hibler, Weeks, and Mock [1972] have studied the statistical aspect of the sea-ice-ridge distribution of the Central Arctic Basin. They found the average sail height of the ridges to be 1.31 m, whereas our estimated sail heights in this ice field range from 3.45 m to 0.78 m. Their average keel depth is 9.6 m, and our results range from 22 m to 3 m (no under-ice features for ridge F). The average ratio of sail height to keel depth has been reported by Kovacs [1972] to be 1/4.5 and by Kovacs et al. [1972] to be 1/3. Francois and Nodland [1972] obtained the ratio of 1/7 at Colby Bay area. Our result is 1/7.6.

ARCTIC OPERATIONS

The side-scanning sonar is satisfactory for under-ice reconnaissance surveys such as an air-mobile operation from small aircraft. The equipment, two pilots, and two scientists flew to a pack-ice site north of Barrow, Alaska, in two Cessna 180s [Berkson, Clay, and Kan, 1973] and finished the complete survey in four hours on the ice.

Several improvements can be made. We could remove the uncertainties in the shadow zone and have more independent information to help our analyses by overlapping the sonar surveys. By introducing a pencil-like beam transducer, we could improve the accuracy of keel depth determination. A tripod and hand winch would make insertion and retrieval of the sonar transducer and pipes much easier.
The only serious trouble we had in the field was the highly variable gain of the germanium transistors in the cold environment. It caused the receiver gain to be unstable and affected our interpretation. The sweep speed of the graphic recorder seemed to vary. Since returning, we have built a sonar calibrator which generates sequentially attenuated signals to calibrate the TVG receiver and the sweep speed of the recorder.

ACKNOWLEDGMENT

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ARCTIC SEA ICE RIDGE FREQUENCY DISTRIBUTIONS
DERIVED FROM LASER PROFILES

by

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Washington, D. C. 20373

INTRODUCTION

From April 1970 to February 1973, the Polar Oceanography Division of the U. S. Naval Oceanographic Office (NAVOCEANO) collected airborne laser sea-ice profile observations on a regular basis. The data collection, reduction, and preliminary analysis were done in support of the Advanced Research Projects Agency's Surface Effects Vehicle program. The work performed by NAVOCEANO in this program was funded by the U. S. Army Cold Regions Research and Engineering Laboratory (CRREL) under contract numbers MIPR CRREL 70-13, MIPR CRREL 72-41, and MIPR CRREL 73-13. Primary objectives of the program were to establish effective methods of data acquisition and to evaluate roughness characteristics of the sea-ice surface using basic statistical techniques applied to the laser profile data.

This report presents results of one part of the study, that of ice ridge frequency distributions. Ridge frequencies for 3600 km of polar ice are included together with a preliminary analysis. Contour charts illustrate seasonal differences in ice ridging.

DATA COLLECTION

Technical aspects of the airborne laser profilometer system have been discussed by Ketchum [1972]. The capability of the airborne laser to profile the sea-ice surface has been demonstrated by Tooma and Tucker [1973]. Sections of the profile data, 35 to 40 km in length, have been chosen from 26 geographical regions within the Arctic Basin (Fig. 1). The 93 sections
that constitute the 3600 km of data were acquired during various seasons between April 1970 and February 1973 (Table 1).

DATA REDUCTION

From the original analog magnetic tapes collected in the field, all data were digitized at a rate of 200 samples per second (approximately 0.46 m spacing on the ice surface at aircraft ground speed of 180 knots)
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</table>
on a Digi-Data data logging system. A computer program then converted the laser signals from voltages to ranges in feet while simultaneously editing the data.

Long-period aircraft altitude variations contained in the laser profile record were removed by a computer program employing a three-step filter process as described by Hibler [1972]. Basically the procedure entailed the construction of an aircraft motion curve by three applications of digital filtering. This curve was subtracted from the original profile to effectively remove the long-period variation and establish a level datum. The data were also resampled at a rate of 66.67 samples per second (approximately 1.39 m) during the aircraft motion removal process.

COMPUTATION OF RIDGES

Ridge counts and basic statistics such as RMS, variance, skewness, and kurtosis have been tabulated for the 93 sections by another computer program. Since no standard definition for a pressure ridge existed in terms of actual physical dimensions (elevation, width, slope, etc.), some method of ridge delineation had to be devised for rapid computer identification. Results using the method described below agreed well with ridge counts made from visual examination of laser plots.

A simple computer subroutine inspected the data for a particular sample with an elevation higher than the samples immediately adjacent to it on either side. When these maxima were detected, data backwards and forwards for a distance of 50 samples (approximately 69.5 m) were searched for a drop of at least 0.61 m (2.0 feet) from the peak elevation. If a higher peak was found in the forward search process prior to locating this elevation difference, it replaced the former, and a new search process was begun with this higher peak as the central peak. If the 0.61 m difference was found on both sides of the peak, it was recorded as a ridge. The recorded ridge height was merely that peak elevation value, since a zero datum had been established with the aircraft motion removal process.
RESULTS AND DISCUSSION

Table 2 gives the average number of ridges per kilometer greater than 1.22 m (4.0 feet) in elevation for each 35-to-40 km section. The 1.22 m minimum was chosen for this study to eliminate the possibility that small rubble and snow features would be included in the ridge tabulations.

Figures 2 through 5 show the authors' interpretation of the ridging contours through several seasons. Dashed lines represent the possible location of additional contours.

TABLE 2
AVERAGE NUMBER OF RIDGES (> 1.22 METERS) PER KILOMETER FOR EACH SECTION (35-40 KILOMETERS)

<table>
<thead>
<tr>
<th>NO. OF SECTION</th>
<th>RIDGES/Km.</th>
<th>NO. OF SECTION</th>
<th>RIDGES/Km.</th>
<th>NO. OF SECTION</th>
<th>RIDGES/Km.</th>
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<tbody>
<tr>
<td>1</td>
<td>4.28</td>
<td>2</td>
<td>6.36</td>
<td>4</td>
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<tr>
<td>2</td>
<td>5.70</td>
<td>3</td>
<td>2.46</td>
<td>5</td>
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<tr>
<td>3</td>
<td>2.52</td>
<td>4</td>
<td>2.46</td>
<td>6</td>
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<tr>
<td>4</td>
<td>3.09</td>
<td>5</td>
<td>2.25</td>
<td>7</td>
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<td>2.39</td>
<td>7</td>
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<td>31</td>
<td>2.91</td>
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The winter chart (Fig. 2) should be the most valid because it was constructed from data from five individual missions that span three years. The late winter - early spring chart (Fig. 3) was constructed from the May 1971 data. The May data were not grouped with the winter data, as they had been in the past [Wittmann and Schule, 1966], because the statistics and ridge frequencies appeared quite different from those of other winter months. The summer and fall contour charts (Figs. 4 and 5) were also constructed from data of one mission.
General agreement with Wittmann and Schule [1966] is apparent as far as areas of ridging intensity are concerned; however, numbers cannot be directly compared, because our work deals only with ridges greater than 1.22 m in elevation. The location of general areas of roughness is in agreement with the winter sonar ridge/keel height distributions of Hibler et al. [1972] for the area north and west of the Canadian Archipelago.

It should be realized that these ridging contour charts have been derived from a relatively small amount of data; however, they are the only
quantitative and objective data of this nature presently available. An average variability of 0.94 ridges per kilometer was found for the winter data in areas where cross checks were available. A maximum variability of 4.26 ridges per kilometer for the winter data occurred in geographic region 11 between data collected in January 1971 and data collected in March 1971. Satellite imagery, however, has shown this area (west of Banks Island) to be a zone of large-scale fracturing, and large variabilities in any statistic computed for the ice in this area should be expected. Absolute values of ridges for a specific area should be used with variability in mind; this
Further evidence of the spatial variation of ridging intensity was observed when all sections (regardless of season) were arranged into groups based solely on the average number of ridges per kilometer. Of the sections containing fewer than 0.5 ridges per kilometer, 90 percent were located off the Alaskan coast below 77° north latitude (geographic region number 9-17). However, during certain seasons, geographic regions 13 and 15 may have been...
ice free. Of the sections with more than 5.0 ridges per kilometer, 82 per-
cent occurred in the nearshore areas extending from northeastern Greenland
through the Lincoln Sea and along the Canadian Archipelago to Banks Island
(geographic region numbers 1-3 and 18-26). The middle group (0.5 to 5.0
ridges per kilometer), into which the large majority of the sections fell,
showed a much more general spatial distribution.

Present work is concerned with more detailed analyses of the available
data. New ridge frequency contour charts using a minimum ridge height of
greater than 1.22 m are being constructed. Another ongoing study groups
ridges into certain height ranges and examines the percentage of the total
number of ridges in these ranges. Studies of these range-percentage
changes from a standpoint of location and season variability should prove
interesting. Data collection, although much less frequent, proceeds, with
emphasis on particular regions such as the northern Greenland - Canadian
Archipelago. Once processed, these additional data will modify the contour
charts presented here.

REFERENCES

Hibler, W. D., III. 1972. Removal of aircraft altitude variation from
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Research* 77(35): 7190-7195.

aspects of sea ice ridge distributions. *Journal of Geophysical
Research* 77(30): 5954-5970.

Ketchum, R. D., Jr. 1971. Airborne laser profiling of the arctic pack

of Environment*. (In press.)

Wittmann, W. I., and J. J. Schule, Jr. 1966. Comments on the mass budget
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Budget and Atmospheric Circulation*, The RAND Corporation, Santa Monica,
AIDJEX PICTORIAL DATA

In addition to the digital data listed in previous Bulletins (19 and 20), the AIDJEX Data Bank also has a file of photographs and films related to AIDJEX. They are listed and described on the following pages.

These pictorial data are available for perusal at the AIDJEX office during regular business hours. Please direct requests for copies to Murray J. Stateman, AIDJEX Data Manager. Such requests will be negotiated on an individual basis, the cost depending on form, size, and quantity desired.

CORRECTION: In AIDJEX Bulletin No. 19, the McGill micrometeorological data include Alaska Standard Time, not Greenwich mean time as reported on pages 130, 131, and 142. AST is ten hours earlier than GMT.

If you note other errors in the data bank indices, please contact Murray J. Stateman, AIDJEX Data Manager, 4059 Roosevelt Way N.E., Seattle, Washington 98105.
AERIAL PHOTOGRAPHS OF PACK ICE BY PRINCIPAL INVESTIGATOR

NASA - CONVAIR 990 "GALILEO"

Cruise: AIDJEX Pilot Study 1971
Roll no.: 1
Flight no.: 6
Date: 8 March 1971
Total frames: 126
Flight path: Low altitude flight over Brooks Range, Alaska

Cruise: AIDJEX Pilot Study 1971
Roll no.: 2
Flight no.: 7
Date: 9 March 1971
Total frames: 30
Flight path: Three 83-km runs over AIDJEX area: 2 northbound, 1 southbound.
Absol. alt.: 35,000 ft (12,000 m)
Weather: Haze, undercast

Cruise: AIDJEX Pilot Study 1971
Roll no.: 3
Flight no.: 8
Date: 11 March 1971
Total frames: 154
Flight path: High altitude pattern over AIDJEX area, 83 x 60 km, consisting of ten 83-km runs, 5 northbound and 5 southbound. Low altitude double-cross pattern, 4 legs, each 17 km (north-south, east-west, northwest-southeast, and southwest-northeast). Also high altitude pass over main camp.
Absol. alt.: 35,000 ft (12,000 m) for 83 x 60 km pattern and pass over main camp; 3,300 ft (1,000 m) for double-cross pattern.
Weather: Almost total undercast
AERIAL PHOTOGRAPHS BY PRINCIPAL INVESTIGATOR

NASA - CONVAIR 990 "GALILEO"

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<td>Flight no.:</td>
<td>9</td>
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<td>Date:</td>
<td>12 March 1971</td>
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<tr>
<td>Total frames:</td>
<td>43</td>
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<tr>
<td>Flight path:</td>
<td>Five runs of 83 x 60 km high alt. pattern (3 northbound, 2 southbound), each 83 km long. Low alt. return to Eielson AFB from Camp 200 (main camp).</td>
</tr>
<tr>
<td>Absol. alt.:</td>
<td>35,000 ft (12,000 m) on 83 x 60 km pattern 5,000 ft (1,700 m) on return to Eielson</td>
</tr>
<tr>
<td>Weather:</td>
<td>Total undercast for high alt. pattern, hazy for low alt. return.</td>
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<td>10</td>
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<td>Total frames:</td>
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<td>Flight path:</td>
<td>High altitude 83 x 60 km pattern over AIDJEX area, consisting of 5 northbound and 5 southbound runs, each 83 km long. Low altitude pass over Camp 200 (main camp). Double-cross pattern (4 17-km legs) at low altitude. Return from Camp 200 to Eielson AFB, low altitude.</td>
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<tr>
<td>Absol. alt.:</td>
<td>35,000 ft (12,000 m) for 83 x 60 pattern 3,500 ft (1,200 m) for double-cross pattern 5,000 ft (1,700 m) for return flight</td>
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<td>Weather:</td>
<td>Clear, patchy clouds</td>
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<tr>
<td>Date:</td>
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<tr>
<td>Total frames:</td>
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</table>
AERIAL PHOTOGRAPHS BY PRINCIPAL INVESTIGATOR

NASA - CONVAIR 990 "GALILEO"

Flight path: High altitude flight from Eielson AFB to Camp 200 (main camp). High altitude 83 x 60 km pattern over AIDJEX area, for which we have no photos due to complete cloud undercover. Return to Eielson AFB at low altitude (continued on Roll 6).

Absol. alt.: 33,000 ft (10,000 m) from Eielson to Camp 200
3,500-5,000 ft (1,200-1,700 m) for return flight

Weather: Overcast

Cruise: AIDJEX Pilot Study 1971
Roll no.: 6
Flight no.: 11
Date: 16 March 1971
Total frames: 86

Flight path: Continuation of return from Camp 200 to Eielson AFB (from Roll 5)
Absol. alt.: 3,500-5,000 ft (1,200-1,700 m)
Weather: Overcast

Cruise: AIDJEX Pilot Study 1971
Roll no.: 7
Flight no.: 12
Date: 17 March 1971
Total frames: 195

Flight path: Return to Moffett AFB, Sacramento, California, from Alaska; flying along coast of California.
Absol. alt.: 5,000-10,000 ft (1,700-3,500 m)
Weather: Clear
<table>
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<tr>
<td>Roll no.:</td>
<td>2-72-3</td>
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<td>Date:</td>
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<tr>
<td>Total frames:</td>
<td>241</td>
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<tr>
<td>Flight path:</td>
<td>Low altitude triangle of AIDJEX area, each leg approx. 60 miles long. Leg 1 - Brass Monkey (northerly sat. camp) to Blue Dog (westerly sat. camp). Leg 2 - Blue Dog to Jumpsuit (main camp).</td>
</tr>
<tr>
<td>Absol. alt.:</td>
<td>5,000 ft (1,700 m)</td>
</tr>
<tr>
<td>Weather:</td>
<td>Hazy</td>
</tr>
<tr>
<td>Remarks:</td>
<td>Blue Dog in frames 105-107</td>
</tr>
<tr>
<td></td>
<td>Jumpsuit in frames 221-222</td>
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<td>Roll no.:</td>
<td>2-72-4</td>
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<tr>
<td>Date:</td>
<td>25 March 1972</td>
</tr>
<tr>
<td>Total frames:</td>
<td>279</td>
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<tr>
<td>Flight path:</td>
<td>6 parallel runs over AIDJEX area, each run approx. 20 miles (see Figure 1).</td>
</tr>
<tr>
<td>Absol. alt.:</td>
<td>6,500 ft (2,000 m)</td>
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<tr>
<td>Total frames:</td>
<td>279</td>
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<tr>
<td>Flight path:</td>
<td>5 parallel and 3 cross-field runs over AIDJEX area, each run approx. 20 miles (see Figure 1). Includes shots of fracture east of Jumpsuit and pass over Jumpsuit.</td>
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<tr>
<td>Absol. alt.:</td>
<td>6,500 ft (2,000 m) for first 8 runs 2,000 ft (700 m) for shots of fracture and Jumpsuit</td>
</tr>
<tr>
<td>Weather:</td>
<td>Hazy</td>
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</table>
Fig. 1. Flight path for Roll 2-72-4: lines 8, 10, 11, 9, 7, and 1. Flight path for Roll 2-72-5: lines 6, 2, 4, 5, 3, 12, 13, and 14.
AERIAL PHOTOGRAPHS BY PRINCIPAL INVESTIGATOR

U.S. NAVAL OCEANOGRAPHIC OFFICE - CONSTELLATION "BIRDSEYE"

Cruise: AIDJEX Pilot Study 1972
Roll no.: 2-72-6
Date: 26 March 1972
Total frames: 282
Flight path: 11 parallel runs, 1 cross-field run, over AIDJEX area, each run approx. 20 miles long (see Figure 2).
Absol. alt.: 10,000 ft (3,500 m)
Weather: Light haze

Cruise: AIDJEX Pilot Study 1972
Roll no.: 2-72-7
Date: 26 March 1972
Total frames: 282
Flight path: Last 2 cross-field runs of Figure 2 (end of Roll 2-72-6), approx. 15 miles long. Also first 2 legs of low altitude macroscale triangle, each leg approx. 60 miles (second leg incomplete, continued on Roll 2-72-8). Triangle leg 1 - Jumpsuit (main camp) to Brass Monkey (northerly sat. camp). Triangle leg 2 - Brass Monkey to Blue Dog (westerly sat. camp).
Absol. alt.: 10,000 ft (3,500 m) for cross-field runs
5,000 ft (1,700 m) for macroscale triangle
Weather: Light haze
Remarks: Jumpsuit in frames 040-041
Brass Monkey in frames 162-163
Fig. 2. Flight path for Roll 2-72-6: lines 4, 6, 8, 10, 11, 9, 7, 5, 3, 1, 2, and 14. Flight path for Roll 2-72-7: lines 13 and 12.
AERIAL PHOTOGRAPHS BY PRINCIPAL INVESTIGATOR

U.S. NAVAL OCEANOGRAPHIC OFFICE - CONSTELLATION "BIRDSEYE"

Cruise: AIDJEX Pilot Study 1972
Roll no.: 2-72-8
Date: 27 March 1972
Total frames: 201
Flight path: Low altitude macroscale triangle continued from Roll 2-72-7; finish leg 2. Leg 3 - Blue Dog (westerly sat. camp) to Jumpsuit (main camp). Shots of fracture east of Jumpsuit, low altitude pass over Jumpsuit, and shots of coastal ice.
Absol. alt.: 5,000 ft (1,700 m) for macroscale triangle
2,000 ft (700 m) for shots of fracture and Jumpsuit
1,000 ft (350 m) for shots of coastal ice
Weather: Light haze
Remarks: Blue Dog in frames 005-006, 025-027
Jumpsuit in frame 168

Cruise: AIDJEX Pilot Study 1972
Roll no.: 2-72-9
Date: 29 March 1972
Total frames: 268
Flight path: Two legs of low altitude macroscale triangle of AIDJEX area. Leg 1 - Jumpsuit to Brass Monkey. Leg 2 - Brass Monkey to Blue Dog.
Absol. alt.: 5,000 ft (1,700 m)
Weather: Light haze
Remarks: Jumpsuit in frame 005
Brass Monkey in frames 119-121
Blue Dog in frames 257-258
AERIAL PHOTOGRAPHS BY PRINCIPAL INVESTIGATOR

U.S. NAVAL OCEANOGRAPHIC OFFICE - CONSTELLATION "BIRDSEYE"

Cruise: AIDJEX Pilot Study 1972
Roll no.: 2-72-10
Date: 29 March 1972
Total frames: 287
Flight path: Last leg of macroscale triangle, Blue Dog to Jumpsuit (continued from Roll 2-72-9). 7 parallel runs over AIDJEX area, each run approx. 20 miles long (see Figure 3).
Absol. alt.: 5,000 ft (1,700 m) for leg of triangle
10,000 ft (3,500 m) for parallel runs
Weather: Light haze
Remarks: Blue Dog in frame 004
Jumpsuit in frames 128-129

Cruise: AIDJEX Pilot Study 1972
Roll no.: 2-72-11
Date: 29 March 1972
Total frames: 286
Flight path: 4 parallel, 3 cross-field runs over AIDJEX area, each run approx. 20 miles long (continued from Roll 2-72-10; see Figure 3). Also shots of fracture east of Jumpsuit (main camp).
Absol. alt.: 10,000 ft (3,500 m) for runs over AIDJEX area
2,000 ft (700 m) for shots of fracture
Weather: Light haze
Remarks: Film run-off over Brooks Range, Alaska
Fig. 3. Flight path for Roll 2-72-10: lines 4, 6, 8, 10, 9, 5, and 7. Flight path for Roll 2-72-11: lines 3, 1, 2, 12, 13, 14, and 11.
MOSAICS OF AERIAL PHOTOGRAPHS BY SPONSORING AGENCY

NASA - CONVAIR 990 "GALILEO"

Cruise: AIDJEX Pilot Study 1971
Roll no.: 2
Flight no.: 7
Date: 9 March 1971
Flight path: 83 x 60 km high altitude pattern over AIDJEX area, 10 parallel runs: 5 north-south, 5 south-north.
Area covered: 130°00'W to 132°00'W
73°15'N to 74°15'N
Absol. alt.: 35,000 feet (12,000 m)
Weather: Much undercast

Cruise: AIDJEX Pilot Study 1971
Roll no.: 3
Flight no.: 8
Date: 11 March 1971
Flight path: 83 x 60 km high altitude pattern over AIDJEX area, 10 parallel runs: 5 north-south, 5 south-north.
Area covered: 129°45'W to 132°00'W and 73°45'N to 74°30'N
Absol. alt.: 35,000 feet (12,000 m)
Weather: Hazy

Cruise: AIDJEX Pilot Study 1971
Roll no.: 3
Flight no.: 8
Date: 11 March 1971
Flight path: Low altitude triangle over AIDJEX area, each leg 17 km.
Absol. alt.: 3,500 feet (1000 m)
Weather: Overcast
MOSAICS OF AERIAL PHOTOGRAPHS BY SPONSORING AGENCY

NASA - CONVAIR 990 "GALILEO"

Cruise: AIDJEX Pilot Study 1971
Roll no.: 4
Flight no.: 10
Date: 15 March 1971
Flight path: 83 x 60 km high altitude pattern over AIDJEX area, 10 parallel runs: 5 north-south, 5 south-north.
Area covered: 130°00'W to 132°15'W and 73°45'N to 74°30'N.
Absol. alt.: 35,000 feet (12,000 m)
Weather: Undercast
MOSAICS OF AERIAL PHOTOGRAPHS BY SPONSORING AGENCY

NASA - CONVAIR 990 "GALILEO"

Cruise: AIDJEX Pilot Study 1972
Flight no.: 11
Date: 4 April 1972
Flight path: High altitude pattern over AIDJEX area, approx. 100 x 120 km, 10 parallel runs: 5 north-south, 5 south-north.
Area covered: 75°00'N to 76°15'N and 148°30'W to 152°10'W.
Absol. alt.: 32,000 feet (10,000 m)
Weather: Light haze

Cruise: AIDJEX Pilot Study 1972
Flight no.: 12
Date: 7 April 1972
Flight path: Low altitude pattern over AIDJEX area, approx. 85 x 85 km, 14 parallel runs: 7 north-south, 7 south-north.
Area covered: 74°45'N to 75°20'N and 147°75'W to 149°45'W
Absol. alt.: 10,000 feet (3,000 m)
Weather: Clear

Cruise: AIDJEX Pilot Study 1972
Flight no.: 13
Date: 12 April 1972
Flight path: High altitude pattern over AIDJEX area, approx. 100 x 120 km, 10 parallel runs: 5 north-south, 5 south-north.
Area covered: 75°00'N to 76°10'N and 148°55'W to 153°20'W
Absol. alt.: 32,000 feet (10,000 m)
Weather: Hazy, patches of undercast
MOSAICS OF AERIAL PHOTOGRAPHS BY SPONSORING AGENCY

NASA - CONV AIR 990 "GALILEO"

Cruise: AIDJEX Pilot Study 1972
Flight no.: 14
Date: 15 April 1972
Flight path: High altitude pattern over AIDJEX area, approx. 100 x 120 km, 10 parallel runs: 5 north-south, 5 south-north.
Area covered: 75°00'N to 76°10'N and 150°27'W to 154°50'W
Absol. alt.: 30,000 feet (10,000 m)
Weather: Hazy, patches of undercast

Cruise: AIDJEX Pilot Study 1972
Flight no.: 15
Date: 18 April 1972
Flight path: Four long legs between 75°N and 81°N and between 150°W and 152°30'W.
Absol. alt.: 35,000 feet (12,000 m)
Weather: Clear
Remarks: Mosaic consists of 12 pieces, each 15" x 20".

Cruise: AIDJEX Pilot Study 1972
Flight no.: 17
Date: 23 April 1972
Flight path: High altitude pattern over AIDJEX area, approx. 100 x 120 km, 10 parallel runs: 5 north-south, 5 south-north.
Area covered: 75°00'N to 76°10'N and 151°10'W to 155°20'W.
Absol. alt.: 35,000 feet (12,000 m)
Weather: Mostly clear, some undercast
MOSAICS OF AERIAL PHOTOGRAPHS BY SPONSORING AGENCY

CRREL - "TWIN OTTER"

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MOSAICS OF AERIAL PHOTOGRAPHS BY SPONSORING AGENCY

CRREL - TWIN OTTER"

Cruise: AIDJEX Pilot Study 1972
Date: 13 April 1972
Roll no.: 5
Site: 10 x 10 km Pattern over AIDJEX area.
Absol. alt.: 5,000 ft (1,700 m)
Remarks: Jumpsuit visible in center of mosaic.

Cruise: AIDJEX Pilot Study 1972
Date: 20 April 1972
Roll no.: 6
Site: 10 x 10 km Pattern over AIDJEX area.
Absol. alt.: 5,000 ft (1,700 m)
Remarks: Jumpsuit visible in center of mosaic.
## FILMS IN AIDJEX DATA BANK BY YEAR AND CRUISE

### 1957


### 1968

| Title: ALL IN ORDER AT THE ICE STATION | Cruise: Severnyi Polyus-16 (North Pole-16) | Dates: May-October 1968 | Length: 30-45 minutes | Subject: Documentary on the ice station "North Pole-16". Covers practically all aspects of life on the ice, including air logistics, setting up camp and building runway, problems of summer melt, scientific operations, and daily life. Courtesy of A. Treshnikov. | Remarks: In color, with English soundtrack. |
FILMS BY YEAR AND CRUISE

1970

Title: PATHS OF DRIFTING ICE STATIONS
Date: 1970
Length: 500 feet (approx.)
Subject: Computer-generated film showing paths of about 30 U.S. and Russian ice stations as they drifted around the Arctic Ocean.
Remarks: In color, no sound.

Title: DRIFT ICE - PUZZLE OF THE FROZEN SEA
Date: 1970
Length: 30 minutes
Subject: Layman's introduction to sea ice; about the structure, formation and movements of drift ice in the Sea of Okhotsk and Tabata's work on radar observation of sea ice.
Remarks: In color, English soundtrack. Courtesy of Dr. Tabata, Institute of Low Temperature Science, University of Hokkaido, Japan.

1971

Title: CAMP 200
Cruise: AIDJEX Pilot Study 1971
Dates: March-April, 1971
Length: 600 feet
Subject: All usable footage taken during 1971 pilot study.
Remarks: On loan to Polar Continental Shelf Project.

1972

Title: OUTDOOR ACTIVITY AT JUMPSUIT
Cruise: AIDJEX Pilot Study 1972
Dates: March-April, 1972
Length: 1175 feet
Subject: People at work, buildings, machines, experimental devices, and instruments.
Remarks: In color, no sound.
### FILMS BY YEAR AND CRUISE

#### 1972 (Continued)

|-----------------------|---------------------------------|--------------------------|------------------|---------------------------------------------------------------------------------|----------------------------|

|---------------------------|---------------------------------|--------------------------|------------------|---------------------------------------------------------------------------------|----------------------------|

|-------------------------|---------------------------------|--------------------------|------------------|---------------------------------------------------------------------------------|----------------------------|

|-------------------------------|---------------------------------|--------------------------|------------------|---------------------------------------------------------------------------------|----------------------------|
FILMS BY YEAR AND CRUISE

1972 (Continued)

Title: RUSSIAN VISIT TO JUMPSUIT
Cruise: AIDJEX Pilot Study 1972
Dates: March-April, 1972
Length: 750 feet
Subject: 3 Russian scientists (Venediktov, Bouchouev, Balakin) visit Jumpsuit at end of pilot study; includes scenes from T-3 and Pt. Barrow, Alaska.
Remarks: In color, no sound.

Title: SURFACE VIEWS OF ICE NEAR JUMPSUIT
Cruise: AIDJEX Pilot Study 1972
Dates: March-April, 1972
Length: 300 feet
Subject: Mostly shots of lead near Jumpsuit, some shots of the camp.
Remarks: In color, no sound.

Title: AERIAL VIEWS OF ICE
Cruise: AIDJEX Pilot Study 1972
Dates: March-April, 1972
Length: 110 feet
Subject: Shots taken enroute from Jumpsuit to Blue Dog; shots of leads and prominent features, coming in over Jumpsuit. (Taken from helicopter).
Remarks: In color, no sound.

Title: MISCELLANEOUS & HUMAN INTEREST
Cruise: AIDJEX Pilot Study 1972
Dates: March-April, 1972
Length: 100 feet
Subject: Molly, the Wonder Dog; sunsets at Jumpsuit.
Remarks: In color, no sound.
FILMS BY YEAR AND CRUISE

1972 (Continued)

Title: SEA-ICE RADAR NETWORK
Dates: 1972
Length: 500 feet (approx.)
Subject: Concerns the network of 3 sea-ice radar stations on the Sea of Okhotsk; includes radar reflection patterns for 4 periods during 1969-1971.
Remarks: In color, no sound.
Courtesy of Dr. Tabata, Institute of Low Temperature Science, University of Hokkaido, Japan.

Title: AIDJEX 1972
Cruise: AIDJEX Pilot Study 1972
Dates: March-April, 1972
Subject: Documentary on the AIDJEX 1972 Pilot Study. This covers most of the aspects of life on the ice, including air logistics, setting up camp, construction of housing and various camp activities. Descriptions of the main scientific experiments.
Remarks: In color, with sound.
### ERTS PHOTOGRAPHS IN THE AIDJEX DATA BANK

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## ERTS Photographs on Order

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ERTS PHOTOGRAPHS ON ORDER

Center: 79°46'N, 118°03'W
Orbit: 072
Cloud cover: 10%
Band/quality:
- MSS1 - good
- MSS2 - good
- MSS3 - good
- MSS4 - good

Center: 80°06'N, 160°09'W
Orbit: 0771
Cloud cover: 00%
Band/quality:
- MSS1 - good
- MSS2 - good
- MSS3 - good
- MSS4 - good

Center: 80°07'N, 131°16'W
Orbit: 0742
Cloud cover: 00%
Band/quality:
- MSS1 - good
- MSS2 - good
- MSS3 - good
- MSS4 - good
A NEW ARCTIC JOURNAL NOW IN PRESS

This summer marks the appearance of a new quarterly journal, the Arctic Bulletin. Published by the National Science Foundation for the Interagency Arctic Research Coordinating Committee, the Bulletin is aimed at an international audience of government officials, science program managers, scientists, and educators. It will summarize past, current, and planned arctic research programs sponsored by the U.S. and other nations and provide a historical record of U.S. research in the Arctic. The first issue, now in press, carries a lead article by Herman Pollack, head of the Bureau of International Scientific and Technological Affairs, Department of State, outlining the official U.S. policy on the Arctic.

According to the author's guide, the Arctic Bulletin emphasizes summary and survey articles on federally funded research rather than technical reports and therefore does not compete with the standard scientific journals. Anyone who wishes to contribute an article or receive the Bulletin should write to

Polar Information Service
Office of Polar Programs
National Science Foundation
Washington, D.C. 20550

Although television pictures from Earth satellites have been used for over ten years to detect major sea ice features, direct photo-interpretation methods have been supplemented with a fully automated technique employing Composite Minimum Brightness (CMB) charts. Lack of on-board calibration has prevented quantitative use of the CMB method. In a newly developed procedure the satellite brightness measurements taken over selected areas are used for external calibration. The calibrated data were used to study sea ice conditions in the North American Arctic. Characteristic brightness levels were found corresponding to the following: 1) compact or very close pack, snow covered; 2) compact or very close pack, without snow but with little or no puddling; 3) very close to close pack with much puddling; 4) open pack, generally with much puddling and rotten ice; 5) very open pack or ice-free conditions.


This paper surveys the use of the high-resolution, multispectral data from ERTS-1 (Earth Resources Technology Satellite) for mapping Arctic sea ice. Methods for detecting ice and for distinguishing between ice and cloud are discussed, and examples of ERTS data showing ice distributions in northern Hudson Bay, M'Clure Strait, the eastern Beaufort Sea, and the Greenland Sea are presented. The results of the initial analysis of ERTS data indicate that the locations of ice edges and ice concentrations can be accurately mapped, and that considerable information on ice type can be derived through use of the various spectral bands. Ice features as small as 80-100 m width can be mapped.


Wind velocity fluctuations have been recorded using a sonic anemometer over sea ice at a number of locations in the Arctic Ocean and in Robeson Channel, and used to compute surface stresses.
and drag coefficients. The wind drag coefficient is found to correlate well with the rms elevation of the ice and snow surface at wavelengths shorter than 13 meters. A formula for the estimation of drag coefficients from surface profiles is given. The form drag of ridges can be of similar magnitude to the measured surface drag and should be allowed for. Gust factors are examined. Spectra and other turbulence parameters are found to be in agreement with other boundary layer turbulence measurements over ice, water, and land.


An analytical solution of the vorticity equation is obtained for a stationary, linear, planetary flow in a circular basin rotating around its center. An inflow and an outflow are prescribed on the circumference. The inflow deflects leftward (westward in the Northern Hemisphere) immediately after entering the basin and is divided into two branches connecting the inlet to the outlet: one is a cyclonic flow and the other is an anticyclonic flow along the lateral wall. The result is confirmed by a simple model experiment on a turntable.
OUT-OF-PRINT BULLETINS NOW AVAILABLE

If you have ever tried to obtain past issues of the AIDJEX Bulletin, you know that most of them are out of print and no longer available from AIDJEX. Now the situation has improved. All Bulletins—except issues 5, 7, and 8—have been incorporated into the National Technical Information Service system. To order any Bulletin, use the identifying NTIS number given below, enclose payment of $3.00 for each issue, and address the order to

National Technical Information Service
5285 Port Royal Road
Springfield, Virginia 22151

The Bulletin tables of contents which were printed in the last Bulletin will serve as a reference.

We are still trying to add Bulletins 5, 7, and 8 to the system, but we have not yet been able to amass the minimum number of copies to send to NTIS. If extra copies of these Bulletins are languishing unused on your shelf, we should be most grateful to have them back.

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