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Front cover: Atmospheric instrumentation at the artificial lead experiment on Elson Lagoon in March 1974.
Back cover: Semifinalist in the annual arctic punting event.
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Arctic Ice Dynamics Joint Experiment
Division of Marine Resources
University of Washington
Seattle, Washington 98105
The AIDJEX Bulletin aims to provide both a forum for discussing AIDJEX problems and a source of information pertinent to all AIDJEX participants. Issues—numbered, dated, and sometimes subtitled—contain technical material closely related to AIDJEX, informal reports on theoretical and field work, translations of relevant scientific reports, and discussions of interim AIDJEX results.

The first article in Bulletin No. 26 describes the program and progress of AIDJEX since it was organized in 1970. Other papers cover oceanographic and ice deformation investigations from the 1972 pilot study, as well as further reports from the modeling group. Editorial optimism misled us, in the last issue, to promise that Bulletin No. 26 would carry reports from the spring lead experiment when, in fact, all we had in hand were the logistics report from Andy Heiberg (which we print here) and the oceanographic narrative from Jim Smith. Clayton Paulson returns this month from his search for the Universal Constant, and so perhaps for Bulletin No. 27...perhaps....But no promises.

Lest it be lost within this Bulletin, we note here a correction to be made four lines from the bottom of page 46 in Bulletin No. 23: it should read "a draft $h_L$ and a freeboard $h_u$," not the other way around.

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INTRODUCTION

During recent years, the terrestrial cryosphere has received increased attention within an evolving new climatology. We know from geomorphic evidence that the earth's climatic conditions must have undergone dramatic variations over the ages, and even now within the span of modern science we can observe and record small but perceptible changes of the average conditions of the atmosphere and the oceans. The mounting volume of "proxy data" and direct observations and the increased ability to simulate terrestrial fluid systems with high-speed computer models have made the advancement of a theory of climate one of the most promising areas of research in earth science. With world population and resource management on a collision course, the need for such a theory is unquestioned. The massive efforts proposed in several countries for climate research support the contention that any decisions made in the future affecting the management of arable land, the oceans, and fresh water must be based in part on a better understanding of climate and its variations.

The study of the cryosphere—that part of our environment dominated by snow and ice—is an important component of climate research. Because the mean temperature of the earth's surface is close to the freezing point of water, the extent of the cryosphere is a sensitive indicator of climatic change: the massive ice sheets of Antarctica and Greenland respond to

climatic changes on the order of a thousand years or longer, while the thin but large masses of snow on the ground and ice on the sea respond to much more rapid fluctuations in climate, on the order of weeks to years. Furthermore, the presence of snow and ice is a boundary condition for models of the fluid ocean and the atmosphere.

Sea ice, that part of the cryosphere to which this discussion is limited, can be studied on three spatial scales. On the smallest scale, one is concerned primarily with the characteristics of the ice itself (such as salinity, crystal structure, thermal and electrical properties, and elastic-plastic properties). These characteristics can be studied in small field samples and in the laboratory and are therefore relatively well known.

On the medium scale, research deals primarily with "floe-to-floe" interaction. (A floe in this context can be from one meter to several kilometers in size.) Here, engineering mechanics finds its most useful applications and its most significant progress, as, for example, in the recent studies made of buckling, ridging, rafting, the bearing capacity of floating plates, and the calculation of the forces that moving ice exerts on rigid structures.

The large-scale properties of sea ice are virtually unknown. We believe there is a length scale large enough to include a great number of medium-scale features such as cracks, leads, ridges, and ice of varying thickness. This allows the average behavior of a large-scale element to become well defined, even though behavior of the individual medium-scale features is complex and highly variable.

The most important variables on this large scale are the extent and thickness of the ice cover and the long-term flow patterns. A model describing these variables must treat the thermodynamic and the mechanical processes involving ice interaction with the ocean and the atmosphere. The Arctic Ice Dynamics Joint Experiment (AIDJEX), established in 1970, is building such a model with all the physics believed to be important but aimed at solving field equations for only a limited portion of the Arctic Basin. If this limited goal is achieved and if the proposed coverage by relatively unsophisticated data from a larger area materializes, there is little doubt that a realistic model of sea ice can be developed.
AIDJEX PROGRAM

AIDJEX's specific purpose is to find a quantitative relationship between large-scale stress and strain fields in sea ice. Given such a relationship and suitable methods of finding the external stresses from wind and water currents, the state of stress in the ice and the ice velocity fields can be determined. The ability to perform such calculations will have the following applications:

- Since ice velocity adjusts to the principal driving force of surface wind within a few hours, it will be possible to interpret a synoptic atmospheric pressure map diagnostically for ice convergence, divergence, or shear. This information will be needed when off-shore drilling and surface shipping in ice-covered seas become a reality.

- It will be possible to predict ice motion (ridging, lead formation, etc.) to the extent that atmospheric pressure fields can be forecast. It is fortunate that the predictive models of the atmosphere in use forecast pressure better than any other meteorological parameter and are considerably accurate up to five days.

- The dynamic ice model being developed will be an important step toward understanding the long-term climatic interaction between the atmosphere, cryosphere, and hydrosphere. It will lay the groundwork for a comprehensive analysis of the role of ice-covered seas, the primary objective of the international Polar Experiment (POLEX).

In addition, AIDJEX will continue to yield specific technical and scientific results on the energy transfer in a predominantly stable atmospheric boundary layer, heat and momentum exchange in the upper ocean, sea ice morphology, pressure ridging mechanics, and data buoy technology.

Experiment Design

The scientific plan for AIDJEX that evolved from the initial 1969 version was published in the AIDJEX Bulletin, 15, in July 1972. Its basic concepts have withstood the onslaught of real data from pilot studies and the slings and arrows of budgetary cuts. These cuts made it necessary to
modify the original plan and develop a "most austere" experiment, to be deployed in spring 1975 and run continuously until late spring 1976 (Fig. 1). Its design calls for a minimum of four manned stations about 100 kilometers apart, surrounded by a ring of at least eight automatic drifting buoys. At the manned stations, observations will include position, atmospheric pressure, wind (air stress), relative ocean current (water stress), and geostrophic flow in the ocean. The automatic buoys will report their position, atmospheric pressure, and air temperature to a central data acquisition system.

An important component of the original experiment design was a study of the shear zone, a region close to shore where (especially along the Alaska and Siberia coasts) a band of open water is frequently found in summer. During the cold season, a zone of high shear deformation exists between the shorefast ice and the more mobile ice 5-100 kilometers farther from shore. All previous attempts to model the dynamics of sea ice, including steady-state models, have suffered from our total lack of knowledge of ice mechanics in that zone.

In changing the original design to the "most austere" experiment, the shear zone experiment was eliminated. The consensus of the experts in the reviewing process was that a good interior flow experiment should precede a boundary shear zone study. Because of the imminence of offshore prospecting in the Arctic, there is high interest in doing a preliminary shear zone experiment as soon as possible.

Field Operations Before the Main Experiment

In preparing for the main experiment, AIDJEX conducted a series of pilot studies and other field exercises to gather essential data, test instrumentation, and refine the scientific program.

Pilot studies. The first pilot study took place in March 1970 in the eastern Beaufort Sea and was devoted entirely to oceanographic observations (AIDJEX Bulletin, 4, 1971). The second study, in spring 1971, was a more ambitious undertaking in which a remote sensing and ground truth experiment was added to the continuing oceanic investigations. Several U.S. and Canadian groups participated in this study, with logistics provided by the Canadian Polar Continental Shelf Project (AIDJEX Bulletin, 8, 1971).
Fig. 1. ○ Approximate starting location of the manned stations of the "most austere" AIDJEX 1975-76

○ Approximate starting location of the automatic data buoys AIDJEX 1975-76

▲ Initial and final location of the triangle of manned stations of the 1972 pilot study

Starting location and drift of data buoys deployed in conjunction with the 1972 pilot study

Buoy no. 1, operating 4/72-7/72: 76 days
2 4/72-4/73: 355 days
3 4/72-2/74: 667 days
4 4/72-6/72: 84 days
5 4/72-12/73: 603 days
6 4/72-6/73: 417 days
8 10/72-3/74: 533 days

+ Originally proposed array of data buoys (now proposed as part of the U.S. contribution to POLEX)
The third AIDJEX pilot study, from late February to late April 1972, was the most complex and sophisticated experiment ever performed on sea ice. It employed an array of manned stations and automatic buoys, and utilized some of the most advanced technology available. Operations, preliminary scientific results, and data are reported in the AIDJEX Bulletin, 14, 18-22.

**Lead experiment.** During most of the year, extremely large temperature differences exist between the surface of thick ice and the surface of open water (leads and polynyas) produced by the deformation of ice. Because this variable area of open water may be the controlling parameter in the overall heat and ice balance during parts of the year, it was proposed in the 1972 AIDJEX scientific plan to repeat an experiment conducted in 1961 to elucidate the role of leads in the total heat balance of the Arctic Ocean.

This spring, scientific groups from three universities participated in the AIDJEX lead experiment on the sea ice north of Point Barrow, acquiring data on the modification of the atmospheric and oceanic boundary layers near an open lead. The rapidly shifting ice, the quickly freezing water during the arctic spring, and the sophistication of the instruments used in the experiment required painstaking preparations, high-precision logistics, and luck. As in previous operations, the Naval Arctic Research Laboratory proved to be a vital source of logistics support. Detailed accounts of the lead experiment by the principal investigators will appear in a forthcoming AIDJEX Bulletin.

**RESULTS**

**Automatic data buoys**

A detailed description of the 1972 buoy test program is given in AIDJEX Bulletin, 22, 1973. Six Interrogation, Recording, and Location System (IRLS) buoys equipped with atmospheric pressure sensors and temperature sensors at two levels above the ice were installed about 400 km from the 1972 main camp (Fig. 1). The buoys were located by the IRLS system aboard the Nimbus D satellite, a year past its designed life time at the time of the experiment. The specific IRLS buoy configuration for ice deployment was developed by the
University of Washington's Applied Physics Laboratory under contract with the National Data Buoy Office at the National Oceanic and Atmospheric Administration, Department of Commerce. The last operating IRLS buoy was recovered at a location west of Banks Island (72°N, 130°W) on March 6, 1974, with the support of the Polar Continental Shelf Project, and is now providing useful technical data with respect to battery performance, electronic component aging, and deterioration of the buoy structure.

Under development with the National Data Buoy Office, the principal buoy for the 1975-76 main experiment will have a radio data link to a central facility at one of the manned camps. The motion of the buoys will be measured using the Navy Navigation Satellite System (NavSat), with an expected accuracy of 100 meters.

**High accuracy navigation**

Satellite navigation systems were operated during the 1972 pilot study at the three manned camps. The root mean square error was 80 m. From these data, the components of the two-dimensional strain tensor were derived to an accuracy of approximately 1 in 1000. As an example, Figure 2 shows the change with time of the station triangle area in AIDJEX 1972. The NavSat system for the main experiment is being designed to automatically select the best passes from the six navigation satellites presently in orbit. The distance measurements (relative position) by the NavSat system will be on the order of 10 meters. During the main experiment its computer will be part of an integrated data acquisition system, serving the atmosphere and ocean sensors as well.

**Space and time scales of ice motion**

Prior to the 1972 AIDJEX field experiment, two crucial questions were unanswered: What is an appropriate high-frequency cut-off for measurements of ice motion? On what space scale does the pack ice begin behaving as a continuum?

The first question was answered using data from the satellite navigation system and the acoustic bottom referencing (ABR) system (AIDJEX Bulletin, 14,
Fig. 2. The total area of the AIDJEX 1972 station triangle, determined by the Navy NavSat system, is shown for March 14 to April 25, 1972. The net ice divergence during the 40 days of observations was 5.5 percent.

1972. The high sampling rate (0.01 per second) and the high precision of the ABR (5-10 m) were adequate to resolve any rapid fluctuations in the ice movement. However, the data showed that for the velocity spectrum, a measure of the amplitude of the velocity fluctuations as a function of their frequencies, drops off dramatically for high frequencies. Ice motions with a period of 12 and 24 hours are documented clearly in the spectral analysis, but their amplitudes are small compared to the low-frequency wind-driven motions. Similar results for strain rather than ice velocity were obtained independently by an extensive set of laser ranging data from a mesoscale strain network (AIDJEX Bulletin, 21, 1973), operated by the U.S. Army Cold Regions Research and Engineering Laboratory (CRREL).

As for the second question, experimental determination of an adequate space scale is nearly impossible because of the immense logistics difficulties
in measuring densely in space. Nevertheless, results obtained in 1972 on a 100 km scale supported the theoretical contention in the scientific plan that if an appropriate scale exists, it should lie between 10 and 1000 km. Viewed on this scale by remote sensing techniques, the pack ice becomes fairly homogeneous or at least smoothly varying. The strain measurements on this scale appear continuous and differentiable, making a continuum hypothesis plausible (Figs. 3 and 4).

![Graph showing ice velocity power spectra](image)

This contention received additional support by a recent analysis of ERTS-1 images of Beaufort Sea pack ice. Figure 5 is an example of several curves obtained for the displacement of a straight line connecting a large number of recognizable pack ice features. On the large scale (spacing of manned stations and data buoys during the main experiment), the deformation
Fig. 4. Power spectra of ice acceleration derived from the same data as in Figure 3. In neither case is there evidence of high-frequency motions with appreciable amplitude.

Fig. 5. Displacement $u$ versus position $x$ for points on a straight line in the Beaufort Sea pack ice, during a two-day period in March 1973. Here $u$ is the component of displacement parallel to $x$. The circles show measured displacements of recognizable ice features. Derived from ERTS-1 images.
seems to be that of a continuum. The medium-scale perturbations (10^1-10^2 km) represent the mechanical interactions of smaller "floe" ensembles. In its most sophisticated form, the AIDJEX model (Fig. 6) should be capable of representing these medium-scale motions. This should be of particular value for engineering and surface shipping applications. Through suitable parameterizations, less detailed versions of the model will be developed for integration with the global fluid models.

**Air stress**

An operational dynamic pack ice model requires that the field of wind stress be known. This involves both theoretical and practical problems.

The gradient wind on top of the planetary boundary layer can be derived from maps of atmospheric surface pressure. The problem is to relate the surface stress to the gradient wind, accounting for the influences of roughness and stratification. This is one of the oldest, most fundamental, and most widely researched problems of geophysical fluid dynamics. All theoretical treatments of this problem are approximate or require data of a degree of completeness and sophistication that generally cannot be achieved. In each case, the investigator must choose the theoretical concepts producing the best results for the particular circumstances. An extensive review of the problem was given in *AIDJEX Bulletin*, 20, 1973. In an additional article *(AIDJEX Bulletin, 23, 1974)*, the same author shows that a usable relationship exists between the geostrophic drag and the stratification of the boundary layer, and between the stratification and the angle of turning of the wind through the boundary layer. The main experiment observations will enable us to choose a model that is consistent with the data.

The problem of acquiring accurate wind fields for ice dynamics application would be greatly alleviated if direct surface wind observations were available from the automatic data buoys. The frequent deposition of rime on all exposed surfaces and the lack of sufficiently large electrical power sources for de-riming make it impossible to use conventional anemometers. Although development of new all-weather wind sensors is underway, including suitable devices for maintaining a reference azimuth to compensate for buoy rotation, the sensors will not be available for some time.
Fig. 6. The AIDJEX dynamic ice model and its relationship to the atmosphere and the ocean global models. The nature of the expected model output and potential applications to practical problems are also indicated.
Water stress and general oceanography

A large amount of data has been gathered on the mixed layer structure beneath the pack ice. The scientific problems associated with the fluxes of momentum and heat in the layer are analogous to those in the atmospheric boundary layer, with the exception of the salt flux. However, while the planetary boundary layer in the atmosphere has a thickness of approximately $10^3$ m and can be observed only from tall masts, balloons, aircraft, or by remote sensing methods, the oceans' planetary boundary layer is only about $10^1$ m thick and is easily observed in its entirety.

Closely spaced horizontal current observations in the mixed layer like those from the 1972 pilot study can be used to evaluate the classic Ekman equation, in a form integrated with respect to the vertical coordinate, to obtain the stress exerted by the water on the ice. This method is extremely convenient because it considers only the time change of the total horizontal momentum in the Ekman layer and requires none of the complex theory needed in the atmospheric boundary layer.

It has often been stated that the pack ice provides a stable platform for observations of general interest in oceanography that are extremely difficult and costly in the open ocean. During the multi-station observations of AIDJEX, it was found that baroclinic eddies exist at depths between 50 and 300 m (AIDJEX Bulletin, 23, 1974). These eddies have diameters of 10-20 km and are believed to develop at the boundary between water masses influenced by the Pacific and by the local arctic regimes. Eddies of this kind have been found and studied in the central Atlantic Ocean. Dynamically, they are analogous to the cyclonic eddies of the mid- and high-latitude troposphere, but their role in the large-scale oceanic mixing is unclear.

Remote sensing

AIDJEX has profited from the remote sensing programs of the National Aeronautics and Space Administration (NASA) and NOAA. The discovery that the emissivities in the 19 and 37 GHz bands are significantly different for first-year and multiyear sea ice has opened unprecedented possibilities of monitoring and studying the global sea ice balance (by satellite-borne
microwave imagers) and the detailed composition of the pack ice in a specific region (by airborne observations). During the 1972 pilot study, a joint experiment was conducted by NASA, the Department of the Interior's U.S. Geological Survey (USGS), Environment Canada, and AIDJEX to provide ground truth for the remote sensing overflights of the NASA "Galileo" aircraft (AIDJEX Bulletin, 18, 1973).

The images from ERTS and aerial photographs have provided for the first time a synoptic view of the large-scale morphological features of the pack ice. A new side-looking radar and other remote sensing instruments were tested in spring 1974 in a series of flights across the shear zone north of Barrow in a USGS and CRREL project.

In remote sensing technology, it is difficult to separate the items that can be done from those that should be done. The inaccessibility of the polar regions makes polar research an obvious prime user of remotely sensed data, but the published literature clearly indicates that efforts toward analyses and interpretations of the data must be increased so rapid progress of technology does not cause the method to overtake the purpose.

The most important information to be obtained for AIDJEX by remote sensing methods is the ice thickness distribution (see the following section). No single and immediately applicable method exists to obtain this information. The plan for remote-sensing observations during the main experiment is being formulated. It will provide for a combination of photography, infrared emission profiles and images, and possibly microwave and radar data which should collectively give us an occasional check (several times during the year) on the ice thickness distribution computed by the dynamic model.

We hope and expect that, in addition to the remote sensing data specifically required for the pack ice model, a wide range of additional data will be acquired for sea ice research in general. It is, for example, highly desirable to have a few sets of sequential photographic or ERTS-type images of the ice to determine the strain field in much greater spatial detail (but with less numerical precision) than will be possible using the position measurements at the manned stations.
Modeling

In addition to management, operations, and technical coordination, an important activity of the AIDJEX office is the development of a numerical model, utilizing the data to be acquired during the main experiment. In its present state, this model has evolved from numerous studies and a continuous exchange of ideas between the AIDJEX modeling group and those at the Department of Interior, U.S. Geological Survey (USGS), CRREL, and other institutions.

On the basis of several theoretical studies of the mechanics of pressure ridge formation, it was decided that pack ice as a geophysical material can best be described by modeling a number of mesoscale phenomena by a composite constitutive relationship realistically describing large-scale ice behavior.

The AIDJEX modeling group's work plan is to perfect the ice model to a point where the field observations beginning in spring 1975 can be utilized with a minimum delay. The field data will both drive and check the model. The ice model structure and its relation to global atmosphere and ocean models and to applications are shown in Figure 7.

A mathematical representation of the physical behavior of any medium must include:

- equations of motions
- a constitutive law for the material
- conservation of mass (including sinks or sources)
- the first law of thermodynamics

Before AIDJEX began, sea-ice models used clear and realistic formulations only for the first item listed. The second item is now tentatively formulated as an elastic-plastic law, with a property similar to strain hardening. On a subgrid scale, the material is assumed to be densely fractured by processes not considered in the model. Furthermore, the strength of the material is determined by a statistical thickness distribution that includes open water.

An important step forward was the identification of this thickness distribution as the key state variable of the pack ice. By means of an ice growth and decay model developed earlier, it was possible to establish a
a statement of the conservation of mass and to formalize the way in which
the mechanical behavior and the thickness distribution affect each other
(conservation of mass).

The fourth requirement, to satisfy the first law of thermodynamics,
is difficult to meet. Work done on the ice by deformation is dissipated
into surface energy by fracturing, floe-to-floe friction and, most important,
the potential energy stored in pressure ridges. The present model assumes
that the entire strain work goes into ridge building.

A detailed progress report setting forth the theoretical foundation
of the model and presenting the results of preliminary calculations was

ORGANIZATIONAL STRUCTURE AND PARTICIPANTS

AIDJEX is structured as shown in Figure 7. A Joint Panel on AIDJEX
(now transformed into the Joint Panel on POLEX, with AIDJEX as a secondary
task) was established by the National Academy of Sciences (NAS), as
requested by the National Science Foundation (NSF). This panel advises
the federal funding agencies and the AIDJEX coordinator and evaluates and
adjudicates in scientific matters. The AIDJEX Committee is composed of
participating scientists and agency representatives. It advises the AIDJEX
coordinator in scientific and operational matters and reviews research
proposals for relevance to the AIDJEX objectives.

The staff of the AIDJEX office at the University of Washington is
grouped into three functional categories: the management staff, with the
coordinator as a project director on behalf of NSF and an employee of the
university; a technical staff responsible for logistics, hardware (data
acquisition system and buoys), and the data bank; and a scientific staff
responsible for the comprehensive model expected to be the main result of
AIDJEX and for developing concepts of theory and procedures required for
the main experiment.

The J in AIDJEX represents the participating projects as shown in
Figure 7. The number reached 26 in the 1972 pilot study (AIDJEX Bulletin,
Fig. 7. Organizational structure of the AIDJEX program.
14, 1972). The budget cuts and an intentional concentration on the scientific focus of AIDJEX have reduced the number of participants. Major participants in the 1975-76 main experiment will be the Lamont-Doherty Geological Observatory of Columbia University, the Cold Regions Research and Engineering Laboratory, the U.S. Geological Survey, NASA, the National Data Buoy Office, the University of Alaska, and several Canadian participants supported through the Polar Continental Shelf Project.

Establishing the AIDJEX office and modeling group at a large university with strong programs in earth sciences and engineering has proved to be highly beneficial. Its faculty has been a convenient source of scientific talent for the program elements or for consultation, and its graduate students provide a pool of trained personnel to work part-time on the program while furthering their own training. Because state universities have a primary responsibility to undergraduate education, legislators, regents, and college administrators are sometimes not entirely sympathetic to highly task-oriented research projects. On the other hand it is, with some exceptions, those large universities that have the diversified intellectual resources to tackle complex and multidisciplinary research tasks. We believe that, with the assistance (and sometimes forbearance) of administrators both in the federal agencies and in the universities, AIDJEX can help find optimal compromises in the operation of such projects.

The initial AIDJEX scientific plan was formulated in 1969 and was the outgrowth of a variety of research projects supported since IGY 1957-58, principally by the Office of Naval Research (ONR) Arctic Program. It was enthusiastically received by the community of polar scientists informally convened in November 1969. Experienced science administrators have observed that research work in snow and ice and in the polar regions attracts a type of scientist who is typically competent (though rarely brilliant), enterprising, resourceful, hardy, and easy to get along with. Several examples could be given to show that national and international panels or committees engaged in snow and ice research are generally among the most productive and least quarrelsome. From its inception, it was the virtually unanimous consensus of this community of scientists that AIDJEX was the only way in which a quantum jump in understanding sea-air interaction in ice-covered seas could be achieved.
AIDJEX Bulletin

In view of the cooperative nature of the project and its diversified participation and support, it was important that all activities be documented and distributed rapidly. Initiated in September 1970, the AIDJEX Bulletin attempts to strike a balance between quality and speed. It contains original scientific papers (often pre-publications of regular journal articles), status and progress reports on specific activities, plans and narratives of field exercises, performance evaluations of hardware, reports on conferences and working sessions, raw data of immediate use to investigators, translations of interest, and an occasional joke. Twenty-five AIDJEX Bulletins totaling about 3,000 pages were published by July 1974. There are 700 copies printed for current editions. Back issues are available from the National Technical Information Service.

Data bank

In November 1971, a data bank was established. It is managed by a full-time member of the AIDJEX office staff and uses the University of Washington computer center and the computer terminal located in the AIDJEX office for data handling and retrieval. The data bank's inventories and acquisitions are periodically reported in the AIDJEX Bulletin. With the concurrence of the AIDJEX Committee, the National Academy of Sciences Joint Panel for AIDJEX, and the federal funding agencies, the AIDJEX office has issued a statement explicitly describing the basic obligations of principal investigators and the working policies of the data bank. To date, the operation of the data bank and its individual investigators and national data depositories have been highly satisfactory.

Financial support and budgetary summary

As mentioned earlier, much of the research leading up to AIDJEX was supported by ONR. When NSF was designated lead agency for the extension of arctic research in 1969, it awarded the first substantial contract to the University of Washington for establishing the AIDJEX office in the Division of Marine Resources. Since then, the support from ONR and, particularly,
NSF has increased substantially. While the support from NSF is split almost evenly between science and operations, a large and vital part of the ONR assistance has been operational and logistics support through the Naval Arctic Research Laboratory. Through its National Data Buoy Office, the National Oceanic and Atmospheric Administration is supporting the development of a buoy especially designed for various applications in ice-covered waters.

Primarily, the NASA contribution has been remote-sensing aircraft overflights during field experiments. Coordination of the NASA flights with the AIDJEX program and support of modeling work has been done by USGS. ARPA's original support was for field experiments, but it is now directing efforts toward the modeling effort.

Canadian participation has mainly been through the Polar Continental Shelf Project, including scientific investigations and logistics support.

Table 1 summarizes funding support for AIDJEX through FY 1973 and shows the projected funding estimates through completion of the field program in FY 1976. The table also shows the favorable ratio between scientific programs and logistics support, despite working in a remote and difficult environment.

CONCLUSION

It should be evident from the preceding pages that AIDJEX has been a group effort in every respect. The participants have enjoyed the benefits and paid the costs which result from such an effort. The number of dedicated scientists and administrators who have advanced the project this far is so large, and their individual contributions are so varied, that they can be neither weighed nor enumerated at this time. The critical period for AIDJEX lies just ahead. The task of maintaining a sound and legitimate scientific focus while accommodating the individual motivation and competence of the participating scientists (all within tight budgetary constraints) will require the continuing support and goodwill we have enjoyed to date.
### TABLE 1

**AIDJEX FUNDING (IN THOUSANDS OF DOLLARS)**

<table>
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Note: Amounts for FY 1970-73 represent actual expenditures. Estimated totals for FY 1974-76 are shown. Breakdown of these totals will not be available until accounts are completed.
AIDJEX LEAD EXPERIMENT, SPRING 1974
FIELD OPERATIONS REPORT

by

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University of Washington, Seattle, Wash. 98105

In March and April 1974, scientific groups from three universities participated in a lead experiment on the sea ice north of Point Barrow. The experiment, a part of AIDJEX, was designed to gather information about the interaction between the ocean and the atmosphere through open and freezing leads as well as to test equipment that will be used in the main AIDJEX year, 1975-76.

The Office of Polar Programs, National Science Foundation, funded the atmospheric research program through a grant to the School of Oceanography at Oregon State University (C. Paulson, principal investigator), which in turn subcontracted with the University of Washington Department of Atmospheric Sciences (J. Businger, principal investigator) for part of the work. The Office of Naval Research funded the oceanographic research program through a contract with the UW Department of Oceanography (J. D. Smith, principal investigator), the investigation of air-sea exchange of CO₂ through a contract with the Institute of Marine Science at the University of Alaska (J. Kelley, principal investigator), and the radiation program through a contract with the UA Geophysical Institute (B. Holmgren, principal investigator). Logistics for the experiment was provided by OPP through the AIDJEX Office and by ONR through the Naval Arctic Research Laboratory at Barrow.
ARTIFICIAL LEAD EXPERIMENT

The first phase of the field operations involved the atmospheric, CO₂, and radiation programs in an artificial lead experiment near NARL to calibrate and check equipment and to collect data under controlled conditions. Two artificial leads were constructed on Elson Lagoon by laying out semicircular enclosures of tubular visquine, filling the tubes with water, and allowing them to freeze. The smaller "lead" (10 m radius) was laid directly on the snow-covered ice of the lagoon, and one ring of tubing sufficed as the perimeter. The larger enclosure (20 m radius) required three rings of tubing and bulldozer leveling of the surface. After the tubes had frozen into an effective shoring, a 6,000 gal/min pump filled the ponds. Continuous pumping offset by drainage from a hole on the pond side opposite the pump allowed a steady flow that kept the pond from freezing over quickly.

NATURAL LEAD EXPERIMENT

In the second phase of the field operations, all the research parties, including the oceanographic group, were deployed to natural leads in the pack ice. The criteria for an acceptable lead were stringent: it had to be 10-100 m wide; it had to lie perpendicular to the wind; it had to be in water at least 100 m deep; it had to pass through thick (old) ice to provide close, safe campsites, but be near smooth (young) ice on both sides to allow safe landing by Cessna; and it had to be free of ridges or obstructions on the upwind side for undisturbed air flow across the lead. As is apparent from the field schedule at the end of this report, the ideal lead was hard to find and compromises had to be made.

The Camps

A camp at its full complement comprised five fully instrumented, self-contained shelters which, once deployed by helicopter, required only a few hours of rigging to render them operational. The helicopter hauling the 4000-pound huts in a sling to the campsite carried enough fuel to fly a maximum distance of 30 miles. Three shelters were lightweight army helo-huts,
11x17x7 feet; two were 12x12x7-foot plywood structures. Each was outfitted with scientific equipment, individual AC power generators and one day's fuel, propane heat, three bunks, and the rude essentials for safety and comfort. Once the huts were landed, back-up provisions were flown from NARL in one or two supply flights. VHF homer beacons enabled the helicopter to find the camp in poor weather.

**Communications**

There were usually two HF transceivers operating in camp, one on each side of the lead. At times they were used to communicate across a wide lead (although walkie-talkies and megaphones were more frequently employed); but in general they served to make scheduled, twice-daily contact with the USCG communications center at NARL, which maintained an around-the-clock radio watch to direct traffic when people were in the field. Four frequencies were available: 8975, 6530, 4625, and 3411 KHz. Of these, the two lower were tested and 4625 proved most satisfactory. Three HF transceivers were tried: RF 301, RF 2200, and RF 1400. The 301 and 2200 worked very well with the double antenna, but the 1400 was only marginal for camp-to-shore communication.

The walkie-talkies proved to be less than satisfactory. It seemed that every time one side tried to make contact with the other side, the batteries were dead, the vip antenna broken, or the called party inside the hut or not listening. The battery-powered megaphones, on the other hand, worked very well.

**Deployment**

The nature of the experiment demanded that the data collection get under way quickly once a suitable lead was found. The open water froze over so fast, especially in the early stage of the program, that usually less than 24 hours was available to locate a lead, deploy the camps, and start making measurements; by the next day, an ice crust had formed, closing the lead. For this reason, the search and the first stage of camp deployment were flown together.
The day before an anticipated deployment, the huts were prepared, equipment checked, and everything gathered in the launching area on the beach in front of NARL. Manifests and aircraft load schedules were issued and all parties informed of the plan. On the morning of the search day, two Cessna aircraft flew the mission, one carrying the ice observers (usually C. Paulson, J. D. Smith, and K. Toovak, the native ice expert), the other hauling survival camp gear and one oceanographer with hydrohole drilling equipment. This gave the oceanographic group a needed head start on drilling the holes, which had to be finished by the time the helicopter arrived with the huts that straddled the hydroholes.

The Cessnas searched an area between 10 and 25 miles north and east of Barrow, the distance dictated by ice conditions, water depth, and helicopter range limitations. Shortly after the Cessnas took off, the helicopter moved up to the launch area and stood by. A VHF station maintained contact between the beach and the search crew. When a good lead was spotted, the Cessnas, before landing, relayed to the beach their position fix which they had obtained from a local DEWline radar station. The helicopter immediately began to deploy the huts, the two atmospheric research units leaving first to give the oceanographers enough time to drill the hydroholes. The Cessnas returned to the beach to transport personnel to the lead.

Seven helicopter flights and six Cessna flights were usually required to move everything to the ice. On the average, the helicopter slingloads went out at a rate of one flight per 75 minutes. The atmospheric group needed two helicopter flights before it became operational, the oceanographic group three, and the University of Alaska group two.

The experiment at each camp continued until the ice crust over the lead became too thick to obtain meaningful data, usually within three days after deployment. The helicopter and Cessnas then cleared the campsite, moving personnel, gear, and huts back to NARL. After one or two days of rest, equipment overhaul, and replenishment of supplies, a new search began for an open lead.

It was necessary more than once to deviate from the deployment scheme just described. At times only one Cessna was available for the search
mission, and the helicopter had to be used to fly cover and carry additional equipment. At other times, only part of the science program was deployed. The following abbreviated record of activities shows how the field operation was actually performed.

OPERATIONS JOURNAL

23 February - 3 March

Preparation for artificial lead experiment on Elson Lagoon proceeding without difficulties. Program three days behind schedule due to late arrival of instrumentation. Deployment to lagoon scheduled for 4 March. Aerial reconnaissance 1 March over ice north of Barrow encouraging. Several good leads observed 10 miles out. Deployment to ice anticipated around 11 March.

4 March

Experimental pond for artificial lead laid out on Elson Lagoon 4 miles northeast of camp. OSU and UW huts, 4000 lb. each, lifted to site by helicopter. Helicopter to be stripped down for increased lift capability.

5 March

Instruments set up at pond by OSU and UW. Smith's group assembling instrumentation in preparation for deployment to ice next week. OSU and UW trouble-shooting sensor installation at lagoon.

6 March

Aerial reconnaissance to ice. Conditions continue favorable. UA ready for deployment to lagoon. OSU and UW still trouble-shooting.

7 March

UA set up at lagoon. Artificial lead still dry, awaiting solution to OSU and UW instrumentation problem. Modification to electronics circuitry in progress. Aerial reconnaissance continues, open water freezes over in less than 24 hours at present ambient conditions.

8 March

OSU and UW equipment operational, pond to be flooded and artificial lead experiment to commence tomorrow. Deployment to ice presently scheduled for middle of next week.

9 March

Small pond established on lagoon for UA research. Ten m (radius) pond for OSU and UW filled with water at noon and steady flow established. Pump froze up shortly afterward and was pulled back to camp for thawing out.
Wind direction shifted and made existing pond unsuitable. New 10 m enclosure laid out. Helicopter on ice reconnaissance. Ice thickness tested adjacent to leads.

10 March

Smith's group assembling and installing equipment in 12x12 huts.

11 March

New 10 m pond for OSU and UW filled with water and good data are collected through the night. UA still having pumping problems in small pond. Holmgren collecting data over large pond.

12 March

OSU, UW, and Holmgren finished work at second pond in early morning. Wind direction changed again and equipment was moved back to first pond, where the experiment got going by early afternoon.

13 March

Reconnaissance to ice in R4D. UA experiment running at lagoon. OSU and UW finished at lagoon in late afternoon. Briefing by helicopter pilot and Coast Guard radio preparation for Friday deployment.

14 March

Helo huts moved back from lagoon and all buildings test-lifted by helicopter. Two huts too heavy; extra helicopter flight planned. Two Cessnas loaded and readied for next day's deployment. Late evening, the oceanographic team develops computer problems and deployment is postponed till Saturday, 16 March.

15 March

Computer problems solved, everybody ready, manifests made up for 7 helicopter flights and 6 Cessna flights.

16 March

Final briefing at 7:30. Cessnas with Paulson, Smith, Toovak, and Lee off at 9:20. No suitable leads spotted by 11:00 within 25 miles of Barrow. New effort scheduled for tomorrow.

17 March

Cessnas off at 8:45 to search for lead. Same manifests as yesterday. Buildings and helicopters on standby at the launching place on the beach. By noon the search area was covered and no lead spotted. This mission was called off. Weather patterns appear stable and no significant ice action is anticipated for the next 48 hours. If no lead is found tomorrow, oceanographic group will be deployed to a floe to check out equipment.
18 March

No lead by 12:30. Smith's group deployed to floe 17 miles north of Barrow. Problems with powerheads and auger for drilling holes caused delay in deployment. By 7:00 p.m., Camp 1 is in operation, occupied by five oceanographers and one native support assistant.

19 March

Cessnas off at 9:30 to search for lead. At 11:00 they set down 20 miles north of Barrow. The lead is about 50 m wide and one mile long. UW, OSU, and UA programs deployed and in operation at Camp 2 by 4:00 p.m. Smith is at Camp 1 checking out his instrumentation. Weather calm, -30°F.

20 March

Helicopter flight to both camps to fix radios and supply Smith with spares. At Camp 2, measurement continuing over thin ice crust. Late evening wind picked up and opened the lead again.

21 March

Experiment at camps 1 and 2 completed. At 3:00 p.m. everything was lifted back to NARL in seven helicopter and six Cessna flights.

22, 23 March

Investigators checking and overhauling instrumentation in preparation for next deployment, scheduled for 24 March.

24 March

Cessnas off at 9:15, one with Paulson, Smith, and Toovak; the other with camp gear and Smith's drilling equipment. Target search area is a 24-mile sector north of NARL. By 10:30 no suitable lead is found; one Cessna loses alternator and mission is called off. Air Force arrive with three HH-3 helicopters for simulated airplane crash maneuver.

25 March

Cessnas off at 8:45. Same configuration as yesterday. Cessnas report lead 18 miles north of Barrow at 10:00. By 4:00 p.m., six helicopter and seven Cessna flights have shuttled everything to the ice except the UA generator and backup supplies. OSU and UW set up and running by 1:00. Five buildings and 17 people on the ice, including a radio repair man and a native support person. At 4:00 the camp radios for helicopter assistance. The lead is closing, the ice is ridging and threatening the downwind buildings. The helicopter is on its way to relocate buildings and start evacuating. The Air Force is asked to assist in evacuating the camps if necessary. Things quiet down, however, and it is decided to evacuate the downwind side only. No Air Force support is required. By 7:00 p.m. the last flight to the ice carries UA generator in a sling. Components of generator tear off in flight and UA crew, being without power, return to NARL. Three huts, seven people left on the upwind side. Smith and Badgley collecting data through the night.
26 March
UA helo hut evacuated. Scientific programs terminated on the ice at noon. Everything is back on the beach by 2:00. Kelley's program on the ice terminated, Holmgren still involved.

27 March
Cessnas off at 9:00. No leads.

28 March
Only one Cessna available. Search with one Cessna and helicopter. No lead in the morning. Reconnaissance in the afternoon found no leads either.

29 March
Search in the morning and again in the afternoon. No lead.

30 March
Search in the morning and afternoon produces no lead. Still no change in weather. Winds out of northeast at 10 knots. Contingency plan worked out: If no lead by next week, OSU and UW will go back to the lagoon for a 20-m artificial lead experiment, and Smith will be deployed to a floe to take STD and current data. Ohtake from UA flies ice crystal sampler. A pond enclosure is constructed on the lagoon.

31 March
No change in weather, search produces no leads. Ohtake flies crystal sampler.

1 April
No weather change and no lead discovered in the morning. Ohtake flies crystal sampler. UW and OSU huts moved to lagoon.

2 April
No leads in the morning. Weather stable. Smith is deployed to a floe 24 miles from Barrow. UW and OSU in operation at the 20-m pond on the lagoon by early afternoon. Holmgren flies crystal sampler in the morning and terminates his program. OSU and UW collecting data through the night.

3 April
Twenty-m pond frozen over and the program on the lagoon terminated by noon. Smith collected STD and current data on the floe down to the bottom (100 m). UW and OSU calibrating in the afternoon.

4 April
No leads. The RF 2200 radio in Smith's camp operates satisfactorily with double antenna, frequency 4625.
5 April

No leads. Smith moved back to NARL. Paulson, Smith give seminar on lead experiment in the NARL lounge.

6 April

No weather change. Search in the morning produces no lead.

7 April

Wind picks up, 20 knots at 90 degrees. Search in the morning indicates that the ice is moving; no acceptable leads, however.

8 April

Big lead (300 m) discovered at 018° and 28 miles. Too wide for OSU and UW, who terminates their program and start packing. Smith's group deployed and in operation by 5:00.

10 April

Smith's group evacuated after a successful experiment at the lead. The Lead Experiment is terminated. Scheduled departure from Barrow 12 April.
MEASUREMENTS OF ARCTIC OCEAN ICE DEFORMATION AND FRACTURE PATTERNS FROM SATELLITE IMAGERY

by

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U.S. Army Cold Regions Research and Engineering Laboratory
Hanover, New Hampshire 03755

ABSTRACT

Imagery of sea ice was analyzed to (1) measure ice deformation by remote sensing techniques and (2) estimate correlation of the deformation with atmospheric driving forces so that the primary winter contribution to the atmosphere-ocean exchange can be calculated.

The distribution of ice openings from VHRR infrared images obtained in March 1973 by the NOAA-2 satellite over the Beaufort Sea was compared with the changes in the atmospheric pressure field during this period. Measured divergence rates were an order of magnitude higher (1% h\(^{-1}\)) than previously seen in this area. The general divergence of the pack correlated with the presence of a prolonged high pressure system in the region. It is concluded that the passage of several systems of this type could significantly change the ice mass balance of a large region for a given year by increasing the amount of thin ice available that is subsequently piled up in pressure ridges.

INTRODUCTION

Earth-orbiting satellites that provide high-resolution regional views of the earth have become an important source of data on sea ice. From the ESMR (Electronically Scanned Microwave Radiometer) on NIMBUS 5, for example, have come microwave maps of the polar ice [Gloersen et al., 1973]; and ERTS has made it possible to delineate ice features [Barnes and Bowley, 1973] and to study the movement of sea ice [Crowder et al., 1973; Hibler et al., 1974; Shapiro and Burns, 1973].

Two other satellites have recently provided data on sea ice that complement and, for some applications, improve on those from ESMR and ERTS. These are the visible and infrared images from NOAA-2 [DeRycke, 1973] and the infrared images from DAPP [Streten, 1974]. Their primary advantage is the increased frequency of coverage, generally every two days for NOAA compared with every 18 days for ERTS and once a month for ESMR. Although the resolution of NOAA-2 and DAPP (~0.5 to 1 km) is not as good as ERTS (0.1 km) the major leads are visible, and these are the features of greatest concern for the polar ocean-atmosphere coupling problem. The ESMR does not have sufficient resolution for leads unless they exceed 32 km in width. The IR images of NOAA-2 and DAPP are available throughout the year, while ERTS is available only when daylight is adequate and the images cover a relatively small region.

Satellite data studies of sea ice features alone provide some useful information for research and operations, but it is important not only to delineate the major motions and features, but also to correlate them with the known driving forces. In this manner, the satellite data as a time series can be optimally applied to ongoing modeling efforts of the drift and dynamics of sea ice and ocean-atmosphere coupling in the presence of an ice cover.

This paper examines 15 NOAA-2 images of sea ice obtained during March 1973 over the Beaufort Sea region and discusses the relationship between the observed deformation and the atmospheric pressure field present at the same time.

IMAGERY INTERPRETATION AND MEASUREMENTS

The major information we can derive from NOAA-2 satellite photography about deformation patterns in the interior areas of pack ice is the changing amount of open water with time. The hypothesis here is that the change in area of open water measures directly the net ice divergence in a given time interval. Present models rely on this information to develop constitutive relationships between ice deformation rates and the driving forces. In particular, the constants in the constitutive relationships, either bulk and shear viscosities in the viscous models or yield relationships in the plastic models, depend on the strain history, i.e., the presence or absence of thin
ice and open water prior to the deformational event. Unfortunately, in obtaining this information from satellite imagery, a number of interpretative problems arise. Chief among these is that the size of the features is often below that resolvable on the imagery. This problem exists for high-level aerial photography as well as for satellite imagery. For example, preliminary comparisons of strain results on the ground with aerial photography taken at 10 km (35,000 ft) altitude from the 1972 AIDJEX study sometimes indicated no visible leads when ground-based measurements showed ice divergence. Other problems include the presence of clouds and the difficulty of differentiating thin ice from open water, since they are only slightly different in the grey tones of IR images.

One semiquantitative measure of the amount of open water is the total length of fracture or lead per unit area, hereafter called the lead density. While not completely defining the amount of open water, the time variation of the lead density should correlate well with the ice divergence, since it seems reasonable that narrower fractures will widen and be seen on the imagery as time proceeds if the ice is diverging and that fractures will disappear from view as the ice is converging. Therefore, some relationship probably exists between the total length of leads observed on an image and the net divergence of the ice compared to some reference level.

To help understand the nature of the dynamical interaction of the sea ice with the atmosphere it is useful to correlate the ice divergence rate and the lead density with the Laplacian of the atmospheric pressure field. The Laplacian of the pressure field provides a measure of the divergence of the wind velocity field, with a negative Laplacian representing a diverging wind field. In particular, analytical calculations of expected ice deformation using a linear drift theory and a viscous ice model [Hibler, 1973] indicate that in the limit of high viscosity the ice divergence rate would be expected to follow the wind divergence, with the rates increasing inversely with viscosity magnitude. Also, as the viscosity becomes very small and the water and Coriolis forces become more noticeable, the divergence rate changes sign and proceeds in opposition to the wind divergence rate. Although a viscous model is a crude representation of the true ice rheology, describing the ice pack with a variable viscosity does give some quantitative feel for the ice dynamics and has been found to explain observed deformation reasonably well.
To quantify the deformation and compare it with atmospheric forces, we first overlaid a grid with ~350 km point spacing on successive satellite images. Measurements of the lead density for each 350×350 km location were then carried out. To calculate the Laplacian, corresponding measurements of the atmospheric pressure at each location were taken from Northern Hemisphere maps prepared by the National Weather Service. The total change in the area surrounding a large polynya was also measured directly to yield divergence and divergence rate for one region.

RESULTS

The qualitative relationships between the deformation patterns and the pressure system are shown in Figures 1 and 2. Figure 1 shows the changes in the fracture patterns for five days in March 1973 and Figure 2 the corresponding pressure field on the same days. From Figure 1 we see that several long N-S leads developed on or about 11 March and that large-scale movements were observed in the polynya just northwest of Alaska. This pattern remained relatively constant, with considerable movement in the polynya, until 19 March, when several more long N-S leads opened. A curved lead appeared at the base of these leads and joined the polynya north of the Barrow, Alaska, region. Deformation proceeded rapidly, apparently as a slippage along the curved lead at the base of the N-S leads.

The deformation proceeded to increase through 25 March, the lead orientations changing to NW-SE as the deformation became centered in the middle of the Beaufort Sea. The divergence generally decreased in the western and northern regions and increased in the southeast later in the month. A second large curved lead roughly parallel to the first one developed between 29 and 30 March in the southeast. Slippage appeared to take place there that was similar to the slippage observed farther west on 19 and 21 March.

The large triangular polynya that developed at the center of the Beaufort Sea between 21 and 30 March clearly shows the slippage. At its maximum extent, the polynya was approximately 200 km long and 75 km across at its widest point.

The increasing size of this polynya with time allows a measure of the deformation rates during its formation. First evidence of the polynya was seen
Fig. 1. Ice fracture patterns taken from NOAA-2 infrared imagery on the days indicated. The points 1, 2, 3, 4 on Figures 1c and 1e indicate the reference points for quantitative deformation measurements shown in Figure 3.
on 21 March, and so measurements were taken from this date for the first opening. To compare the rates with previous values obtained from deformation measurements in 1972 [Hibler et al., 1973] we initially "straddle" the polynya with four recognizable points. The position of these points is shown in Figure 1c and 1e and the initial and final lengths of the lines are given in Table 1.

<p>| TABLE 1 |
| INITIAL AND FINAL LENGTHS OF DEFORMATION LINES SHOWN IN FIGURE 1c AND 1e |</p>
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<th>Initial Length (21 March)</th>
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<td>69.3 ± 6.2 km</td>
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<tr>
<td>3-4</td>
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</table>

The net divergence is then the change in area ΔA of the quadrilateral divided by the initial area, and the average divergence rate for the period $t_i - t_{i-1}$ is given by

$$\nabla \cdot u = \frac{A_i - A_{i-1}}{A_{i-1}} \frac{1}{(t_i - t_{i-1})}$$

(1)

Figure 3 gives a plot of the divergence rate obtained in this manner, the negative Laplacian of the pressure field, and the net divergence. The Laplacian was calculated by taking the pressure given by the IRLS buoy (Fig. 4) and using four points equidistant from it to obtain the Laplacian. The Laplacian is then given by

$$\nabla^2 p = \frac{P_1 + P_2 + P_3 + P_4 - 4P_0}{a^2}$$
Fig. 2. Atmospheric pressure field for the same days shown in Figure 1.
Fig. 3. Comparisons between the negative Laplacian of the atmospheric pressure field (a) with the ice divergence rate (b) and the net ice divergences (c) for the latter part of March 1973.

where $P_i$ is the pressure at distance $a$ (~500 km) from the center point, $P_0$. Only the variations are needed, so $a = 1$ for the values in Figure 3. The pressures $P_i$ were obtained from the Northern Hemisphere surface charts for this period. There is reasonable correlation between the divergence rate and the negative Laplacian, in agreement with previous results reported by Hibler [1973].

The magnitudes of the divergence rate, however, represent a distinct departure from previous measurements, in that the maximum rate observed (0.218/day, or 0.009 hr$^{-1}$, 21 to 25 March) is nearly an order of magnitude higher than the maximum rate previously observed for this region in 1972 [Hibler et al., 1973]. Since this rate is "smoothed" over a four-day period, even larger rates could be reasonably expected to have occurred during the four-day period. The net divergence of 121% over ten days is also much larger than the 5% observed over a six-week period in March–April 1972.
In most cases, the observed deformations cannot be resolved as well as they can where, as in Figure 3, quantitative measurements of reasonable accuracy and usefulness can be obtained. However, we can make a semi-quantitative estimate by using the lead density previously described. Figure 4 gives the lead density measures for four squares of the grid used to quantify the area shown. The coordinates of the grid point centers are given in Table 2. Figure 4d corresponds to the same region measured quantitatively in Figure 3. As shown here, the lead density changes generally follow the divergence rate information given in Figure 3. The correspondence with the negative Laplacian is also similar; the large values of lead density correlate well with prolonged high values of the negative Laplacian in the later part of the month. The detailed correlation for the entire month is not good, however (correlation coefficient 0.2 to 0.3), because of two error sources.

Fig. 4. Comparisons between negative Laplacian of the atmospheric pressure field and the lead density (length of lead per unit area) for March 1973. Geographic coordinates of the grid locations are given in Table 2.
TABLE 2
COORDINATE CENTERS OF GRID SQUARES (Figure 4)

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</table>

The first source of error is, as discussed earlier, the resolution problem of the satellite imagery. For prolonged diverging events, the ability of the imagery to follow the event improves because the leads are large. However, for more rapidly fluctuating converging and diverging regions, the leads of interest may be on the border area of resolution and the changes in density will not correlate as well.

The second problem is in the calculation of the pressure field Laplacian. Generally the pressure field is not defined on a sufficiently fine grid to enable accurate calculation, since the only data input from the oceanic areas of interest are the few data buoys or manned drifting stations. A feeling for this problem can be gained by examination of the Laplacian records shown in Figure 4. The Laplacian of Figure 4d was calculated using the IRLS buoy data as the pressure at the center and interpolating the other pressure from surface charts, while Figure 4a, 4b, and 4c were completely interpolated from the charts and show somewhat different behavior. At any given time an error of ±2 mb can probably be assigned to the real pressure compared with that given on the charts. When the differences are added up in the calculation of the Laplacian, the error is at least as large as the expected value and makes correlation with another high error time series, the lead density, quite difficult.

DISCUSSION

A qualitative assessment of the driving mechanism for deformation may be made by comparing Figure 1, the deformation pattern from the imagery, with Figure 2, the atmospheric pressure field for the same days. The sequence
of events was the movement of a high pressure system into the Beaufort Sea during mid-March and its relatively stationary position during the period from 19 March to nearly the end of the month before it drifted slowly off to the southeast.

The system was characterized by pressure exceeding 1040 mb at the center for this period, which was longer (~ 10 days) than the usual two-to-three-day passage time for systems of this type. The anomalous behavior of the weather system for this region is shown by the pressure record of the IRLS buoy (Fig. 5), which was located in the region of interest (77°N, 152°W) during 1972-1973. From Figure 5 we see that the period of March 1973 was anomalous both for the intensity of the high (> 1040 mb) and for its prolonged duration. The entire record for 1973 is quite different from that for 1972; it shows several high pressure systems that were prolonged relative to similar periods in 1972. The detailed IRLS record is shown for March 1973 in Figure 6.

Such a system apparently causes the ice to open. Because the leads are much wider than usual due to the prolonged high pressure, the resistance between floes is generally lessened. An adequate portrayal may be given either by a time-dependent viscosity in the linear model used by Hibler [1973] or by plastic yielding in the anticipated AIDJEX model, which allows strain to proceed rapidly once a yield stress is reached. This results in a much higher strain rate for a given stress once this state is attained; and, as shown here, the strain rates can easily fluctuate up to 1% hr⁻¹ even during winter conditions of closely compacted ice. A limit in this situation is the expected reversal of this behavior when the viscosity drops very low (summer conditions) and water drag causes the ice to converge in a high rather than diverge, as observed under winter conditions.

CONCLUSIONS

Analysis of satellite imagery from the center of the Beaufort Sea in March has indicated that large deformational events can occur under the influence of prolonged atmospheric high pressure. These events led to ice divergence rates (0.009 hr⁻¹) an order of magnitude higher than those
Fig. 5. IRLS buoy pressure and temperature records 1972-73, approximate position, 77°N, 154°W (after Haugen and Kerut, 1973).
observed in AIDJEX 1972. In effect, the initial high pressure system appears to open the ice. Once opened, the "effective" viscosity is lowered and the opening is accelerated by the persistence of a system with a diverging wind field. The consequence of a number of these occurrences may be large fluctuations in ice production and heat exchange between the ocean and the atmosphere on a year-to-year basis. Increased divergence rates may also cause increased pressure ridging since new ice is more easily deformed. Consequently the results in this paper are commensurate with the large year-to-year variability in ridging intensity [Hibler, Mock, and Tucker, 1973] observed in the western Arctic Basin using airborne laser profilometry. In general, these estimates indicate the strong relationship between the atmospheric driving forces and the ice cover dynamics, in that persistently diverging wind fields yield abnormally large ice divergence rates.

This paper also illustrates the advantages offered by NOAA-2 imagery over that from other satellites: (1) enough detail to resolve some of the deformation parameters, (2) a large-scale view to gain regional perspective, and (3) year-around coverage to examine the seasonal dependence of deformation parameters.

Fig. 6. IRLS buoy pressure record for March, 1973.
ACKNOWLEDGMENT

Our thanks to Dr. E. Paul McClain of NOAA for providing the satellite images for analysis and Mr. Murray Stateman of the AIDJEX Office for the IRLS data buoy record. This work was supported by NSF Grant AG-492.

REFERENCES


AUTHORS' CAVEAT...

Since the following report is preprint of a paper appearing in the Proceedings of the First International Conference on Computational Methods in Nonlinear Mechanics, we have not changed it for presentation in the Bulletin. However, the reader should be forewarned that the elastic response is not consistent with that introduced by Coon et al. in AIDJEX Bulletin 24 [1974]: \( M_1 \) as described here is two times the bulk modulus presented in that work. The notation within our paper is consistent, and results are correct if we assume that Poisson's ratio is zero.

If the more normal choice of \( \nu = 1/3 \) were used (as in Coon et al.), Figure 2 would have to be changed to

![Diagram with stresses](image)

with the elastic response showing a contribution in the normal stress \( \sigma_{yy} \).

R. Pritchard and R. Colony
ONE-DIMENSIONAL DIFFERENCE SCHEME FOR AN ELASTIC-PLASTIC SEA ICE MODEL

by

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Abstract

A difference scheme presented in this paper describes how arctic pack ice responds to the driving forces found in its environment. The material response of this plate-like body is assumed to be elastic-plastic. Material strength depends on the amount and thickness of the thin ice present, measured by the ice thickness distribution. The numerical method is derived for one-dimensional motions, that is, motions which vary in only one coordinate direction. The spatial region is partitioned into cells and the leap-frog method is used to integrate the governing equations forward in time. This ice thickness distribution function at point \( x \) depends on the independent thickness variable \( h \). Within each cell the thickness distribution is integrated forward in time by following the characteristics defined in \((h,t)\) space. Stable calculations show the effect of an air stress across the top surface. Yield stress either increases or decreases; with softening, the material becomes unstable and violates Drucker's postulate. The continuum model may help in understanding the mechanical response of pack ice and also in predicting motion. The present assumption, which neglects elastic strain when plastic flow occurs, is not satisfactory and must be generalized for future calculations.

Introduction

The AIDJEX (Arctic Ice Dynamics Joint Experiment) modeling group has developed a mathematical model to aid in the understanding and prediction of the motion of Arctic pack ice in its environment [Coon et al., 1974]. In this paper we present a method for solving some of the initial-boundary value problems which occur and for which the model is expected to provide meaningful results.

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Pack Ice Model

The configurations that pack ice may occupy are limited to thicknesses of the order of three meters and horizontal extent of the order of 1000 kilometers. Vertical motions are constrained so that the material floats at the surface of the Arctic Ocean. Horizontal motions are much larger than vertical motions with typical velocities of 15 cm sec\(^{-1}\). These kinematic constraints dictate a two-dimensional mathematical model. Thus particle velocity \(\mathbf{v}(\mathbf{x}, t)\) is the time rate of change of position, \(\mathbf{x} = \mathbf{x}(\mathbf{X}, t)\), where \(\mathbf{x}\) and \(t\) indicate the particle \(\mathbf{X}\) at time \(t\) and \(\mathbf{v} = \dot{\mathbf{x}}\). It is further assumed that the curvature of the earth is small enough to allow an analysis in planar form. The velocity gradient \(\mathbf{L}\) is a second-order tensor which has four components in the two-dimensional problems. The stretching is obtained as \(\mathbf{D} = \frac{1}{2}(\mathbf{L} + \mathbf{L}^T)\).

The model is intended to resolve the mechanical response on length scales of 100 km and time scales of one day. Within typical 100 km elements the material is densely fractured. It is assumed that cracks are formed by processes independent of the motions described by the model. Deformation of the large-scale element occurs when leads open from these pre-existing cracks and when pressure and shear ridges form from the thin ice accumulated in the frozen leads. These features of the response and the mechanism of the ridge formation [Parmeuter and Coon, 1973] are satisfied by postulating the large-scale response to be that of an elastic-plastic material.

The elastic response is

\[
\varepsilon_e = \frac{1}{4M_1} \mathrm{tr} \sigma \mathbf{1} + \frac{1}{2M_2} \sigma' \]

where \(\varepsilon_e\) is the elastic strain tensor (two-dimensional), \(\sigma\) is the stress resultant tensor (in excess of isostatic balance), \(M_1\) and \(M_2\) are the elastic moduli, \(\mathbf{1}\) is the identity tensor, and \(\sigma'\) is equal to \((\sigma - \frac{1}{2} \mathrm{tr} \sigma \mathbf{1})\), the stress deviator.

The yield constraint used in this work is

\[
\psi(\sigma, p^*) = II' + I(I/2 + p^*) \leq 0
\]

where \(I = \mathrm{tr} \sigma\), and \(II' = \mathrm{tr} \sigma' \sigma'\). Here \(p^*\) is a hardening parameter which varies in time at each material particle. The variable \(p^*\) is the yield strength of the material.

Plastic flow is governed by the associated flow rule

\[
\dot{\mathbf{D}} = \lambda \frac{\partial \psi}{\partial \sigma}
\]

which assures uniqueness of the solutions when the material hardens in the course of deforming. We point out that the model presented here may either harden or soften, and there is no limitation on the plastic area change (usually \(\mathrm{tr} \dot{\mathbf{D}}\) is a negligible contribution in three-dimensional theories).

The elastic moduli are large enough that the response may be thought
of as nearly rigid plastic. Elastic strains can therefore be neglected when plastic flow occurs, resulting in the kinematic relation.

\[
\frac{D}{\rho} = \begin{cases} 
\frac{\epsilon}{\epsilon_e} & \text{if elastic, } \phi < 0 \text{ or neutrally plastic, } \phi = 0 \text{ and } \dot{\phi} \leq 0 \\
D_p & \text{if plastic, } \phi = 0 \text{ and } \dot{\phi} > 0
\end{cases}
\]

The yield strength and the elastic moduli of this material must depend on both the amount and the actual thickness of the thin ice present. We find it convenient to introduce a thickness distribution function [Thorndike and Maykut, 1973] as a measure of the thin ice. This function \( G(x, h, t) \) is the fractional area of ice thinner than \( h \) at each point \( x \). To use this definition, we envision pack ice as a mixture whose constituents are classified by their thicknesses. The thickness distribution function is a measure of the abundance of each constituent in the mixture.

Mass balance (including mass sources due to accretion and ablation of ice at the upper and lower surfaces) provides the appropriate differential equation for the thickness distribution

\[
\dot{G} + f \frac{\partial G}{\partial h} = \Psi - G \text{ tr } \frac{\partial h}{\partial h}
\]

where it is assumed that the mass density of all constituents is a constant. The scalar \( f = f(x, h, t) \) represents the growth rate of each category of ice. It is defined as the monthly climatological average at various locations in the Arctic Basin [Coon et al., 1974; Maykut and Untersteiner, 1971]. The redistribution function \( \Psi = \Psi(G, D_p) \) is a functional of the present ice thickness distribution and varies with the plastic stretching. The rate of production of ice by redistribution in all categories thinner than \( h \) is given by \( \Psi(h) \).

The yield constraint and the redistribution function are restricted by energy considerations [Rothrock, 1974b]. It is assumed that plastic work equals the time rate of change of gravitational potential energy of the sea ice floating on the Arctic Ocean [Rothrock, 1974a]. The functional relations which allow computation of \( \rho \) and \( \Psi \) in terms of \( G \) and \( D_p \) have been given by Coon et al. (1974).

The principle of momentum conservation is written as

\[
m \dot{v} = \mathbf{f_c} + \mathbf{f_w} + \text{div } \mathbf{\sigma} - m \mathbf{f_c} \cdot \mathbf{k} \times \mathbf{v} - mg \frac{\partial H}{\partial z}
\]

where \( m = \rho \) the mass per unit area of the material, \( \rho \) is mass density

\( f_c \) the Coriolis parameter

\( k \) unit base vector perpendicular to the \((x,y)\) plane

\( g \) acceleration of gravity

\( H \) height of sea surface; the gradient \( \partial H/\partial z \) gives the force
contributed by sea surface tilt

\( \tau_a \) traction on the upper surface induced by the atmospheric boundary layer

\( \tau_w = -A,; \tau_w \) is the traction induced on the lower surface by the oceanic boundary layer and \( A \) is a constant.

The characteristic time scales of interest are dictated by the atmospheric driving force. Although the mathematical model is capable of predicting rapid response (even shock waves), two features preclude such responses. First, the frequency content of the air stress is of the order of one cycle per day. Second, the ocean drag damps the response, so that all wave-like motions are reduced to insignificant amplitudes over distances of 100 km. Therefore, structural responses are desired rather than wave-like solutions. The present difference scheme does not take advantage of this feature, but time step limitations are not too severe.

We choose to specify the velocity at the boundary, since that is the only boundary condition presently of interest to AIDJEX. The initial data are given by specifying the velocity field, the stress field, the thickness distribution field, and the initial configuration.

The response of this elastic-or-plastic constitutive relation has been studied by Coon and Pritchard [1974] and by Coon et al. [1974]. In these works a strain rate history is assumed which allows the thickness distribution and the stress histories to be evaluated at one point without recourse to momentum balance. The stress behavior is seen to be discontinuous even when strain rates are continuous. However, such is to be expected from the chosen elastic-or-plastic model. The material either hardens or softens (violating Drucker's postulate [Drucker, 1950]), depending on the initial thickness distribution. We must expect unstable behavior because of this softening property.

**One-Dimensional Motions**

We restrict our attention to problems that can be solved independently of one of the two spatial coordinates. This independence simplifies the resulting difference scheme, yet contains essentially all the features of the general two-dimensional model.

Consideration of each of the governing equations shows that only the evaluation of the velocity gradient and the expression of the momentum balance change under the one-dimensional restriction. The elastic-plastic constitutive law and the thickness distribution equation (mass balance) must be considered in their general forms. Components of the velocity gradient are \( L_{xx} = \partial u / \partial x; L_{yx} = \partial v / \partial x; L_{xy} = L_{yy} = 0 \) where velocity has components \( u, v \) and spatial variations are in the \( x \) direction.

Momentum balance still requires that we satisfy two scalar equations, but these reduce to

\[
m \frac{du}{dt} = \tau_a x + \tau_w x + \frac{\partial \sigma_{xx}}{\partial x} + m_f c \frac{\partial H}{\partial x} - mg \frac{\partial H}{\partial x}
\]
\[ m \frac{d\nu}{dt} = \tau_w \nu + \tau_a \nu + \frac{\partial \sigma}{\partial x} - m f_c u \]

where \( \tau_w \nu = -Au \) and \( \tau_a \nu = -Av \). The atmospheric driving force is also limited to variations in \((x,t)\). In this work, properties of the model and its response to typical driving forces have been studied. We have not tried to predict pack ice motion in a real situation. For these cases the Coriolis term and the contribution to sea surface tilt have been neglected.

The Difference Scheme

All of the dependent variables are functions of position \( x \) and time \( t \). The thickness distribution function depends on the additional independent variable \( h \). It is the only variable to do so. We are able to describe the difference scheme by first ignoring the dependence on \( h \) and describing the method, and then at each location integrating the entire thickness distribution function forward in time.

The elastic-plastic constitutive laws and momentum balance equations are integrated by using the leap-frog scheme [Richtmyer and Morton, 1967]. To describe this method, we divide the body into cells by choosing the mesh \( \{x_j, x_j = \Delta X(j - 1), j = 1, 2, \ldots J\} \) which coincides with the position \( x_j \) in the initial configuration at \( t = t^0 \) and \( t^n = t^0 + n\Delta t \) for \( n = 1, 2, \ldots, N \). Unequal mesh sizes could have been chosen as easily. The spatial location of this material point at later times \( t^n \) is indicated by

\[ x^j_n = x(x_j, t^n), \quad y^j_n = y(x_j, t^n) \]

Since we allow no variations with respect to \( y \), the second equation is not evaluated during actual calculations. The sub- and superscript notation just presented is used throughout to indicate at which particle \((j)\) and at which time \((n)\) the function is evaluated. As the difference approximations are presented, it is suggested that the reader refer to Fig. 1, a schematic diagram showing where each of the variables is evaluated in the \((X,t)\) plane.

Position is obtained by the central time difference

\[ x^j_n - x^{j-1}_n = \Delta t \frac{u^{n-1/2}_j}{u^{n+1/2}_j} \]  

(1)

The velocity gradient is evaluated as a central difference in space relative to the changing configuration of the cell.

\[ (L_{xx})^{n-1/2}_j = \frac{u^{n-1/2}_j - u^{n+1/2}_j}{x^{n+1}_j - x^n_j} \]
\[ (L_{yy})^{n-1/2}_j = \frac{v^{n-1/2}_j - v^{n+1/2}_j}{y^{n+1}_j - y^n_j} \]

\[ (L_{xy})^{n-1/2}_j = \frac{v^{n-1/2}_j - v^{n+1/2}_j}{x^{n+1}_j - x^n_j} \]

where the cell size \( x^{n+1}_j - x^n_j \) is evaluated at \( t^n \) instead of centered time \( t^{n-1/2} \).
The stress state is computed by first making an elastic estimate. We rewrite the elastic response in incremental form and evaluate $\sigma_j^{n+1}$. Since all variables are evaluated at $x_j^{n+1}$, we suppress the subscript notation. Finally, we neglect changes in the elastic moduli, and solve:

$$f_n = + 2\Delta t M_1 \text{tr} \sigma_j^{n-1/2} \quad (\check{\sigma}^{n} = (\sigma')^{n-1} + 2\Delta t M_2 (\sigma')^{n-1/2})$$

The yield constraint is then checked to see if the elastic estimate is admissible:

$$\phi[\sigma_j^{n}, (\check{\sigma}^{n}), (p^{*})^{n-1}] \leq 0$$

If admissible, then $\sigma_j^{n} = \sigma_j^{n}$; if not, we branch to the plastic flow rule, which requires

$$\text{tr} \sigma_j^{n-1/2} = 2\lambda \left( \frac{\partial \phi}{\partial \check{\sigma}} \right)^n; \quad (\sigma')^{n-1/2} = 2\lambda \left( \frac{\partial \phi}{\partial \check{\sigma}} \right)^n (\sigma')^{n}; \quad \phi[I^n, (\sigma')^{n}, (p^{*})^{n-1}] = 0$$

This system of nonlinear algebraic equations is solved using Newton's method [Isaacson and Keller, 1966] with the added restriction that $\lambda > 0$.

At this point the plastic stretching is known so that the ice thickness distribution may be integrated forward in time. We again suppress the index $j^{n+1/2}$. To accurately treat discontinuities in the response, we use the method of characteristics to integrate the thickness distribution. Characteristic curves in $h,t$ space are defined by $t - \frac{h}{f} = \text{constant}$. We then write $\xi = t + \frac{h}{f}$, and the change of variable gives

$$\frac{DG}{D\xi} = \Psi - G \text{tr} D$$

where the derivative $DG/D\xi$ means rate of change of $G$ along the characteristic. The $h$-axis is divided into intervals by the mesh $h_k$, $k = 1, 2, \ldots, K$ where $h_1 = 0$. As a first step, the shapes of the characteristics are found as $h_k = h_{k}^{n-1} + f \Delta t$ where $f$ is evaluated according to position,
thickness, and time. Then the thickness distribution is advanced forward in time using the modified Euler method, a two-step scheme:

\[
\hat{G}_k = G_k^{n-1} + \frac{\Delta t}{2}(\psi_k^{n-1} - G_k^{n-1} \text{ tr } D);
\]

where \(\psi_k^{n-1}\) is evaluated using \(\hat{G}_k\) and \(\Delta \xi = \Delta t\) is used. As a final step, the characteristic mesh function is interpolated back to the original set of mesh points \(h_k\). We then call the interpolated solutions \(G_k^n\). Boundary conditions on the \(h\)-axis are specified several ways. The point \(h_k^{n+1}\) is chosen so that it is larger than any point for which \(G\) may change. Then \(G_k^{n+1} = 1\). At the left-hand boundary we require [Coon et al., 1974] that

\[
G_k^n = \begin{cases} 0 & \text{if } f(h=0) > 0 \\ \text{unspecified} & \text{if } f(h=0) \leq 0 \end{cases}
\]

Momentum balance is the final step in the cycle of computation. The velocity is evaluated at the forward half time step \(t^{n+1/2}\) as the solution of

\[
\frac{u_j^{n+1/2} - u_j^{n-1/2}}{\Delta t} = \frac{m}{a}\left[\frac{u_j^{n+1} + u_j^{n-1}}{2} + \frac{(\sigma_{xx})_{j+1/2}^n - (\sigma_{xx})_{j-1/2}^n}{\Delta x_j}\right]
\]

\[
\frac{v_j^{n+1/2} - v_j^{n-1/2}}{\Delta t} = \frac{m}{a}\left[\frac{v_j^{n+1} + v_j^{n-1}}{2} + \frac{(\sigma_{xy})_{j+1/2}^n - (\sigma_{xy})_{j-1/2}^n}{\Delta x_j}\right]
\]

where \(\Delta x_j = x_{j+1/2}^n - x_{j-1/2}^n\) for \(x_{j+1/2}^n = 1/2(x_{j+1}^n + x_j^n)\) and \(m\) is assumed constant rather than a variable according to definition.

The above sequence of calculations is made for each cell in the interior of the region. At boundaries the velocity history is given - i.e., \(u_1^{n+1/2}, v_1^{n+1/2}, u_1^{n+1}, v_1^{n+1}\) -- and the position may be calculated using eq.1.

We point out that the scheme is consistent to second order [Richtmyer and Morton, 1967] except when plastic flow occurs. Then the constitutive law has a truncation error of order \(\Delta t\). The particular way these quantities are evaluated is chosen to assure stability of the difference scheme for reasonable time steps. Although the analysis of stability is incomplete, it is necessary to satisfy the Courant-Friedrichs-Lewy criterion that \(c \Delta t/\Delta x \leq 1\) where \(c\) is a sound speed depending on elastic properties and the mass. It is chosen here as \(c = \sqrt{M_1/m}\).

We are at present not using a special starting technique to begin the calculations, and this again introduces errors of order \(\Delta t\).

**Results**

In the test problems, we have arbitrarily chosen a basin length of 1000 km and an initial cell size of \(\Delta X = 20\) km. The initial ice thickness distribution is given in Table 1. This function represents a typical distribution.
Table 1. Initial Ice Thickness Distribution

<table>
<thead>
<tr>
<th>$h$ (m)</th>
<th>0.0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.5</th>
<th>0.7</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>3.0</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G$ (%)</td>
<td>0.0</td>
<td>0.5</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>8</td>
<td>10</td>
<td>30</td>
<td>80</td>
<td>100</td>
</tr>
</tbody>
</table>

The body is initially at rest and stress free. When a uniform wind stress is applied and held constant in time ($\tau_{0x}^y(x, t) = 1$ dyn/cm$^{-2}$) the response has three distinct regions. At the left-hand boundary, the rightward motion induces opening. The stress state must be plastic and in tension (Fig. 2). Any tendency to open from stress state 1 causes plastic flow, so that the stress must lie at state 2. This observed discontinuity has caused many difficulties.

![Fig. 2. Normalized principal stress space.](image)

In the central region of the body the stress remains elastic (Fig. 3). The stress rises linearly due to the increasing fetch of the air stress. The elastic stress states lie along line 1-3 in this region. That stress $\sigma_{yy} = 0$ is a consequence of the choice of elastic moduli $M_1/M_2 = 1/2$. The stiff elastic moduli, $M_1 = 10^9$ dyn/cm, hold elastic strains to very low levels (<0.3%). This causes the central region to appear rigid.

At the right-hand boundary, plastic flow occurs in about one hour. The stress jumps from state 3 to state 4 at that time. The continuing compression hardens the ice as shown in Fig. 4 and slows the rigid body motion in the local region where flow is plastic. The elastic-plastic interface moves leftward with time as the plastic region grows. As this progresses, the motion of the material in the plastic region slows down (Fig. 5).

![Fig. 3. Normal stress $\sigma_{xx}$ (10$^6$ dyn/cm) vs. distance (1000 km).](image)
Conclusions

The AIDJEX ice model is a useful tool for understanding the response of arctic pack ice to its environment. The leap-frog difference scheme provides a viable method for computing accurate solutions to the model. The ice thickness distribution is accurately found by using the modified Euler integration formula along characteristic curves.

The assumption that material response is either elastic or plastic gives discontinuous stress histories even for continuous stretching histories. This causes numerical trouble and can mask the entire solution with ringing. The elastic strain when plastic flow occurs will be incorporated into the next model.

We have developed a difference scheme which considers one-dimensional motions. Many other schemes would do equally well; we have, however, chosen the leap-frog scheme because it generalizes readily to the two-dimensional case. This generalization will be necessary if we are to predict motion of arctic sea ice.

Acknowledgments

The authors thank the AIDJEX staff and particularly other members of the modeling group--M. D. Coon, B. Mukherji, D. A. Rothrock, and A. S. Thorndike—and point out that development of the AIDJEX ice model as a means to obtain solutions has been a group effort. Special thanks are also due to J. Fitzgerald, N. Fukuyama, A. Johnson and C. Plank for helping us.
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References


WHAT? STRAIN? WHAT STRAIN?

by

R. S. Pritchard

AIDJEX

ABSTRACT

To interpret sea ice motion and to evaluate theoretical models for such motions, we separate deformation into its basic parts of translation, rotation, and stretch. Sea ice (on length scales of 100 km) has a material response that is independent of strain relative to a fixed reference configuration. Although this independence limits the usefulness of strain interpretation, strain is one important property of the strain rate history, a quantity which helps to define the state of the material.

Using position data from the 1972 AIDJEX pilot study, we compute the right stretch tensor and the infinitesimal strain tensor and compare the two. We also compare the rotation with an infinitesimal approximation. For the 41 days of the pilot study, the data compare reasonably well, but the error increases with time. Since the state of the material is independent of strain and rotation, this error can increase to unacceptable levels. We therefore recommend a large-deformation theory rather than an infinitesimal theory be used to describe and evaluate the deformation of sea ice, since this approach eliminates one unnecessary source of error.

INTRODUCTION

In this paper we seek to determine how the position data from the 1975-76 AIDJEX Main Experiment might be used most effectively to check the AIDJEX ice model. More generally, we also wish to address the question of what strain measures are useful in interpreting the motion of arctic sea ice. Data from the 1972 AIDJEX pilot study are used in this work and are assumed to be typical of those expected from the Main Experiment.

We attempt to identify all kinematic quantities that make it possible to compare theory with experiment. This work is intended to answer two
questions pertinent to the interpretation of data:

1. What is the physical meaning of strain which is calculated from the history of observed positions of an array of points on arctic pack ice?

2. Of the many possible strain measures, which one is best to use?

The first question requires consideration of a constitutive assumption, and as such the answer becomes rather subjective. The problem is that strain is defined relative to a certain (though arbitrary) reference configuration and the resulting value of strain is just as sensitive to choice of reference configuration as it is to a motion (change of its final configuration).

The second question will be answered only in a limited sense. We will determine whether or not the classical infinitesimal strain tensors provide an accurate measure of strain in pack ice. Thus, the data will show either that (1) all strain measures (including infinitesimal approximations) provide essentially the same information and any one may be used, or that (2) significant differences arise depending on the definition of strain, so that care must be exercised in interpreting what is meant by the amount of strain.

In the course of answering this second question we require a convenient indicator of rotation (of a deforming body), one that is well defined and also reasonably simple to determine.

BASIC KINEMATIC RELATIONS

Position and particle velocity (time rate of change of position) are two important variables that may be compared directly with each other. To introduce other variables, we must define the kinematics more completely. We call the set of points in space that are occupied by the body at any time the configuration of the body [Truesdell and Toupin, 1960; Malvern, 1969]. The family of configurations occupied by the body at different times is the motion:

\[ \mathbf{x} = \mathbf{x}(x, t) \]  

(1)
where \( x \) is the position in space relative to the origin of the reference frame; \( X \) is the particle identifier, each point in the body having a unique identifying value; and \( t \) is time. A simple way to identify each particle \( X \) is to prescribe its location in a reference configuration \( \bar{x} \) such that

\[
\bar{x} = \bar{k}(x)
\]  

(2)

where \( \bar{x} \) is the position in space of \( x \) in the reference configuration. The deformation \( \Gamma \) is defined to provide a measure of the amount of change from one configuration to another configuration:

\[
\bar{x} = \chi[\bar{k}^{-1}(\bar{x}), t] = \Gamma(\bar{x}, t)
\]  

(3)

Equation 3 says that the deformation \( \Gamma \) gives the spatial location \( \bar{x} \) at time \( t \) of the particle which occupies position \( \bar{x} \) in the reference configuration. Velocity of a material point \( x \) is called particle velocity and is given by

\[
y = \frac{\partial}{\partial t} \chi(x, t)
\]  

(4a)

or by

\[
y = \frac{\partial}{\partial t} \Gamma(x, t)
\]  

(4b)

and is the rate of change of position of a material point.

We may apply these definitions to the 1972 AIDJEX pilot study data by identifying the previously defined concepts as they relate to real data from the 1972 AIDJEX pilot study, shown in Figure 1. We first identify three particles \( X \): Jumpsuit, Blue Dog, and Brass Monkey. Then we consider the motion of these three points (the three bold lines). Finally we indicate the configuration at three different times by the three triangles; three camps provide a unique configuration, assuming the deformation varies linearly within the triangle.
Fig. 1. Motion of AIDJEX camps during 1972 pilot study relative to 75.5°N latitude, 150°W longitude.

Although positions of the particles are valuable for comparison, it is possible to decompose the motion into three parts, each of which has an important interpretation. These parts of the deformation are rigid-body translation, rotation, and strain. Rotation and strain are both required to describe the material response, and both may be determined from the deformation gradient $\mathbf{F}$:

$$\mathbf{F} = \frac{\partial \mathbf{r}}{\partial \mathbf{x}}$$

In equation 5 a symbolic notation is used for the gradient operator [Truesdell and Noll, 1965]. A vector in the reference configuration $\mathbf{N}$ "deforms into" a vector $\mathbf{n}$ under the operator $\mathbf{F}$:

$$\mathbf{n} = \mathbf{F} \mathbf{N}$$
By "deforms into" we mean that the length and orientation of $N$ at a material point $X$ in the reference configuration change to the length and orientation of $n$ at the same material point in the final configuration. We see that translation is not accounted for by the deformation gradient. It is readily measured by the displacement

$$u = x - X$$

By the polar decomposition theorem [cf. Truesdell and Toupin, 1960; Appendix by Erickson] the nonsingular tensor $F$ may be uniquely rewritten as

$$F = R U$$

where $R$ is the rotation operator (orthogonal, $R^{-1} = R^T$) and $U$ is the right stretch tensor. Eigenvalues of $U$ (a symmetric positive-definite operator) are the ratio of final to initial length. Eigenvectors specify the orientation of the material lines in the reference configuration. Finally, the rotation operator $R$ has elements which are the direction cosines of the angles between the eigenvectors in the reference and final configurations. These concepts may be demonstrated by writing

$$U N_\alpha = \lambda_\alpha N_\alpha \quad \text{(no sum on } \alpha) \quad \alpha = 1,2$$

where $\lambda_\alpha$ is an eigenvalue of $U$ and $N_\alpha$ is the associated eigenvector. For this choice of $\overline{N}$, equation 5 becomes

$$\frac{N_\alpha}{\lambda_\alpha} = R N_\alpha \quad \text{(no sum on } \alpha) \quad \alpha = 1,2$$

Thus, by equation 9 we see that $U$ stretches the material lines $N_\alpha$ by the amount $\lambda_\alpha$. Similarly, by equation 10 we see that $R$ rotates those lines into $N_\alpha/\lambda_\alpha$, a unit length vector. These two quantities provide a means to measure both rotation and strain. Other rotation and strain measures will be compared with these.

The classical strain tensor is usually introduced as a measure of change in length squared. In the present notation this is written

$$\overline{E} = \frac{1}{2}(\overline{F}^T \overline{F} - I)$$

(11)
This quantity is a strain measure equivalent to $\mathcal{U}$, since it is easily related to $\mathcal{U}$ by

$$
\mathcal{E} = \frac{1}{2}(\mathcal{U}^2 - 1)
$$

It is seen that $\mathcal{E}$ and $\mathcal{U}$ have the same eigenvectors and the eigenvalues are related as shown:

$$
\lambda_{\mathcal{E}} = \frac{1}{2}(\lambda_{\mathcal{U}}^2 - 1).
$$

However, it is not $\mathcal{E}$ which is used in classical infinitesimal elasticity; it is $\mathcal{E}$, a linear approximation. It is instructive to express $\mathcal{E}$ in terms of $\mathcal{E}$ so that $\mathcal{E}$ and $\mathcal{E}$ may be compared:

$$
\mathcal{E} = \frac{1}{2}(\mathcal{E} + \mathcal{E}^T) - 1
$$

Equation 12 approximates equation 11 when $\mathcal{E} - 1$ is "small." (The meaning of this statement is left mathematically vague because there is little to be gained by being more explicit and because the meaning of "small" is reasonably understood from physical grounds.)

One purpose of this work is to evaluate $\mathcal{E}$ during the 1972 AIDJEX pilot study to see if it is an accurate approximation to $\mathcal{E}$, or to $\mathcal{U} - 1$. Thorndike [1974] has used $\mathcal{E}$ to present the 1972 AIDJEX pilot study data. It should be pointed out that it is often not $\mathcal{E}$, but $\mathcal{E}$,

$$
\varepsilon = \frac{1}{2} \left( \frac{\partial \mathcal{U}}{\partial \mathcal{X}} + \left( \frac{\partial \mathcal{U}}{\partial \mathcal{X}} \right)^T \right)
$$

which has been used in classical elasticity and in reducing 1972 AIDJEX pilot study data [Hibler et al., 1973]. This strain is related to the deformation gradient by

$$
\varepsilon = 1 - \frac{1}{2} \left[ \mathcal{E}^{-1} + (\mathcal{E}^{-1})^T \right]
$$

The gradients $\partial \mathcal{U}/\partial \mathcal{X}$ and $\partial \mathcal{U}/\partial \mathcal{X}$ are related by

$$
\frac{\partial \mathcal{U}}{\partial \mathcal{X}} = \frac{\partial \mathcal{U}}{\partial \mathcal{X}} \mathcal{E}
$$
We relax rigor for compactness of notation and use $y$ to indicate displacement when either $x$ or $z$ is the independent variable. The difference between $e$ and $\hat{y}$ is of the same order as differences between $\hat{e}$ and $y - \frac{1}{2}$. Any conclusions reached when comparing $\hat{y}$ to $y - \frac{1}{2}$ will be pertinent to the comparison of $e$ with $y - \frac{1}{2}$.

The classical approximation to rotation is the antisymmetric part of $\hat{R}$, namely,

$$\hat{R} = \frac{1}{4}(\hat{R} - \hat{R}^t)$$

(16)

Consideration of equation 8 shows that $\hat{R}$ is an approximation to $R$ with errors of the same order of magnitude as the infinitesimal strain tensor.

MEANING OF STRAIN AND STRAIN RATE

In the previous paragraphs we have discussed several strain measures. All variables were defined with respect to an arbitrary but fixed reference configuration. Components of each strain measure may be interpreted in terms of physically meaningful and measurable quantities. The only questions that remain, therefore, are (1) which reference configuration provides a measure of strain that is useful in interpreting observed motion? and (2) which is an important quantity to compare in a theoretical calculation? The choice of reference configuration is a constitutive assumption.

If we turn our attention specifically to arctic pack ice, the question of choice of reference configuration is overshadowed by the question of whether any reference configuration is meaningful. Theoretical models to describe pack ice have characterized the material response as fluid [Campbell and Rasmussen, 1972; Hibler, 1973; Rothrock, 1973]. According to Truesdell and Noll [1965], a simple fluid may be defined as a material that has "no permanent memory for any particular state." This means that there are no preferred configurations and thus no reference configuration that arises naturally. In recent efforts, the AIDJEX modeling group [Coon et al., 1974] has assumed that a model for arctic pack ice motions should have elastic-plastic response. In the AIDJEX model it is assumed that the entire
history of the motion is described by the present ice thickness distribution. In the mechanical constitutive law, the material properties and the yield curve are assumed to depend on the history only through the ice thickness distribution at the present time. The constitutive law is formulated by considering such microscale mechanistic processes as formation of leads and ridges. Even at the microscale level any preferred configuration is "forgotten by the material" whenever a lead or ridge is formed. In the mathematical description of the constitutive law, the absence of a preferred configuration is evidenced by the fact that strain rate, not strain, influences the stress state. It should be pointed out that the elastic response does depend on a strain measure. This strain, however, is measured relative to a reference configuration that changes in time and depends on the plastic flow.

All the examples of constitutive laws that have been used to represent pack ice indicate that no fixed reference configuration is preferred by the material. Therefore, any strain measure is chosen arbitrarily and does not represent a measure of the state of the material. In contrast, however, it is valid to select a fixed reference configuration arbitrarily and then to compare the experimentally observed and the theoretically computed values of the rotation and strain relative to that reference configuration.

The theoretical models discussed here do have the state of the material dependent on the strain rate. The actual measure of strain rate which may appear in a viscous constitutive law is the stretching \( \dot{D} \) defined as the symmetric part of the velocity gradient \( \dot{L} \):

\[
\dot{L} = \frac{\partial D}{\partial x} \\
\dot{D} = \frac{1}{2}(\dot{L} + \dot{L}^T)
\]

The antisymmetric part of \( \dot{L} \) is the spin \( \dot{W} \). Therefore, we find that the velocity gradient is an important quantity to compare. This is true because (1) the symmetric part \( \dot{D} \) helps define the state of the material in most models used to date, (2) the antisymmetric part \( \dot{W} \) helps define the rotation, and (3) knowledge of the history of \( \dot{L} \) is adequate to allow computation of the deformation gradient, given proper initial conditions. The governing ordinary differential equations are

\[
\dot{\dot{Z}} = \dot{L} \dot{F} 
\]
where

\[ \dot{\mathbf{F}} = \frac{\partial}{\partial t} \mathbf{F}(\mathbf{X}, t) \bigg|_{\mathbf{X}=\text{constant}} \]

is the material rate of change of \( \mathbf{F} \).

We note that the important decomposition of the velocity gradient is linear, i.e., separated into symmetric and antisymmetric parts. This decomposition has been used by Thorndike [1974] on the 1972 AIDJEX pilot study data. Thus, no questions arise in the velocity gradient decomposition as they did when a decomposition of \( \mathbf{F} \) was considered, and no need is found in this work to present such data again.

RESULTS

In this section we use the observed motion of pack ice to determine how much error is introduced by using the infinitesimal approximation to calculate strain and rotation.

Motion of the three manned stations for two months during the 1972 AIDJEX pilot study was obtained from Thorndike [1974]. These histories represent filtered responses determined from the raw data. Test data were smoothed, and the processed data points used in this work were taken at uniform intervals of time from the smoothed motion histories. However, during the time interval from approximately 2375 hours to 2500 hours, one of the AIDJEX camps (Blue Dog) lacks position data. The processed data provide numbers throughout this period, but we will be extremely careful about depending on the corresponding results. Since this study compares two different strain measures using the same position data, the position data need not be known to a particularly high degree of accuracy. It is important, however, that the motions be typical of those that actually occur in the pack ice. The data processing that obtained the motion histories used in this work satisfies this criterion.

The deformation gradient \( \mathbf{F} \) (a two-dimensional operator) is uniquely determined at each time from the position of three points (AIDJEX camps...
Jumpsuit, Brass Monkey, and Blue Dog) by assuming that the deformation is a linear function of position within the triangle.

The reference configuration was arbitrarily chosen to be the configuration occupied by the body at time $t = 1776$ hours. The time scales are consecutive in hours, with 1776 occurring at midnight GMT on 14 March 1972. Positions were given at six-hour intervals, with all three positions known for 41 days to a maximum time of $t = 2760$ hours (midnight GMT on 25 April 1972). Displacement is measured in kilometers relative to the point at 150°W longitude and 75.5°N latitude. Geographic positions have been spherically projected outward onto the osculating plane at 75.5°N, 150°W. The deformation gradient is calculated as a two-dimensional operator assuming planar motion.

The motion of each of the three AIDJEX camps is presented in Figure 1. The particle path of each camp is shown by a solid bold line. The position of each camp at ten-day intervals (14 March, 24 March, 3 April, 13 April, and 23 April) is indicated by the circles on each path. The triangles connecting the camps at a fixed time help to visualize the deformations that occur. It is apparent that rigid-body translation is the dominant mode of motion. Displacements after 40 days' elapsed time are on the order of 100 km, which is the same as the original gauge length over which strain is measured. Rotation is also evident from the change in orientation of the triangles. The amount of rigid-body translation or rotation depends on the frame of reference from which positions are observed. For this analysis the reference has its origin fixed on the sea surface at the original centroid of the three camps. Orientation of one direction is always vertical (parallel to gravitational attraction), and the remaining directions are always aligned with geographic coordinates at the origin. This reference frame is a consequence of projecting the position data onto the osculating plane rather than using raw geographic coordinates on the earth's spherical surface. A discussion of this is presented by Nye [1974].

For two-dimensional motions, the rotation operator is characterized by one angle. That angle is the amount a line rotates with respect to the Cartesian coordinates associated with the reference configuration. Components of $\hat{R}$ are
\[ R = \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \]  

(19)

and $\beta$ provides a measure of the rotation of the pack ice contained in the triangle defined by the AIDJEX camps. These data are presented in Figure 2. All rotations that occur are clockwise, as expected in the anticyclonic Beaufort Sea gyre. Also presented in Figure 2 is $\hat{R}$, the infinitesimal approximation to $R$. To compare the two tensors, one orthogonal and the other antisymmetric, we use the angle of rotation $\beta$. Expressed in radians, $\beta$ is the off-diagonal term in $\hat{R}$. Thus

\[ \hat{R} = \begin{pmatrix} 0 & -\beta \\ \beta & 0 \end{pmatrix} \]  

(20)

It is seen that $\hat{R}$ provides a very good measure of the rotation during the chosen time period. The maximum error is about three percent when the maximum rotation of 0.12 radian is achieved.

Fig. 2. Rotation during 1972 AIDJEX pilot study.
Strain history data are presented in Figure 3. Comparison of components of \( U - \frac{1}{2} \), \( \hat{E} \), and \( \hat{c} \) indicates that motions were small enough during the 1972 AIDJEX pilot study that the infinitesimal strain measures provide reasonable indications of the deformation of the body. The components have the same pattern of response. It is difficult to state a meaningful quantitative error, and so we will say only that components agree to within reasonable limits. The disturbing feature, however, is that the strain measures disagree more and more as time passes. This is true even though the amount of strain is not particularly large during the indicated period of time. The problem may be associated with the rotation that accompanies the deformation. Furthermore, one may expect the error to continue to increase because the state of the material does not depend on the arbitrarily chosen reference configuration, and so deformations and rotation will probably continue to increase with time.

To support the statement that rotation is the main cause of disagreement between various strain measures, we look more closely at the two strain measures, \( U - \frac{1}{2} \) and \( \hat{E} \). The first invariants are related by

\[
\text{tr} \hat{E} = \cos \beta \text{tr} U - 2
\]

and the second invariants of the deviators are equal

\[
\text{tr} \hat{E}' \hat{E}' = \text{tr} U'U'.
\]

The results in equation 22 show that the deviators \( \hat{E}' \) and \( U' \) may be related by an appropriate rotation. Thus,

\[
\hat{E}' = Q U' Q^T
\]

where the orthogonal operator \( Q \) is given by

\[
Q = \begin{pmatrix}
\cos \beta/2 & \sin \beta/2 \\
-\sin \beta/2 & \cos \beta/2
\end{pmatrix}
\]

and thus the infinitesimal strain \( \hat{E} \) may be expressed directly in terms of the rotation and the right stretch tensor. The only term that introduces a difference between \( \hat{E} \) and \( U - \frac{1}{2} \) is the rotation \( \beta \) (operator \( \underline{R} \)).
Fig. 3. Strain components during 1972 AIDJEX pilot study.
It should be pointed out that a more exhaustive analysis than ours would further increase our understanding of the motion; for example, it would be illustrative to present the first and second invariants and the principal values and associated principal directions. However, this task is not necessary to satisfy the aims of this paper. Here we seek only to compare different measures of strain and to point out the arbitrariness of the choice of reference configuration.

CONCLUSION

Position data from the 1975-76 AIDJEX Main Experiment will provide a useful check of the theoretical model. In addition to the positions, other derived variables provide useful information. One of these quantities is particle velocity. From the particle velocity the velocity gradient should be determined. This is the most important measure of deformation or of deformation rate of sea ice, because the velocity gradient helps to define the state of the material and measure the rotation. The assumption that strain rate, not strain, defines the state of the material is a constitutive assumption. However, consideration of previous models indicates that most modelers consider this to be a reasonable assumption.

It is doubtful that any significant physical meaning can be given to strain, because the measure of strain depends on choice of reference configuration and there appears to be no preferred reference configuration for pack ice.

Although strain does not define the state of the material, it does nevertheless provide a measure of the history of the motion. Since the strain measure is arbitrary and does not affect the state of the material, one may select the reference configuration arbitrarily. During the 1972 AIDJEX pilot study, deformations were computed relative to the initial configuration. These deformations were small enough that the classical infinitesimal strain measures provide an accurate approximation to the stretch and the classical infinitesimal rotation provides an accurate approximation to the rotation.
Differences between strain measures tend to increase with time. The primary cause of these differences is the rotation. Since the state of the material does not depend on the strain and the rotation, there is no tendency to reduce these quantities. Therefore, it may be expected that during the main experiment even larger differences will arise. For these reasons we believe that the infinitesimal strain assumption should not be used. Instead, the deformation gradient should be used directly and decomposed into its exact measures of rotation and stretch.

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REFERENCES


OCEAN CURRENT OBSERVATIONS AT THE AIDJEX 1972 MAIN CAMP

by

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ABSTRACT

During the 1972 AIDJEX pilot study, mast-mounted current meters were operated continuously for 30 days in the main camp to measure currents at ten depths between 2 m and 100 m below the ice base. The currents were plotted in three forms: relative speed and direction, absolute speed and direction, and progressive vector.

INTRODUCTION

The primary aim of the AIDJEX oceanographic program is the determination of water drag on the base of drifting pack ice. During the AIDJEX pilot program in 1972, ocean currents were monitored continuously at ten levels between the surface and 100 m. This report describes the experimental techniques and data reduction and summarizes the results. An accompanying article in this Bulletin applies these results to water drag.

A secondary objective of the oceanographic program is to explore basic processes in the upper layers of the Arctic Ocean. The current observations presented here and those from the 1971 pilot program have contributed to a better understanding of two phenomena in the upper layers: sub-ice boundary currents and subsurface baroclinic eddies. The sub-ice boundary currents were seen most clearly in the 1971 data and are apparently convection cells driven by brine production during freezing. Subsurface eddies were most clearly observed in the 1972 data presented here. The eddies are 10-20 km in diameter and are found between depths of 50 and 300 m. Speeds may reach 40 cm/sec at the core level, 150 m. These eddies may play an
important part in the horizontal exchange of heat, momentum, and salt between the Arctic Ocean and adjoining seas. Details of the eddies are given in a separate report [Hunkins, 1974].

DATA ACQUISITION

Current meters were suspended on inverted masts with their orientation rigidly fixed to that of the floe on which the station was situated. The current meters (Bendix/Marine Advisers Q-9) were of the Savonius rotor type. Although these meters have a magnetic compass for direction reference, the compasses were locked by a magnet attached to the exterior of the instrument. Current direction was thus referenced to the floe itself. The azimuth of the floe was monitored daily with celestial observations.

The threshold sensitivity of the current meters is conservatively estimated by the manufacturer as 2.6 cm/sec, although our experience indicates that the threshold may be considerably lower. The manufacturer's calibration curve was used for rotor turns versus water speed. It is a general conclusion of most tests on Savonius rotor meters that this is satisfactory for meters manufactured with adequate quality control so long as the bearings are free. The bearings were checked for clearance before launching the instruments by observing rotor spindown in air. Directional accuracy is specified as ±3° by the manufacturer. The current meters weigh 34 pounds in air and 22 pounds in water. Bolted fittings attached the masts to the flanges on the top and bottom of the instruments. The instruments were axially aligned with the mast, forming an extension of it, so that no supports interfered with flow.

Three masts were used to support the ten current meters. Meters were placed on the shallow mast at levels of 2, 4, 8, 12, and 20 m below the ice base. All current meter depths were referenced to the bottom of the ice, nominally taken as 2 m below sea level. (For depth below sea level, add 2 m to indicated depths.) Ice thickness was measured at four locations by the CRREL group around the current meter building. They found ice thicknesses of 2.26, 2.04, 2.11, and 2.08 m below sea level. Meters
were suspended at 30, 40, and 50 m on the intermediate mast and at 70 and 100 m on the deep mast.

The shallow mast, a prototype of the ones to be used in the 1975 experiment, was made of one-inch stainless steel tubing fitted with locking swivel attachments for alignment of individual meters. The intermediate and deep masts were of 1/4" aluminum pipe locked together with collars and pins. Individual instruments were not aligned on the aluminum masts, but their orientations relative to each other were measured. Proper alignment along all masts was assured by punch marks on the tubing or pipe applied in a lathe bed.

The shallow and intermediate masts were installed through separate ice wells 2 m apart in the current meter building; the deep mast was located 30 m away in the Lamont living quarters. The top of each mast emerged to eye level in the buildings and was fitted with an azimuth plate for alignment. Surveys run with a theodolite tied the mast alignment to the floe azimuth.

Signals, carried from the sensors to the surface by electrical cables, were converted to analog voltages by signal conditioners (Bendix/Marine Advisers S-11). Speed and direction at all ten levels were displayed continuously on panel meters in the current meter building during the experiment. The data were recorded on paper strip charts with multipoint servo recorders which printed once per minute on magnetic tape. The digital data acquisition system (Hewlett Packard Model 2012D) included a reed scanner, integrating digital voltmeter, coupler, and incremental magnetic tape recorder. The signal conditioners had a response time of about 30 seconds for the speed channel and 10 seconds for the direction, so that the problem of aliasing high-frequency fluctuations with a one-minute sampling interval should be negligible. The chart recorders operated from 15 March to 26 April, but a failure in the tape recorder delayed the start of digital recording to 28 March. The digital data, essentially complete through the inclusive dates of 29 March to 25 April, are the data described here.

The entire system, except for the current sensors themselves, was calibrated at the beginning, the middle, and the end of the experiment.
Calibration signals were introduced into the signal conditioners and monitored on the chart recorders and digital voltmeter. A precision decade resistance box was used to simulate direction changes of the vane and potentiometer in the actual direction sensor. A square-wave generator simulated output of the magnetic reed switch in the actual speed sensor. The data were corrected on the basis of the calibrations during processing after the field study.

Since the ocean currents are observed from a moving platform, a knowledge of ice drift is required for complete interpretation of the results. Ice position was monitored at irregular intervals of roughly one hour with a U.S. Navy satellite navigation system. The position data were smoothed with a Kalman filter with a response of 50% at a period of 14 hours. The slope of the Kalman filter is somewhat steeper than that of a cosine filter. Details of the treatment of the navigation data are given by Thorndike [1973].

Smoothed ice speed and direction are shown in Figure 1. Since these data have been filtered, it is of interest to know if higher-frequency

Fig. 1. Smoothed ice speed and direction, AIDJEX main camp, 1972.
motions of the ice have been suppressed. To check this point, positions were taken over a period of ten days with an acoustic bottom reference system for comparison with the satellite navigation. The acoustic system yields highly precise position relative to a bottom transponder at intervals of a few minutes. Combined data, incorporating both satellite and acoustic results, presumably represent ice motion of all periods in all essential detail. These combined data were compared with the satellite data alone. The variance of the difference between the two time series was 1000 m² hr⁻² for the east component and 650 for the north. This indicates that the ice velocities are reliable to better than 1 cm/sec.

Inspection of the data plot of Thorndike reveals that much of the variance is associated with oscillations with a period of about 12 hours. This is probably due to inertial oscillations of the ice, which are suppressed by the smoothing. For some purposes, 12-hour means of ice velocity have been used to eliminate these oscillations.

DATA REDUCTION

The data tapes written by the digital data system are formatted BCD tapes that can be read by many computers. Since all the current meter data were to be reduced on a single computer (IBM 1130), the tapes were rewritten in unformatted arrays. This reduced the amount of magnetic tape used from two full 2400-foot reels to about half a reel, with considerable savings in computer access time. These data were expressed in volts, the original field unit. The calibration results were then used to convert the data into speed in cm/sec and degrees from true north. The final tape record contains data in 21 channels: 10 speed channels, 10 direction channels, and a time channel.

Since the data were recorded at one-minute intervals for almost a month, there are more than 40,000 data points per channel. This number was reduced considerably by filtering the data with an 80-minute rectangular filter and sampling every hour. Each hourly value thus represents an average of the preceding and the succeeding 40 minutes. These filtered data
were recorded on disk files to provide rapid random access. The one-minute data have not been used since they were filtered, but the magnetic tapes are still available for studying high-frequency phenomena.

Ice velocity at one-hour intervals was also recorded on disk. These data were then added vectorially to the data for currents measured relative to the ice motion. The true currents thus obtained were also written on disk. The disk therefore contains three types of data, each sampled hourly: ice velocity, relative current velocity at ten depths, and true velocity at ten depths. These are the data sets that have been used to construct the figures and tables in this report.

DATA PRESENTATION

Ocean currents in general form a three-dimensional vector field that changes continuously with time. The current meters used in AIDJEX measured only horizontal currents. Vertical currents are usually several orders of magnitude less than horizontal currents in the ocean. The AIDJEX data are thus two-dimensional vectors in time at discrete levels in the ocean. The location of the measuring site varies with time as the ice station drifts. The station drifted generally westward at a rate of about 2 km/day. At the beginning of digital recording on 29 March the station was at 75°03'N 148°43'W; at the end, on 25 April, it was at 75°06'N 151°32'W. The observations are thus neither exactly in an Eulerian nor exactly in a Lagrangian frame of reference. The first would be from a point fixed relative to the ocean floor, and the second would follow the path of a water particle.

Since a time-varying vector field involves four dimensions, it cannot be simply depicted on a two-dimensional page. Presentations of current data are compromises, and some presentations are better than others for emphasizing particular aspects of current behavior. From the many possibilities, two types of presentation were selected for use here. The first, current speed and direction, either relative or absolute, plotted against time, emphasizes such time-varying current features as tides, inertial oscillations, and storm effects. The second presentation, progressive vector diagrams representing
the displacement of a water particle provided that the currents are unchanging horizontally, emphasizes the net displacement of a water particle or the mean current.

The quantities actually measured were current speed and direction relative to the ice at ten levels. The relative currents are shown in Figures 2-11. Note that the times in these diagrams and in all others in this paper are Alaskan Standard Time, where AST = GMT - 10 hours. Direction is referenced to true north.

The relative current plots provide some indication of the operating condition of the instruments. Any stickiness of the rotor or vane, for example, will be evident. One such fault was detected, not in the field, but in the plot (Fig. 3): a sluggish vane at 4 m depth, whose variation is much less than that of the instruments above and below it. A fault like this would be obscured by any subsequent manipulation of the data to combine speed and direction or to incorporate them into other data. We show the 4 m data here for completeness, but we do not use them in later interpretation. Another error, a misalignment of the current meters at 30 and 40 m, was detected later by comparisons with auxiliary data from other current meters. The directions were corrected during data reduction. All of the other current meters appear to have produced satisfactory results.

Relative currents are useful for drag calculations where the velocity difference between ice and water is important. For other studies of ocean currents it is generally useful to remove the ice drift. In the extreme case when the ice is motionless, the relative observations are equivalent to true ocean currents. In the other extreme case of moving ice but motionless water, the relative currents reflect ice drift and the current meter acts as a pit log. In general, however, both ice and water are in motion, and true ocean currents are obtained by adding the ice velocity to the relative current velocity with due regard for their vectorial nature. In practice, the velocities are separated into their north and east components and the addition is performed separately on each component. Plots of true current speed and direction are shown in Figures 12-21. Progressive vector diagrams for the true currents are shown in Figures 22-31.
The current measurements for this study were made within two ranges of depth, each range characterized by a particular current behavior and physical regime. The first of these ranges, the planetary boundary layer, extends from the surface to a depth of about 35 m. (Salinity and temperature observations showed that the mixed layer also extended down to about 35 m during the observational period, so that these two layers can be considered here as essentially the same.) In the planetary boundary layer, currents move primarily under the influence of friction, horizontal pressure gradients, and the earth's rotation. There is a large shear in the upper part (5-10 m) of this layer during storms, but little shear occurs at any time in the lower part, where the flow is apparently nearly geostrophic. Since the water is also nearly homogeneous, it is essentially a barotropic flow.

The deep current measurements (at 40, 50, 70, and 100 m) were taken in the steep density gradient that extends downward from the planetary boundary layer to a depth of 300 m. In this region the currents also appear to be in nearly geostrophic equilibrium, but the flow is highly baroclinic, with strong vertical shears at all times. The swiftest currents appear at the deepest levels and are associated with baroclinic subsurface eddies. These eddies appear intermittently on the records, attaining speeds as great as 35 cm/sec at 100 m.

Means over periods longer than one hour are useful if currents are reasonably steady over the averaging interval chosen. The longer intervals tend to smooth out effects of turbulence and oscillating currents. A 12-hour interval was chosen to eliminate the effects of inertial and semidiurnal tidal motions, which both have periods close to 12 hours and which are noticeable in the ice velocity records and in some of the water velocity records. Both relative and absolute current means over 12-hour intervals are given at all ten depths in the Appendix. Mean ice velocity is also given.
ACKNOWLEDGMENT

These current experiments were made possible by the logistical support of the AIDJEX Office and the Naval Arctic Research Laboratory. Barry Allen and Allan Gill assisted with the measurements. The cooperation of all of the other investigators on the 1972 pilot study is gratefully acknowledged. This research was supported under contract N00014-67-A-0108-0016 with the Office of Naval Research.

REFERENCES


Figs. 2-11. Relative current speed and direction for specified depths below the base of the ice (sea level = 2 m), plotted at hourly intervals.

Fig. 2. Depth = 2 meters.

Fig. 3. Depth = 4 meters.
Figs. 2-11. Relative current speed and direction for specified depths below the base of the ice (sea level = 2 m), plotted at hourly intervals.

Fig. 4. Depth = 8 meters.

Fig. 5. Depth = 12 meters.
Fig. 6. Depth = 20 meters.

Fig. 7. Depth = 30 meters.

Figs. 2-11. Relative current speed and direction for specified depths below the base of the ice (sea level = 2 m), plotted at hourly intervals.
Figs. 2-11. Relative current speed and direction for specified depths below the base of the ice (sea level = 2 m), plotted at hourly intervals.
Figs. 10-11. Depth = 70 meters. Fig. 11. Depth = 100 meters.

Figs. 2-11. Relative current speed and direction for specified depths below the base of the ice (sea level = 2 m), plotted at hourly intervals.
Fig. 12. Depth = 2 meters.

Fig. 13. Depth = 4 meters.

Figs. 12-21. True current speed and direction for specified depths below the base of the ice (sea level = 2 m), plotted at hourly intervals.
Figs. 12-21. True current speed and direction for specified depths below the base of the ice (sea level = 2 m), plotted at hourly intervals.
Fig. 16. Depth = 20 meters.

Fig. 17. Depth = 30 meters.

Figs. 12-21. True current speed and direction for specified depths below the base of the ice (sea level = 2 m), plotted at hourly intervals.
Figs. 12–21. True current speed and direction for specified depths below the base of the ice (sea level = 2 m), plotted at hourly intervals.
Fig. 20. Depth = 70 meters. Fig. 21. Depth = 100 meters.

Figs. 12-21. True current speed and direction for specified depths below the base of the ice (sea level = 2 m), plotted at hourly intervals.
2. METERS

Fig. 22. Depth = 2 meters.

4 METERS

Fig. 23. Depth = 4 meters.

Figs. 22–31. Progressive vector diagrams for currents at specified depths below the base of the ice (sea level = 2 m), plotted at hourly intervals.
Figs. 22-31. Progressive vector diagrams for currents at specified depths below the base of the ice (sea level = 2 m), plotted at hourly intervals.
Figs. 27-31. Progressive vector diagrams for currents at specified depths below the base of the ice (sea level = 2 m), plotted at hourly intervals.
Figs. 22-31. Progressive vector diagrams for currents at specified depths below the base of the ice (sea level = 2 m), plotted at hourly intervals.
APPENDIX

CURRENT OBSERVATIONS AT THE MAIN CAMP
1972 AIDJEX PILOT STUDY

12-hour mean values
Velocity components in cm/sec
Time is Alaskan Standard Time
(AST = GMT - 10 hr).
Direction is given in terms of true north.
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ABSTRACT

During the periods of storm and rapid ice drift, currents in the upper 15 to 25 m showed development of a modified Ekman spiral during the 1972 AIDJEX pilot study. A momentum integral method is used to evaluate ice-water stress. Maximum hourly mean stress is 1.66 dyn/cm². The balance of forces on the ice for cases when wind speed exceeds 5 m/sec shows a consistent pattern. Ice-water stress, air-water stress, and Coriolis force are all of the same magnitude. The pressure gradient force is, however, much smaller than these. Internal ice resistance is found as a residual from the force diagram and, during the rapid drift periods, is directed about 135° to the left of the ice drift.

The exchange of properties between the polar air and the polar water takes place across the pack ice barrier moving on the interface between atmosphere and ocean. In the case of momentum exchange, the flow is generally downward from air to water. Wind stress on the upper ice surface acts as a driving force and water stress on the lower ice surface acts as a retarding force. Momentum is transported downward from the atmosphere through the ice and into the oceanic planetary boundary layer.

One of the aims of the oceanographic program during the AIDJEX pilot experiment in 1972 was to assess this momentum exchange and measure ice-water stress in the central Arctic Ocean. For this purpose detailed current observations were made in the upper levels of the ocean from the lower ice surface through the planetary boundary layer and below it. The structure of the boundary layer and its changes with time over a period of one month were recorded. From this information the ice-water stress was calculated.
The techniques of current measurement and data reduction are described in an accompanying article in this issue of the AIDJEX Bulletin. Data averages over intervals of 12 hours are used here to minimize fluctuations due to turbulence and inertial oscillations. Over intervals of 12 hours ice drift and ocean currents are generally quite stable, so that we feel justified in treating these as equilibrium situations. Winds were light and ice drift small during much of the four weeks. Most of the drift was driven by the high winds of short-lived storms. Only selected intervals when ice and wind motion was sufficiently high were used for study here. During low winds the pattern is not so evident and is obscured by random experimental errors, but during high winds clear patterns of current structure emerge.

Current hodographs, lines connecting the tips of current vectors, are presented in Figures 1 and 2 for intervals when mean ice drift exceeded

Fig. 1. Hodographs of absolute currents for 12-hour intervals with ice drift greater than 9 cm/sec. Depths in meters below ice base. North at top of page. Local standard time.
9 cm/sec. A mean hodograph over a 60-hour period coinciding with the largest storm is shown in Figure 3. The zero depth level for these measurements is the base of the ice, 2 m below sea level. The hodograph is dashed in the upper 2 m; this is considered to be the surface boundary layer in which frictional forces, but not the earth’s rotation, are important. In the surface boundary layer there is a large vertical current shear in speed with very little change in current direction. Below the 2 m level there is a large current shear in both speed and direction. This is the Ekman layer, in which the effect of the earth’s rotation is manifested by a clockwise rotation of the current direction. A theoretical Ekman spiral decreases exponentially downward in speed, the angle increasing in direct proportion to depth. The spiral in some of the plots tends to continue downward to 50 m. At the center of the decreasing spiral is the geostrophic current vector at the base of the Ekman layer where frictional effects vanish.

Fig. 2. Hodographs of absolute currents for 12-hour intervals with ice drift greater than 9 cm/sec. Depths in meters below ice base.
Fig. 3. Mean current hodograph for 60-hour interval during storm.

The exact depth of an Ekman spiral is not well defined since it spirals infinitely downward. Ekman took the depth to be that at which the current flowed in the direction opposite to that at the surface. Others have chosen the level at which the current speed has decreased by a factor of \(1/e\) from its surface value. The exact choice is not critical for most purposes, since the speed decreases exponentially and changes are small near the base of the layer. The geostrophic current vector would lie in the center of the loop between 25 and 50 m.

The mixed layer revealed by temperature and salinity profiles during the experiment was about 35 m deep. Below this level a steep gradient in these properties and density extended to a depth of 300 m. The upper part of the mixed layer, 0-15 m, was often unstable, with brine-driven convection caused by freezing and brine production at the ice base [Smith, in press]. The lower part of the mixed layer, 15-35 m, was generally neutral or slightly stable. The Ekman layer and frictional exchange of momentum must be limited by the sharp density interface below 35 m and to some extent by the change at about 15 m.
MOMENTUM INTEGRAL METHOD

A technique involving direct evaluation of momentum in the upper water column from current profiles offers several advantages for determining ice-water stress on pack ice. Effects of both friction and form drag are included in the result. The assumptions are few, and none is made about the nature of eddy viscosity. The method can be demonstrated most simply for the case of smooth ice so that the nonlinear and pressure-gradient terms are neglected, i.e., only skin friction is acting. In the appendix it is shown that the method is more general and that with a proper set of current observations the form drag can also be included in the stress result. The horizontal equations of motion in their vertically integrated form are

\[
\frac{\partial M_x}{\partial t} - f M_y = \tau_{o,x}
\]

\[
\frac{\partial M_y}{\partial t} + f M_x = \tau_{o,y}
\]

where the mass transports are given by

\[
M_x = \int_{-H}^{0} \rho(u - u_g)dz
\]

\[
M_y = \int_{-H}^{0} \rho(v - v_g)dz
\]

with \(z\) positive upwards. It is assumed that there is a stress on the planetary boundary layer with components \(\tau_{o,x}\) and \(\tau_{o,y}\) at the ice-water interface but no stress at the base, \(-H\), of the layer. Observed velocity components are \(u, v\), while \(u_g, v_g\) are the geostrophic velocity components at depth \(-H\), where frictional effects become negligible.

The Cartesian system used here is a valid approximation for small areas at some distance from the pole. Very close to the pole a spherical coordinate system would be necessary. A constant Coriolis parameter, \(f\), is also a valid approximation for the relatively small areas considered.
The variation of this parameter with latitude would have to be included if large areas of the earth's surface were being considered.

The geostrophic current is assumed to be barotropic, since the planetary boundary layer coincided with the upper mixed layer in the 1972 experiments so that no baroclinic currents would have been present. A uniform geostrophic current prevails through the Ekman layer. Its value is the observed current at depth \(-H\). At deeper levels there is stratification and there may be vertical shear in the geostrophic current. At shallower depths the observed current is the sum of frictional and geostrophic currents. If density variations are present in the upper layer, hydrographic stations would be necessary to evaluate the baroclinic contribution to the geostrophic current, but for a well-mixed Ekman layer, current observations alone are sufficient to determine ice-water stress by the momentum integral technique.

The method depends, like any experimental method, on the quality of the observations. Sufficient measurements must be made of currents in a vertical profile and spaced closely enough in time that representative means can be formed. The available data described in the accompanying article appear to be adequate for the purpose. From the data a choice of depth, \(-H\), must be made. It is evident in Figures 1 and 2 that a mean Ekman spiral exists over time periods of 12 hours. The depth \(-H\) may be arbitrarily defined as the depth at which the current flows opposite to its surface direction, or it may be taken as the depth at which speed has decreased to \(1/e\) of its surface value. Another approach would be to choose the depth as the first minimum in the north and east velocity profiles. At that level, mean stress would vanish. Still another approach is to examine the hydrographic profiles for the sharp density change at the base of the mixed layer and assume that this coincides with the depth of the Ekman layer. Any one or some combination of these ways of choosing \(-H\) could be objectively stated so that the level could be selected individually for each profile. This may be done in the future, but for the 1972 data a constant level of 25 m was chosen for all profiles. This is approximately the level of current reversal and decrease of speed to \(1/e\) of the value at 2 m relative to the geostrophic current. Since speeds are small near this level, the exact choice of depth does not seem especially critical.
An added guide to the choice of Ekman layer depth is provided by hydrographic profiles which show instability due to brine convection in the 0-15 m layer, neutral or slightly positive stability in the 15-35 m level, and strong stability below 35 m. Convection in the uppermost 15 m will mix momentum thoroughly down to this depth and possibly somewhat below it. The choice of 25 m here is in agreement with these hydrographic measurements.

The equations imply horizontal uniformity, and only a single vertical current meter array was used. Keels of pressure ridges may be expected to extend to an average depth of 10 m at an average spacing of about 200 m. Averaging over these keels is accomplished by the horizontal transport of momentum in the Ekman layer. Effects of keels upstream are carried to the observation site by advection. The distance of influence may be loosely estimated as one to two orders of magnitude of the Ekman layer thickness, that is, 250 m to 2500 m. Another estimate of distance of upstream influence can be found from the response time of an Ekman layer, which is somewhat shorter than one inertial period (12.4 hr at latitude 75°N). At 10 cm/sec a water particle travels a distance of about 2 km in one inertial period.

Ice-water stress was evaluated for the period 29 March to 25 April using hourly current values at levels of 2, 8, 12, 20, and 30 m below the base of the ice. Differences of current velocity between the geostrophic level at 25 m and shallower levels were calculated and integrated vertically using the trapezoidal rule. These are the Ekman mass transports; they assume a water density of 1.0 g/cm³. The time-dependent term was found by taking hourly differences of these transports. The mean time-dependent term was about one order of magnitude less than the Coriolis force term. Note that only relative currents are required for finding Ekman mass transport.

Variations of stress magnitude and direction are shown in Figure 4. The maximum mean stress over 12 hours was 1.10 dyn/cm² for 0000-1200 LST on 12 April. The maximum mean hourly stress was 1.66 dyn/cm² at 0800-0900 LST of the same day. Stress results during the same period were reported as 1.0 dyn/cm² by the University of Washington using an eddy correlation method [Smith, in press]. The eddy correlation method gives only surface friction, and it may be that the difference between the two methods is attributable to form drag. However, the errors in the methods may account for the
Fig. 4. Ice-water (solid line) and air-ice (dashed line) stress at 1972 AIDJEX main camp based on 12-hour means. Direction of air-ice stress reversed for comparison.
difference. For comparison, wind stress on the upper ice surface was calculated from the synoptic observations. The drag law

\[ \tau_a = \rho_a C_D \frac{v^2}{2} \]

was used with \( \rho_a = 0.00125 \text{ g/cm}^3 \) and \( C_D = 1.5 \times 10^{-3} \). The drag coefficient is based on eddy correlation measurements made with a sonic anemometer by the Bedford Institute of Oceanography group. Because of the quadratic nature of the drag law it is not strictly appropriate to use the average wind speed. In this case, however, their use seems justified since the observed winds were fairly stationary over the 12-hour time intervals. The reverse of the wind stress direction is plotted in Figure 4 for easier comparison with water stress direction; the stresses are generally nearly opposed to each other.

Peak stresses on both top and bottom of the ice occurred during the storm of 12-13 April. It is interesting that there is a tendency for the water and wind stress to track each other fairly closely for the first day or two of a storm. After that, water stress decreases although wind stress continues to climb for another 12 hours. This pattern is also evident in the storm of 17-18 April. Maximum mean wind stress over 12 hours exceeded the water stress, reaching a maximum value of 1.875 dyn/cm², which coincided with a wind speed of 10.0 m/sec between 1200 and 2400 LST on 14 April. The water stress maximum of 1.101 dyn/cm² was attained in the preceding 12-hour interval.

**BALANCE OF FORCES**

During periods of strong winds, swift ice motion, and high stresses, conditions are frequently fairly stationary over 12-hour intervals and the forces acting may be considered to be in equilibrium. Information on the surface stresses of wind and water has already been presented. The body forces due to the pressure gradient associated with tilt of the ocean surface and to the Coriolis force associated with the earth's rotation must be determined. Since the current at 25 m has been assumed to be in geostrophic
balance and since the pressure gradient does not vary with depth over the top 25 m of the water column, the pressure gradient found at the base of the Ekman layer is the same as that acting on the ice. $F_{p,E} = -\rho_i h f v_g$ and $F_{p,N} = \rho_i h f u_g$ are the pressure-gradient forces in the east and north directions, respectively, where the following constants are used: ice density, $\rho_i = 0.9 \text{ g/cm}^3$; ice thickness, $h = 2.5 \text{ m}$; and Coriolis parameter, $f = 2\Omega \sin\phi$, where $\phi = 75^\circ$ and $\Omega$ is the earth's angular velocity.

Note that absolute current velocities are required to find the pressure-gradient force. The Coriolis force depends only on ice velocity and is given by

$$F_{c,E} = \rho_i h f V$$
$$F_{c,N} = -\rho_i h f U$$

where $U$ and $V$ are components of ice velocity in the east and north directions, respectively.

These four forces are plotted for selected 12-hour intervals in Figures 5, 6, and 7 and for 60-hour intervals in Figure 8. There is also an inertial force due to the acceleration of the ice, but this term is negligible in the present cases, being too small to plot. The eight cases in these figures are for intervals when mean wind speed exceeded 5 m/sec. With the constants used, this is equivalent to cases when wind stress exceeded 0.5 dyn/cm$^2$. The velocity of the ice is plotted also in these diagrams, although it does not enter into the balance of forces. If it is assumed that the ice is drifting under balanced forces, then an equilibrant is required for balance. This fifth force, not measured directly but determined as a residual, is the internal ice force.

The diagrams indicate that all forces except the pressure-gradient are generally significant. However, on longer time scales, pressure-gradient forces may assume more significance. Over a mean drift of one month for AIDJEX 1972 the Coriolis and pressure-gradient forces are nearly balanced [Newton and Coachman, 1973].

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Fig. 5. Balance of forces on ice for 12-hour intervals when winds exceeded 5 m/sec. Ice velocity, \( V_i \), in cm/sec; air-ice stress, \( \tau_a \); ice-water stress, \( \tau_w \); Coriolis force, \( F_c \); pressure-gradient force, \( F_p \); and internal ice force, \( F_{int} \).
Fig. 6. Balance of forces on ice for 12-hour intervals when winds exceeded 5 m/sec. Ice velocity, \( v_i \), in cm/sec; air-ice stress, \( \tau_a \); ice-water stress, \( \tau_w \); Coriolis force, \( F_C \); pressure-gradient force, \( F_p \); and internal ice force, \( F_{int} \).

Fig. 7. Balance of forces for two cases with westerly winds.
The pattern of stresses is consistent during the highest values of stress (Fig. 5). During some lower values of stress (Fig. 6), deviations from the earlier pattern are seen. The consistency of the pattern during the highest stresses suggests that random errors are not a problem and that averaging in time and space is adequate. The presence of systematic errors is not ruled out, however. All of these were cases of easterly winds. Two cases of westerly winds were examined to see if the pattern changes with azimuth. These cases (Fig. 7) were for winds less than 5 m/sec. The force diagram still shows a similar pattern for these two cases, with air stress about 30° to the left of ice motion, water stress about 135° to the right of ice motion, and internal ice stress about 135° to the left of ice motion.

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APPENDIX

THE INTEGRATED STRESS ON A ROUGH SURFACE IN A ROTATING SYSTEM

Consider a rough surface, such as the underside of pack ice, moving over an ocean of great depth. Frictional effects will vanish at \( H \), the depth of the planetary boundary layer, which in this case is small relative to the total depth of the ocean. The \( x \)-component of the Navier-Stokes equation in a local Cartesian coordinate system is

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} - f v = - \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \tau_{xx} + \frac{\partial}{\partial y} \tau_{yx} + \frac{\partial}{\partial z} \tau_{zx} \tag{1}
\]

where \( u, v, \) and \( w \) are the velocity components in the \( x, y, \) and \( z \) directions, respectively; \( \rho \) is density; \( p \) is pressure; and \( \tau_{xx}, \tau_{yx}, \) and \( \tau_{zx} \) are the viscous stress components. The equation of continuity is

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{2}
\]

for uniform density conditions. We are concerned with flows on scales small enough that we may neglect the variation of \( f \), the Coriolis parameter, with latitude. We are interested in average stress over a distance, \( X \), sufficiently long to include a representative sample of roughness conditions. A weighting factor is defined to average horizontally across the water between solid protuberances. Average values in the \( x \)-direction of a representative dependent variable, \( q \), are defined as

\[
q(y, x, t) = \frac{1}{X} \int_{-X/2}^{X/2} a q(x) dx \tag{3}
\]

The weighting factor \( a \) is unity when the position \( x \) lies within the fluid and is zero when it lies within a solid portion of the rough boundary. Perturbations from the horizontal average are defined by

\[
q' = q - \bar{q} \tag{4}
\]

Substituting equation 3 into equation 1 gives the perturbation equation.
\[
\frac{\partial}{\partial t} (\bar{u} + u') + (\bar{u} + u') \frac{\partial}{\partial x} (\bar{u} + u') + (\bar{v} + v') \frac{\partial}{\partial y} (\bar{u} + u') + (\bar{w} + w') \frac{\partial}{\partial z} (\bar{u} + u') \\
- f(\bar{v} + v') = - \frac{1}{\rho} \frac{\partial}{\partial x} (\bar{p} + p') + \frac{\partial}{\partial x} (\bar{\tau}_{xx} + \tau') + \frac{\partial}{\partial y} (\bar{\tau}_{yx} + \tau') \\
+ \frac{\partial}{\partial z} (\bar{\tau}_{zx} + \tau').
\]

This equation is now averaged over \( x \), noting that the \( x \)-average of an \( x \)-derivative gives values at the points \( x_{1n} \) and \( x_{2n} \) (Fig. 9). For example, the average of \( \partial \bar{p}' / \partial x \) is given by

\[
\frac{\partial \bar{p}'}{\partial x} = \sum_{n} \frac{p'(x_{2n}) - p'(x_{1n})}{\chi} \approx \bar{\Delta p}'.
\]

For \( z < h \), the greatest protuberance depth, the partial \( x \) distance,

\[
\chi_n(x,z) = \int_{-\chi/2}^{\chi/2} a \, dx = \sum_{n} (x_{2n} - x_{1n}),
\]

is the distance occupied by water.

Fig. 9. Scheme for horizontal integration at constant height, \( z \), passing through solid keels. The weighting factor \( a \) is zero within the solid and unity in the water.
The $x$-average of the nonlinear terms in eq. 4 can be rewritten as in the following example:

\[
(\bar{v} + v') \frac{\partial}{\partial y} (\bar{u} + u') = \frac{\partial}{\partial y} \bar{v} u + v' \frac{\partial}{\partial y} \bar{u} + \frac{\partial}{\partial y} \bar{u}' + \frac{\partial}{\partial y} v' + \frac{\partial}{\partial y} \bar{u}'
\]

\[
= \frac{\partial}{\partial y} \bar{v} u + v' \frac{\partial}{\partial y} \bar{u} + \frac{\partial}{\partial y} \bar{u}' + \frac{\partial}{\partial y} v' + \frac{\partial}{\partial y} \bar{u}'
\]

\[
= \frac{\partial}{\partial y} \bar{v} u + v' \frac{\partial}{\partial y} \bar{u}'
\]

since $\bar{u'} = \bar{v'} = 0$ by definition. Thus the left-hand side of equation 4, averaged over $x$, becomes

\[
\frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} + \bar{w} \frac{\partial \bar{u}}{\partial z} + \bar{u}' \frac{\partial \bar{u}'}{\partial x} + \frac{\partial \bar{u}'}{\partial y} + \frac{\partial \bar{u}'}{\partial z}
\]

but

\[
\bar{u}' \frac{\partial \bar{u}'}{\partial x} + v' \frac{\partial \bar{u}'}{\partial y} + \bar{u}' \frac{\partial \bar{u}'}{\partial z} = \frac{\partial}{\partial x} (u'u') + \frac{\partial}{\partial y} (u'v') + \frac{\partial}{\partial z} (u'w')
\]

Substitution of the perturbation velocity values into the equation of continuity (2) shows that the last term in parentheses in eq. 8 is zero.

Equation 5 in $x$-averaged form then becomes

\[
\frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} + \bar{w} \frac{\partial \bar{u}}{\partial z} - f(\bar{v} - \bar{v} g) = - \frac{1}{\rho} \frac{\partial \bar{p}'}{\partial y} + \bar{\tau}'_{xx} + \frac{\partial}{\partial y} (\bar{\tau}'_{yx} - \bar{v}' u') + \frac{\partial}{\partial z} (\bar{\tau}'_{zx} - \bar{w}' u')
\]

The large-scale pressure gradient has been separated out as the geostrophic velocity, $\bar{v} g$. The remaining pressure term is associated only with form drag.

Note that the no-slip boundary condition requires that the term

\[
\frac{\partial}{\partial x} (u'u') = 0.
\]

The $\frac{1}{\rho} \frac{\partial \bar{p}'}{\partial y}$ term represents the net pressure drag across the protuberances, and the $\bar{\tau}'_{xx}$ term represents the net effect of asymmetries in the fluctuating gradient $\frac{\partial}{\partial x} u'/\partial x$ evaluated at positions $x_{1n}$ and $x_{2n}$. The last two terms on the right-hand side of eq. 9 are the horizontally integrated effects of skin friction, including both viscous and turbulent contributions.
Assume now that the variation in the $y$-direction of the average parameters, $\bar{u}$, $\bar{\tau}_{yy}$, and $\bar{v}u'$, is negligible. This implies that conditions are laterally uniform. Also assume that $\bar{w} = 0$. Vertical motion is zero at the upper boundary surface which is rigid, and this condition implies that vertical motion vanishes also at the base of the planetary boundary layer. Ekman divergence effects which can induce vertical velocity at the base of the layer are thus neglected.

Equation 9 is integrated vertically from the base of the planetary boundary layer, $-H$, where geostrophic velocities, $u_g$ and $v_g$, exist upward to the rough surface:

$$\frac{\partial M_x}{\partial t} - f M_y = \int_{-H}^{0} X_n (\rho \frac{\partial \tau_{xx}'}{\partial x} - \Delta p') dz + \int_{-H}^{0} X_n \frac{\partial}{\partial z} [\bar{\tau}_{xz} - v'u'] dz \tag{10}$$

where $X_n$ is the horizontal weighting factor and the horizontal mass transports are given by

$$M_x = \rho \int_{-H}^{0} X_n (\bar{u} - u_g) dz$$

$$M_y = \rho \int_{-H}^{0} X_n (\bar{v} - v_g) dz \tag{11}$$

Only the velocity departure from the geostrophic is used in the case of $M_x$ as well as $M_y$. Since the time rate of change of $M_x$ is involved, adding a nearly constant geostrophic velocity does not affect the result. Since density is constant, the geostrophic velocities do not vary with depth.

The assumption of vanishing stress at the depth $-H$ allows the lower limit of integration in the second integral to be reduced from $-H$ to $-h$. The right-hand side of eq. 10 sums the contributions from turbulent and roughness asymmetries, pressure drag, and skin friction due to both molecular viscosity and turbulent contributions. These effects may be combined into a single integrated boundary stress term with components $\tau_{0,x}$ and $\tau_{0,y}$ so that eq. (11) becomes
where the $y$-equation has been obtained in an analogous manner to the $x$-equation. Thus the total stress on a rough surface in a rotating system may be determined from the mass transports in the planetary boundary layer which have been averaged over a representative horizontal distance. Equation 12 is a statement of the Ekman transport relation including time dependence and with the added generalization that the stress term includes both skin friction and form drag. Vertical integration has freed the result from dependence on the details of flow structure and friction within the Ekman layer.

This derivation is similar to one given by Faller and Mooney [1971] for Ekman boundary layer stress over a rough surface in a rotating tank.

Application of the method requires that the mass transports be averaged over a representative area. Provided that the averaging is adequate, the method will be rigorous even though the protuberances penetrate a large fraction of the planetary boundary layer. In practice, measurements at only a single location have been used for measuring stress below pack ice. This is justified provided that the site is located at an average ice thickness so that no weighting of the velocity values is required and provided that the protuberances are not so deep as to cause extreme variations from one location to another. Camp sites are normally chosen away from large ridges in areas of fairly level ice; this choice is suitable for the method. Details around large ridges are ignored, and for this reason the momentum integral method tends to underestimate drag.
The following two papers consider the problem of internal wave drag on sea ice from substantially different points of view. Rigby, with the aid of a theoretical model, examines the principal factors that affect wave drag on an individual keel, and attempts to define the conditions under which wave drag could be greater than form drag. Hunkins, on the other hand, uses an experimental approach to estimate wave drag on a particular keel, and then applies these results to large-scale estimates of the water stress. Differences between the local wave drag predictions of Hunkins and Rigby appear to arise primarily from different assumptions regarding the depth at which internal waves are generated and not from the different approaches.
THEORETICAL CALCULATIONS OF INTERNAL WAVE DRAG ON SEA ICE

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ABSTRACT

Because of the density stratification in the upper part of the Arctic Ocean, the movement of pressure ridge keels through the mixed layer can create internal waves in the vicinity of the pycnocline. These waves transport energy away from the keels and thereby generate drag on the ice. This study defines the conditions under which wave drag could contribute significantly to the total water stress. The development of internal waves downstream from a semielliptical keel is described by a simple two-dimensional, two-layer ocean model. Results calculated from the model show the dependence of wave drag on keel depth, current speed, depth of the upper layer, and density change across the interface between the layers. Comparison of the relative magnitudes of form drag and wave drag suggests that wave drag is not negligible near large keels when the current is flowing strongly.

INTRODUCTION

Accurate predictions of the dynamic behavior of sea ice require good estimates of the forces that act on the ice. Among the largest of these forces are the atmospheric drag on the upper surface of the ice and the oceanic drag on the lower surface. To determine the magnitude of air drag and water drag in the Arctic, two aspects of the ice/water and ice/air coupling must be dealt with simultaneously: (1) skin friction generated by fluid flow over small-scale roughness elements, and (2) form drag arising from the pressure difference across larger surface obstacles such as pressure ridges.

It has not yet been definitely established which drag—form or skin—dominates regional stress values. A recent air/ice study made by Arya [1973]
states that, contrary to what many investigators have assumed, the form drag exerted by the atmosphere on the upper surface of the ice can be as great as or greater than the skin friction. Because the underside of the ice is rougher than the topside and water is denser than air, we infer from Arya's calculations that form drag on the bottom of the ice should be much larger than skin drag. However, other investigators (e.g., Hunkins, 1974, personal communication), using different assumptions, conclude that skin drag is usually the major part of the water stress.

In the ocean, internal gravity waves are a third source of drag on the ice. We know that the internal waves created by an object passing through a stratified fluid dissipate energy and can greatly increase the drag experienced by the object. This phenomenon occurs in Norway, where the fresh-water discharge of rivers and streams can spread, with almost no mixing, over the surface of a sea-water fjord. Small, slow-moving fishing boats sometimes encounter greatly increased water resistance when they enter such waters [Ekman, 1906; Lamb, 1916].

An analogous situation appears to exist in the Arctic Ocean, where sharp stratifications are found beneath the pack ice. Coachman [1968] and Smith [1974] observed a fairly steep density gradient, beginning about 40-50 meters below the ice, that persists for months and possibly for the whole year. Large internal waves have been noted at a depth of 60 m in the Arctic Ocean [Yearsley, 1966], and it is worthwhile to ask if pressure ridge keels could generate internal waves large enough to make a significant contribution to the water stress on the ice.

THE MODEL

The simplest way to describe the ocean under the ice is by a two-dimensional model consisting of two constant density layers with the lower layer assumed to be infinitely deep. This model approximates the actual conditions rather well: the pycnocline below the ice is sharp and, on the scale of this problem, does approximate an interface; the upper layer is well mixed and of fairly constant density; density increases only slowly with depth in the lower layer,
which, because of the great depth of the Arctic Ocean, is essentially infinite. The fact that pressure ridge keels may be several kilometers long but only tens of meters across tends to support a two-dimensional treatment, although it must be noted that this treatment neglects any effects of the Ekman spiral below the ice as well as any effects that arise when the current is oblique rather than perpendicular to the keel.

Kovacs et al. [1972] have studied a typical, medium-sized pressure ridge and found that the keel below it was roughly semielliptical in cross section with its semimajor axis in the horizontal. Lamb [1932] presents a solution for waves on the surface of a stream that are created by semicircular and semielliptical obstacles on the stream bottom. This is analogous to the simple model we have set up, and Lamb's solution can be adapted easily to describe the development of internal waves beneath the ice.

CALCULATIONS

Figure 1 shows a sketch of the model and the coordinate system used in our calculations. The lower surface of the ice is assumed to be flat except for a semielliptical keel of width $2a$ and depth $b$, centered at $x=0, y=h'$. Here $\rho$ is the density, $c$ is the velocity of the water relative to the ice, and $h$ is the depth of the water. Quantities for the upper layer are primed; those for the lower layer are unprimed.

Fig. 1. Idealized sketch of a semielliptical keel.
Lamb [1932] gives the velocity potential, $\phi(x)$, for two-dimensional laminar flow around an obstacle with a semielliptical cross section as

$$\phi(x) = -c'x - \frac{b(a + b)c'}{2} \int_0^\infty e^{-k(h'-y)} \sin(kx) \, dk + \psi'(x) \]$$

(1)

when the obstacle is oriented with its semimajor axis, $a$, horizontal and its semiminor axis, $b$, vertical; $k$ is the wave number, $\psi'$ is any function that is small in the vicinity of the obstacle, and $b$ is assumed to be small compared to $h'$. For $y \geq 0$, we assume that $\psi'(x)$ has the form

$$\psi'(x) = \int_0^\infty \alpha(k) \, e^{-ky} \sin(kx) \, dk \]$$

(2)

where $\alpha(k)$ is a function to be determined. If we let $\eta(x)$ be the displacement of the interface, $\beta(k)$ is defined by

$$\eta(x) = \int_0^\infty \beta(k) \cos(kx) \, dk . \]$$

(3)

Since we are referring to a keel, we can treat the internal waves behind the keel as a wake. This makes the problem steady, i.e., the phase velocity of the waves is equal and opposite to the water velocity relative to the keel. If we assume that $h'$ is large relative to the amplitude of the waves, we can apply a form of the continuity equation at $y = 0$,

$$- \frac{\partial \phi}{\partial y} = c' \frac{\partial \eta}{\partial x} . \]$$

Hence,

$$\frac{b(a + b)}{2} c' e^{-kh'} + \alpha(k) = -c' \beta(k) \]$$

or

$$\alpha(k) = -c' \left[ \beta(k) + \frac{b(a + b)}{2} e^{-kh'} \right] . \]$$

(4)

Below the interface ($y \leq 0$), we have
\[ \phi(x) = -cx + \psi(x). \] (5)

As before, we define \( \gamma(k) \) by
\[ \psi(x) = \int_0^\infty \gamma(k) e^{ky} \sin(kx) \, dk \] (6)
and at \( y = 0 \)
\[ -\frac{\partial \phi}{\partial y} = c\frac{\partial \eta}{\partial x}, \]
implying that
\[ \gamma(k) = c\beta(k). \] (7)

Having developed these relationships, we can substitute them into Bernoulli's equation
\[ P + \rho g \eta + \rho \left( \frac{\partial \phi}{\partial x} \right)^2 = P' + \rho' g \eta + \rho' \left( \frac{\partial \phi'}{\partial x} \right)^2 + \text{constant} \] (8)
at \( y = 0 \). At the interface, we assume that \( P = P' \) and make the Boussinesq approximation that \( \rho = \rho' \), except where \( \Delta \rho = \rho - \rho' \) appears (\( \rho > \rho' \)). Substituting the preceding equations into equation 8 and dropping all second- and higher-order terms gives
\[ \frac{1}{2}[\rho \sigma^2 - \rho'(\sigma')^2] + \int_0^\infty \left[ \frac{\Delta \rho}{\rho g} \beta(k) - \kappa \sigma^2 \beta(k) \ight.
\[ - \frac{1}{2} kb(a + b)(\sigma')^2 e^{-kh'} + k\sigma' \alpha(k) \right] \cos(kx) \, dk = \text{constant}. \]
Because the integral on the left-hand side of this equation is \( x \)-dependent, its integrand must be zero. From equation 4, we can substitute for \( \alpha(k) \) in the integrand and obtain
\[ \frac{\Delta \rho}{\rho g} \beta(k) - \kappa \sigma^2 \beta(k) - \frac{1}{2} kb(a + b)(\sigma')^2 e^{-kh'}\]
\[ - k(\sigma')^2 \beta(k) - \frac{1}{2} kb(a + b)(\sigma')^2 e^{-kh'} = 0. \]
Hence,

\[ \eta = - \frac{2(a')^2}{[\sigma^2 + (a')^2]} \int_0^\infty \frac{b(a + b)ke^{-\kappa h'}}{2(k - \kappa)} \cos(kx)dk \]  

(9)

where

\[ \kappa = \frac{\Delta \rho g}{\rho [\sigma^2 + (a')^2]} . \]  

(10)

The integral in equation 9 is indeterminate, but its principal part can be evaluated. It has the same form as the solution that Lamb [1932] derived for surface waves. The solution downstream of the keel \((x > 0)\) has the form

\[ \eta = - \frac{2(a')^2}{\sigma^2 + (a')^2} \pi \kappa \frac{b(a + b)}{\sigma^2 + (a')^2} e^{\kappa h'} \sin(\kappa x) . \]  

(11)

The drag resulting from the waves generated is found from the formula

\[ R = \frac{g}{\sigma^2 + (a')^2} \frac{\pi^2 \kappa^2}{[\sigma^2 + (a')^2]^2} \frac{b^2(a + b)^2}{e^{2\kappa h'}} \]  

(12)

This will give the drag per meter of keel, since the model is two-dimensional. The form drag, \(D\), is given by

\[ D = \frac{1}{2} C_d \rho (a')^2 A \]  

(13)

where \(A\) is the cross-sectional area across the current and \(C_d\) is the drag coefficient. In our case we use

\[ D = \frac{1}{2} C_d \rho (a')^2 b \]  

(14)

for form drag per meter of keel. According to Kovacs et al. [1973], \(C_d\) varies from 0.8 for a semicircle to 0.5 for semielliptical forms.

We now apply these formulas to data that approximate conditions found in the Arctic. We take \(\rho = 10^3\) kg/m\(^3\) and \(g = 9.8\) m/sec\(^2\). Salinity measurements beneath perennial ice in the Beaufort Sea [Coachman, 1968; Smith, 1974] indicate that a fairly abrupt salinity change of about 2\(^\circ\)/oo occurs at a depth of 45-50 meters in the ocean. For our calculations, we use \(h' = 50\) m and \(\Delta \rho = 2\(^\circ\)/oo. Selecting a value for the relative water velocity is
difficult, because $c'$ can vary from 0 to about 50 cm/sec. Since we are trying to assess the general conditions under which internal waves can be expected to form, we decided to choose an initial value for $c'$ near the upper end of the observed range, 50 cm/sec. The model described here requires that velocity be constant in each layer, but it allows for a shear across the interface. Shears on the pycnocline have been observed in the Arctic, but they are not always present or of uniform size. For simplicity, we took $c = c'$.

RESULTS

Results of the calculations are sketched in Figures 2 through 6. Figure 2 shows the form drag and wave drag for $c = 50$ cm/sec as a function of keel depth, both for a semicircular keel and for a semielliptical keel ($a = 3b$) similar to the one reported by Kovacs et al. [1973]. It is evident from Figure 2 that for semielliptical keels as shallow as 10 meters the internal wave drag can be a significant fraction of the total drag.

Fig. 2. Form drag (solid curves) and wave drag (dashed curves) as a function of keel depth, $b$, for $c = c' = 0.5$ m/sec, $h' = 50$ m, and $\Delta \rho = 2 \text{o}/\text{oo}$.
Keels 20 or more meters deep are not extraordinary, although they are infrequent. Since the amplitude of the internal waves goes as the fourth power of the keel depth, the contribution of very deep keels to the total drag may be disproportionately large considering their rarity.

In Figure 3 we show the relative importance of wave drag and form drag on a 20 m semielliptical \((a = 3b)\) keel as a function of the relative current speed. We see that \(c = 0.3\) m/sec appears as a kind of threshold velocity below which the wave drag is insignificant, but above which the wave drag rapidly becomes the dominant part of the total drag. The same general relationship between form drag and wave drag is repeated for all keels deeper than about 8 m.

![Fig. 3. Drag on a 20 m semielliptical \((a = 3b)\) keel as a function of relative current speed (solid curve). The dotted curve represents the contribution from form drag, and the dashed curve is the corresponding wave drag contribution; \(h' = 50\) m and \(\Delta\rho = 2^\circ/oo.\)]

To better appreciate the conditions under which wave drag can be important, let us define a "critical velocity," \(v^*\), as the velocity at which form drag and wave drag are equal \((v^* = 0.38\) m/sec for the case described in
Figure 3). Figure 4 shows the dependence of $v^*$ on keel depth, assuming again that the keel is semielliptical with $a = 3b$. The results suggest that under certain circumstances wave drag in the Arctic Ocean is at least equal to form drag; however, with the rapid increase of drag in the vicinity of $v^*$, it would be difficult to achieve or maintain velocities of this magnitude.

![Figure 4. Critical velocity, $v^*$, of a semielliptical ($a = 3b$) keel as a function of keel depth, $b$; $h' = 50$ m and $\Delta \rho = 2^\circ/\text{oo}$.

Although the treatment described here assumes a sharp interface between the two fluids, this is not strictly the case in the Arctic, and there is some question as to what values of $h'$ and $\Delta \rho$ would best describe the development of internal waves beneath the ice. This is an important question, since, according to our calculations, the wave drag is quite sensitive to both $h'$ and $\Delta \rho$. Figure 5 shows the dependence of $R$ on $h'$ for the 20 m semielliptical keel, assuming that $c = c' = 0.5$ m/sec and $\Delta \rho = 2^\circ/\text{oo}$. Figure 6 shows the variation of $R$ with $\Delta \rho$, if the interface is at 50 m. At this point, it is not clear what values are the most appropriate, but internal waves of as much as 10 m in amplitude have been observed at a depth of 60 m [Yearsley, 1966].
While the data of Yearsley are insufficient to determine if there were larger waves above or below this level, the choice of $h' = 50$ m does not appear to be unreasonable. To settle this question, current velocity profiles are needed, as well as direct observations of the wavelength and amplitude of internal waves downstream from a large pressure ridge.

![Graph](image)

Fig. 5. Wave drag on a 20 m semielliptical ($a = 3b$) keel as a function of $h'$; $c = c' = 0.5$ m/sec and $\Delta \rho = 2^\circ/\text{oo}$.

It is important to stress that the calculations described above apply to a highly idealized situation that only approximates conditions under the ice. Other uncertainties besides the proper choice of $h'$ and $\Delta \rho$ could influence the predicted wave drag. The assumption that $c = c'$ is simple, but not necessarily good. The presence of shear with $c > c'$ diminishes the $(c')^2/[c^2 + (c')^2]$ term in equation 11 and thereby decreases $\eta$ and $R$, so that a strong shear across the interface could suppress internal waves. Even a more gradual change of velocity with depth could modify our results. A serious limitation in the present model is that the assumption that keel depth is small compared to $h'$ causes the wave drag estimates to become increasingly crude as keel depth increases.
Fig. 6. Wave drag on a 20 m semielliptical ($a = 3b$) keel as a function of $\Delta \rho$; $c = c' = 0.5$ m/sec and $h' = 50$ m.

Despite the uncertainties in the treatment presented here, it appears that wave drag is significant near large keels when the current is flowing strongly. Even so, it would probably not appreciably alter the large-scale values of water stress. Crucial questions to be resolved center around the relative importance of form drag versus skin drag, and the depth at which internal waves are generated. Further field observations are required to definitely evaluate the role of internal wave drag in the dynamic behavior of pack ice.

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AN ESTIMATE OF INTERNAL WAVE DRAG ON PACK ICE

by

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ABSTRACT

As pack ice drifts in a stratified ocean, the deep keels of pressure ridges generate internal waves which produce a retarding force on the motion. An estimate of this drag force was made by scaling results from experiments performed long ago by Ekman, who towed ship models in a tank with two fluid layers of different densities. For conditions assumed to be typical of the Arctic Ocean, internal wave drag is shown to amount to only 10% of the form drag and only 20% of the skin friction. The two-layered model gives only the effect of one mode. Further studies with multiple layers or with continuous stratification would be valuable to find the drag contribution of higher modes.

The resistance of fluids to moving bodies can usually be divided for descriptive and measurement purposes into three types. Skin drag is caused by the velocity shear near the surface of the body. Form drag is due to the turbulent wake which occurs after the boundary layer has separated from the surface. Both of these types of drag are a function of velocity, usually squared, with a proportionality constant known as the drag coefficient. Both types of drag have been discussed and measured to some extent on pack ice.

If the fluid has a free surface, a third type of resistance, wave drag, may also exist. The loss of energy to the train of gravity waves left behind the object produces drag. Surface wave drag is not important in pack ice where nearly total ice cover prevents the generation of waves on the surface. However, if a fluid is composed of layers with differing densities, internal waves may be generated that can also produce wave drag. Like most oceans, the Arctic Ocean does possess density stratification. Since internal waves
and pack ice both have similar speeds it is possible that the movement of pack ice with deep keels might generate significant internal waves. Little attention has been given to this type of drag on pack ice. This note attempts to provide an estimate for wave drag on pack ice by scaling the results of model experiments.

The first studies of internal wave drag were prompted by the "dead water" resistance met by ships under certain conditions. The effect of internal wave drag on ships is evident at low speeds of about one knot, and in areas where a thin layer of fresh water overlies salt water. Thus it was especially noticeable in the time of slow sailing ships and in areas, such as the Norwegian fjords, where a fresh layer often overlies a more saline layer. Ships caught in dead water were held to the internal wave speed unless they could break free by suddenly increasing their speed. Such a sudden increase is generally possible for motor ships but not for sailing ships.

At the suggestions of Nansen and Bjerknes, Ekman [1906] undertook an extensive study of dead water. Nansen became interested in the subject when, in 1893 off the Taimur Peninsula, the Fram was caught by dead water that reduced the ship's speed from a normal 5 knots to 1 knot. Ekman cited this and many other examples of dead water in an historical summary, and he conducted a series of carefully organized, quantitative experiments which proved conclusively that the high resistance was due to internal waves. In these experiments, ship models were towed in tanks containing a fresh water layer over a salty layer, and the towing resistance was measured as a function of such parameters as speed, layer densities and depths, and hull shape and size.

The atmosphere is also stratified and internal waves frequently develop in the lee of mountain ranges. Lee waves, or mountain waves, are sometimes made visible by cloud bands at the wave crests which remain stationary with respect to the ground despite the wind. Reviews of the subject have been given by Miles [1969] and Turner [1973]. Although a considerable amount of theoretical work and some model studies have been stimulated by the atmospheric problem, the application of the results to the
present problem is not clear. Model studies by Davis [1969], for example, do not show good agreement with theory. The careful model studies by Ekman are still, despite the time that has elapsed, a good starting point for an estimate of internal wave drag on pack ice.

Ekman used models of the Fram on 1/100 and 1/200 scales as well as other hull shapes. He demonstrated that the results from one Fram model could be applied to the other by using the rule of Froude for ship modeling. He also showed that the results could be scaled to give results in close agreement with experience for the full-scale Fram. Froude's rules are based on dimensional considerations and neglect the viscosity. They were originally applied to surface wave resistance, which is the most important drag factor at higher speeds for displacement hulls.

Ekman extended the rules to cover internal wave drag. The scaling in the case of wave drag is in terms of the Froude number \( F \), the ratio of ship speed to the speed of long gravity waves, rather than the Reynolds number \( R \). The drag force is proportional to the displacement of the hull, or cube of linear increase in dimensions, and to the difference in specific gravity between the layers. This allows some of Ekman's major results to be summarized by a plot of normalized drag force versus Froude number, with the ratio of layer to keel depth as a parameter (Fig. 1).

![Fig. 1. Normalized internal wave drag versus Froude number with the ratio of layer to keel depth as a parameter. Curves are based on Ekman's [1906] experiments.](image-url)
Drag results for homogeneous water are also shown for each keel depth. The difference between the homogeneous water curve and the appropriate curve in stratified water represents the drag due to internal waves. Two important aspects of internal wave drag are shown clearly in the figure. First, there is a maximum drag when the Froude number is 0.73, that is, when the boat speed nearly coincides with the internal wave speed. For the case of keel depth less than layer depth, wave drag is greatest for Froude numbers less than one. At that speed the largest waves are generated and drag is highest. Internal wave drag is significant only between Froude numbers of roughly 0.3 and 2.0. Note that for Froude numbers exceeding 0.73 there is an unstable region in which drag decreases as speed increases. In this speed range, ships break free of dead water, increasing their speed suddenly until limited by other types of drag. Second, the curves of Figure 1 show the effect of changing keel depths in relation to the depth of the fresh water layer. Maximum drag occurs when the keel depth is the same as the depth of the fresh water layer.

These curves may be applied to the drag of pack ice provided that layer depth, specific gravity difference, and displacement of the ice keel are known. Data are available that allow one to make reasonable estimates of these parameters. The assumptions are that the shape of the ice keel resembles a boat hull, a round-bottomed one in the case of Figure 1, and that the Arctic Ocean stratification can be represented by a density discontinuity.

Although probably only the keels of old pressure ridges resemble the hulls of ships very closely, the exact form may not be particularly critical for an estimate of internal wave drag. The different hull shapes that Ekman tested showed no remarkable differences in drag. A study by Kovacs et al. [1972] of an old pressure ridge with a keel depth of 13 m showed a smooth, roughly semicylindrical underwater form somewhat resembling a ship's hull. Younger ridges are more likely to be composed of jumbled, unconsolidated blocks with a greater form drag than old ridges, although even in this case internal wave drag may not be affected much.

Although stratification in the Arctic Ocean is actually gradual, its representation by a discontinuity is a reasonable first approximation so long
as only the first mode is being considered. A density profile taken at the AIDJEX main camp in 1972 (Fig. 2) shows a low salinity mixed layer 35 m deep overlying a steep density gradient extending to a depth of 300 m. Below the gradient lies nearly homogeneous water extending to the bottom, which is at 3800 m in this area.

Fig. 2. Vertical profile of temperature (T), salinity (S), density (σ_T) and Väisälä frequency (N), 25 March 1972, 72°07'N 149°00'W, University of Washington hydrographic stations 24 and 26.
Internal wave dispersion curves based on this profile are shown in Figure 3. The curves are based on a matrix method for calculating internal wave properties in a rotating ocean with many layers (Fliegel and Hunkins, in preparation). Note that over the nearly level portion of the curve, phase velocity and group velocity are nearly equal. It is in this region that internal wave drag is greatest. The appropriate modeling parameter in this case is the phase velocity, which is about 200 cm/sec over the level part of the curve for the first mode.

Fig. 3. Internal wave dispersion curves for the first four modes. Calculations are based on an 18-layer model approximating the density curve of Fig. 1. Solid lines are phase velocity; dashed lines are group velocity.

A two-layer model has only one mode; it is equivalent to the first mode of a multilayered model. For the two-layer model, the interface is chosen near the center of the gradient at a density of 1.026 g/cm³. The upper layer \( h \) is 100 m thick with a density \( \rho_1 \) of 1.024 g/cm³, and the deep layer is in effect
infinitely thick with a density \( (\rho_2) \) of 1.028 g/cm\(^3\). The phase velocity is

\[
C = \sqrt{\frac{\rho_2 - \rho_1}{\frac{1}{2}(\rho_1 + \rho_2)}} \cdot gh = \sqrt{\frac{1.028 - 1.024}{1.026}} \times 983 \text{ cm/sec}^2 \times 100 \text{ m}
\]

\[= 195.8 \text{ cm/sec},\]

where \( g \) is the acceleration due to gravity. This is sufficiently close to the multilayered model. Some experiments that Ekman performed on multiple layers showed a change of 14% at most when the density change occurred in several steps rather than in a single discontinuity.

Information on the numbers and average depths of ridge keels comes from U. S. Navy nuclear submarines whose upward-looking sonars give profiles of the bottom of the ice [Lyon, 1961]. Actual keel depths range up to 47 m, the deepest yet observed [Hibler et al., 1972]. The use of an average keel depth appears justified for rough estimates since the drag is approximately proportional to depth over the range of interest. For the central Arctic Ocean an average keel depth is taken to be 10 m with a spacing between ridges of 200 m. The ratio of layer depth to keel depth is thus 10 for the Arctic Ocean. Since the largest ratio tested by Ekman was 2.95, his data must be extrapolated.

The maximum drag appears to be approximately proportional to the reciprocal of this ratio for the range of interest. For a layer/keel depth ratio of 1.14, the maximum internal wave drag, shown by the double-headed arrow, is 72 dyn/g at a Froude number of 0.73. This is multiplied by a factor of 0.1 to compensate for the increased ratio of layer-to-keel depth in the Arctic Ocean, giving 7.2 dyn/g. Assume an increase of 200 in linear dimensions over the 1/100 Fram model: the actual Fram is 35 m \( \times \) 11.5 m \( \times \) 4.4 m; the model ridge is therefore 70 m long, 23 m wide, and 8.8 m deep and travels in the direction of the long axis. The 70 m long ridges would seem to occupy a large fraction of the pack. Actually a shorter hull with the same draught and width might give a similar internal wave drag. In other words, the internal wave drag may not be particularly sensitive to hull length. The actual Fram displaces 800 tons, and so the model ridge displaces \( 2^3 \), or 8 times, that value. Scaling the drag we have
\[ D_{\text{wave}} = 7.2 \text{ dyn/g} \times 8 \times 800 \times 10^6 \text{ g} \times 0.004 = 1.84 \times 10^8 \text{ dyn} \]

Consider an ice pack with one of these ridges spaced at 200 m intervals. The internal wave stress will be

\[ \tau_{\text{wave}} = \frac{1.84 \times 10^8 \text{ dyn}}{(200 \text{ m})^2} = 0.46 \text{ dyn/cm}^2 \]

This represents an extreme upper limit, since the drift speed necessary to attain this value is never reached in nature. This maximum value of internal wave stress is achieved at an ice speed of

\[ V_{\text{ice}} = F \times \text{internal wave speed} = F \times \sqrt{g \times \text{sp. gr. diff.} \times \text{layer depth}} \]

\[ = 0.77 \sqrt{980 \text{ cm/sec}^2 \times 0.004 \times 100 \text{ m}} \]

\[ = 0.77 \times 198 \text{ cm/sec} = 152 \text{ cm/sec} \approx 3 \text{ kt} \]

This is far in excess of actual ice speeds. During the 1972 AIDJEX program, the highest observed speed was 26 cm/sec, or about 1/2 knot. This is equivalent to a Froude number of 0.13. Reference to Figure 1 shows that internal wave effects are becoming negligible at this speed. Ekman's experiments did not extend to such small Froude numbers. Extrapolation of the curve suggests that the internal wave drag is at least an order of magnitude smaller at \( F = 0.13 \) than at its maximum value. Thus under actual conditions, a maximum value is \( \tau_{\text{wave}} \approx 0.05 \text{ dyn/cm}^2 \).

The internal wave drag must be compared with skin and form drag to determine its relative significance. In terms of Prandtl's boundary layer theory, the roughness parameter value at the AIDJEX main camp in 1972 was \( z_0 = 4.17 \). Assuming a reference level of \( z = 2 \text{ m} \), corresponding to the depth of the skin friction layer, the drag coefficient is given by

\[ C_D = \frac{2k^2}{[\ln(z/z_0)]^2} = 0.021 \]

where von Karman's constant, \( k \), is 0.4. The stress due to skin friction during the period of fastest drift in 1972 was then

\[ \tau_{\text{skin}} = \rho C_D u^2/2 = 1.026 \times 0.021 \times (15)^2/2 = 2.4 \text{ dyn/cm}^2 \]
where \( u \) is the velocity in cm/sec. There will also be a contribution from form drag. The Reynolds number in this case will be

\[
R = \frac{\rho u l}{\mu} = 1.026 \times 26 \times 10^3 / 10^{-2} = 2.7 \times 10^6
\]

where \( l \) is a characteristic length and \( \mu \) is the viscosity. Boundary layer separation has occurred for most geometric forms at this Reynolds number, and for a hemispheric shape the drag coefficient will be in the neighborhood of unity or perhaps slightly less. The form drag of a ridge will then be

\[
D_{\text{form}} = \rho C_D A \frac{u^2}{2} = 1.026 \times 1.0 \times 23 \text{ m} \times 8.8 \text{ m} \times (15)^2 / 2
\]

\[
= 2.33 \times 10^8 \text{ dyn}
\]

where \( A \) is the cross-sectional area of the ridge. The stress will be

\[
\tau_{\text{form}} = \frac{(2.33 \times 10^8)}{(200 \text{ m})^2} = 0.6 \text{ dyn/cm}^2
\]

These calculations indicate that internal wave drag on pack ice in the Arctic Ocean is negligible in comparison with other types of drag. This somewhat justifies the neglect of internal wave drag in the AIDJEX field studies where only skin and form drag have been considered.

A more precise estimate of internal wave drag on pack ice would require direct observations in the field, further model studies, theoretical investigations, or some combination of these. Theoretical investigations alone will probably not be sufficient.

The study of internal wave drag under actual conditions in pack ice presents difficulties. The internal waves that produce drag will, under steady conditions, have crests and troughs in a fixed relation to the ice. They cannot be detected from a single observation point as can internal waves generated by other causes which travel relative to the ice. Internal waves moving relative to the ice have been observed from drifting ice with thermistor strings and with echo sounders. However, either a dense horizontal net of fixed sensors or a mobile sensor is required to observe standing internal waves. The unmanned research torpedo developed by the Applied Physics Laboratory of the University of Washington would be an ideal mobile
sensor, since it could be programmed to sample along paths downstream from ridges, searching for the undulations in temperature and salinity indicative of lee waves.

A side-scanning sonar might be another useful technique. It has been shown that acoustic waves with a frequency of 100 kHz are reflected by the sharp boundary at the base of the mixed layer, which was at 35 m in 1972. A side-scanning sonar with high frequency might be able to detect the undulations of this interface over a fairly wide area and give a three-dimensional picture of it in time. Further model testing would be valuable to extend the work of Ekman to specific questions about pack ice drag. Some points to be clarified would be:

1. Drag at small ratios of layer depth to keel depth.
2. Drag at small Froude numbers.
3. Effect of shapes other than ships' hulls.
4. Effect of continuous stratification.

The calculations have been based on two-layered model tests. Although Ekman's results seemed to show otherwise, there is still a possibility that models with more layers will show high drag effects for large ratios of layer/keel depth at certain low speeds. In a two-layered model, only one mode of motion exists. The maximum of vertical motion is at the interface, which moves up and down as horizontal currents in the upper and lower layers flow in opposite directions. Higher modes with more than one maximum of vertical motion in depth may exist in fluids with many layers. The higher modes have slower phase speeds than the fundamental mode.

There may be cases in which a keel extending a small distance into the mixed layer will excite higher mode internal waves of large amplitude when it moves at a certain lower speed. Ekman's limited experiments with density gradients could have missed this effect. Note that the fourth mode has a phase velocity of 45 cm/sec in this region of high drag. It is clear that a phase speed in the range of ice speeds will occur in this flat region within the next few higher modes. These modes will have a complex vertical structure which might be excited by relatively small keels.
Vertical current shear is another complication which could be of importance. In the model tests the fluid has a uniform motion relative to the moving object. The horizontal velocity in the ocean does vary with depth during internal wave generation by pack ice. This effect has been included in various theoretical studies of lee waves in the atmosphere. The production of a flow with horizontal velocity varying in depth might require, in a tank model, considerable experimental ingenuity. Theory may be the simplest method for evaluating the importance of this particular effect.

Although there are still a number of unexamined factors, it appears that the internal wave drag generated by ridge keels is not likely to be an important factor in the general underwater resistance of ice floes in the Arctic Ocean. The assumptions and extrapolations involved in this rough analysis make it desirable to attempt more refined calculations and experiments. However, the neglect of internal wave drag relative to skin and form drag appears at present to be justifiable for large-scale ice motion.

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A SIMPLE MOMENTUM INTEGRAL MODEL

by

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AIDJEX

ABSTRACT

Planetary boundary layer data taken by an instrument package suspended from a tethered balloon during the AIDJEX 1972 pilot study indicate that an adiabatic layer with uniform velocity topped by an inversion frequently forms in the Arctic. A simplified von Karman momentum integral is developed for this case to calculate a drag coefficient with respect to surface velocity, geostrophic velocity, and inversion height.

INTRODUCTION

Although atmospheric stratification, as determined by the temperature difference across the planetary boundary layer, is often stable in the Arctic, turbulence occurs frequently from surface roughness, convective heating, and the dynamic instabilities intrinsic in the flow. There is diffusion by small-scale eddies and advection by secondary flows, even in light to moderate winds, that can result in an adiabatic layer topped by an inversion.

We have studied the possibility of using a gross momentum integral model for well-mixed layers to model this adiabatic layer. Our model was inspired by the relatively uniform data on wind speed and temperature taken during the 1972 AIDJEX pilot study. These data were collected by R. McBeth of NCAR using an instrument package suspended from a kytoon (tethered balloon) that was raised and lowered; the instruments and kytoon make up the standard boundary layer profiler used by the National Center for Atmospheric Research (NCAR).

The purpose of this model is to explore the possibility of relating the surface stress to the inversion height. Initially, this is done by
considering the momentum integral representation of surface stress,

\[ u_\star^2 \equiv \frac{T_0}{\rho} = \int_0^h (\vec{\tau} - \vec{\tau}_g) dz \]  

(1)

where \( T_0 \) is the stress at the surface, \( \rho \) is the density, \( f \) is the Coriolis parameter, \( 2\Omega \sin \text{ (lat.)} \), \( \nu \) and \( \nu_g \) are velocity and geostrophic velocity, and \( h \) is the height of a sharp inversion in which \( \nu + \nu_g \rightarrow \infty \) and \( \tau \rightarrow 0 \). However, the kytoon data, restricted to light to moderate winds, showed a nearly constant velocity beneath the inversion (and above the surface layer). This suggested that a simplified model could be used for these conditions.

In our assumptions, the boundary layer is a thoroughly mixed layer of nearly constant velocity and potential temperature. Any change in velocity (magnitude and direction) takes place at the top of the boundary layer within an inversion. The inversion suppresses instabilities in this strong shearing region by providing high enough temperature gradients that Richardson numbers are greater than critical. The boundary layer then has an effective lid, and the momentum equations can be integrated to produce relations between this height and the surface stress, the geostrophic velocity, and the constant velocity of the boundary layer.

**STRESS COEFFICIENT**

The boundary layer velocity is \( \vec{u} = (u, \nu) \), the freestream (geostrophic flow) is \( \vec{U} = (U, 0) \), and the stress is \( \vec{\tau} = (\tau^x, \tau^y) \). Vertical profiles of these parameters are approximated as shown in Figure 1. The drop to zero velocity is assumed to take place within 10 meters of the surface. In this respect, the model is the same one used by Thompson [1973].

The steady-state momentum equations are

\[ f \nu = \frac{1}{\rho} \frac{d\tau^x}{dz} \]

\[ f(u - U) = - \frac{1}{\rho} \frac{d\tau^y}{dz} \]  

(2)
Integrating to the top of the boundary layer, \( h \), where \( u = U, \ v = 0, \) and \( \nabla = 0 \), we obtain

\[
 f u h = - \frac{T_0}{\rho}
\]

and

\[
 f(u - U)h = \frac{T_0}{\rho}
\]

where \( T_0 = (T_0^x, T_0^y) \) represents the stress in the constant stress layer next to the surface.

If we assume that the stress is aligned with the surface wind, then \( T_0^x/T_0^y \approx u/v \), and

\[
 u^2 + v^2 = uU = |\vec{u}|^2.
\]

Solving for \( T_0 \), we obtain

\[
 \frac{|T_0|}{\rho} = f h(U^2 - uU)^{\frac{3}{2}} = f h U \left[ 1 - \left( \frac{|\vec{u}|}{U} \right)^2 \right]^{\frac{3}{2}}
\]

The stress can then be written with respect to a geostrophic drag coefficient as

\[
 C_g \equiv \frac{u_*}{G} = \left\{ \frac{\vec{f} h U \left[ 1 - \left( \frac{|\vec{u}|}{U} \right)^2 \right]^{\frac{3}{2}}}{U \int \vec{f} h G} \right\}^{\frac{1}{2}}
\]

\[
 u_* = \left[ \int \vec{f} h G \right]^{\frac{1}{2}}
\]
To compare with surface drag values obtained at AIDJEX 1972, the stress can also be written

$$ C_D = \frac{\tau_0}{\rho |\vec{u}|^2} = \frac{\int_0^H \left( \frac{U}{|\vec{u}|} \right)^2 - 1 \, \mathrm{d}h}{|\vec{u}|^2} $$

(7)

EVALUATION OF DRAG COEFFICIENTS

Using equations 6 and 7, we can calculate drag coefficients from the rough kytoon data taken during the 1972 AIDJEX pilot study; they are shown in Table 1. Although it would be possible to calculate the momentum integral using the kytoon profile of wind speed versus height, the observed nearly constant $|u|$ profiles suggest the approximations shown in Figure 1. In most runs the stratification was very nearly neutral up to a well-defined inversion and was stable above the inversion. The heating or cooling tendency of the surface is noted in Table 1. An example of a data run is shown in Figure 2. The value of $|\vec{u}|$ is taken as the mean value below the inversion, while $u_{10}$ is taken as the 10 m value. $C$ is calculated from equation 6 with $u_{10}$ in place of $|\vec{u}|$.

Fig. 2. A representative flight data profile of temperature and wind speed.
| Run Time | Inversion Height (m) | $U$ (m/sec) | $|\Delta|$ (m/sec) | $U_{10}$ | Boundary Layer Stratification | $C_D$ from (6) with $U_{10}$ from (7) |
|----------|----------------------|--------------|-------------------|---------|-------------------------------|-----------------------------------|
| 4 March  | 1330-1340            | 80           | 4.7               | 3.9     | 3.6 adiabatic, warming        | 0.036                             |
| 14 March | 1348-1355            | 45           | 3.1               | 2.7     | 2.2 adiabatic, cooling        | 0.030                             |
| 15 March | 1030-1040            | 110          | 5.3               | 3.5     | 2.8 adiabatic with unstable region in middle | 0.045                             |
| 19 March | 1301-1319            | 80           | 3.2               | 2.7     | 2.5 adiabatic, slightly unstable near surface | 0.046                             |
| 20 March | 1600-1610            | 35           | 2.4               | 1.5     | 1.5 adiabatic, warming        | 0.040                             |
| 21 March | 1600-1628            | 80           | 5.2               | 4.4     | 4.2 adiabatic                 | 0.034                             |
| 26 March | 1300-1320            | 100          | 4.2               | 3.2     | 2.5 adiabatic with slightly unstable region near surface | 0.046                             |
| 17.1     | 2100-2110            | 7            | 3.25              | 1.6     | 3.0 stable                    | 0.010                             |
| 17.2     | 2110-2120            | 5            | 3.3               | 2.5     | 3.3 stable                    | 0.011                             |
Although heating took place in the layer during several runs of the kytoon, as indicated by a change in surface temperature, the layer remained adiabatic and the heat apparently went into changing the inversion height or uniformly heating the layer. These preliminary calculations indicate surprisingly constant values of $C_g = 0.04 \pm 0.004$ for the 18 nearly neutral stratification cases. The largest variation occurred on 14 March when the inversion height went from 90 m to 45 m in less than one-half hour. During the four cases in which the boundary layer was stably stratified, $C_g$ dropped to 0.025, 0.016, 0.013, and 0.011. This effect would be expected to depend on the degree of stratification, which cannot be resolved from the available data. The only example of unstable stratification was associated with very weak winds. The error range for winds of less than 1 m/sec was too large to allow meaningful calculations. When the 10 m wind was used in equation 6 for $|\vec{U}|$, $C_{g_{10}} = 0.043 \pm 0.004$, reflecting an underestimate of $|\vec{U}|$. In run 17, this height was above a ground-based inversion.

There was more scatter in the value of $C_D = 0.0028 \pm 0.0009$. During the AIDJEX pilot study, Banke and Smith [1973] took measurements of air stress with sonic anemometers that yielded a gross average $C_D$ of 0.0018 $\pm 0.0002$ for the March-April period. Langleben and Pounder [1972] found an average $C_D$ of 0.0016 and 0.0017 at two AIDJEX locations using the profile method on data from a cup anemometer mounted on a 4 m tower. W. Goddard also used the profile method on data taken on 19 March 1972 (personal communication) and found $C_D$ to vary from 0.0016 to 0.0022 during the early afternoon. These values for $C_D$ are all based on an extrapolated 10 m wind speed.

DISCUSSION

Kytoon data taken of the boundary layer flow over the arctic ice in March-April 1972 clearly showed the flow to have adiabatic layers of nearly constant velocity topped by an inversion. There was generally a velocity maximum in or immediately above the inversion. These observations led to a simplified momentum integral calculation for the surface stress in which the drag coefficient depends on the Coriolis parameter, the inversion height, and the ratio of velocities above and below the inversion. Since the flow
above the inversion can be estimated by the surface geostrophic flow, and
a representative value of the boundary layer velocity can be measured on a
10 m tower, the height of the inversion remains the only parameter that is
difficult to obtain. Currently, inversion height, as well as wind speed,
may be determined effectively by an acoustic sounder [Beran and Clifford,
1972].

The AIDJEX ice dynamics model requires the surface air stress in terms
of the geostrophic flow and some gross characterization of the boundary layer
stratification. Unfortunately, the only surface stress measurements available
are those made within 4 meters of the surface and in relatively smooth areas.
It seems likely that in regions of moderate ridging, the form drag on the
ridges may be comparable to the "skin friction" drag appearing in locally
determined $u_*$ [Arya, 1974]. One way to incorporate the ridge effect on
stress is to look higher, into the Ekman layer, where the integrated effect
of ridge drag might appear. The kytoon reaches several hundred meters, and
data taken to that height should incorporate the stress effect of ridges up
to several kilometers away. The neutral geostrophic drag coefficient
calculated from AIDJEX 1972 kytoon data (Table 1) is about 30 percent
larger than that determined using surface $u_*$ measurements over arctic ice
[Brown, 1974]. The values of $C_D$ from the 1972 data are about 50 percent
larger than surface-based values. It is difficult to determine whether this
difference represents the ridge drag or is simply a result of the difference
in methods. It is encouraging that the integral values are in the expected
range.

To be of practical value, the momentum integral method must relate
stress to thermal stratification in the boundary layer. Although the kytoon
soundings show that the boundary layer lapse rate is almost always adiabatic,
there is ample evidence that frequently the boundary layer is adjusting to
changes in bulk stratification. A typical diurnal heating cycle occurred
during the spring experiment.

The inevitable problem in discussing the practical effects of stratifi-
cation is to establish an appropriate parameter with which to quantify the
stratification. Most characteristic scales incorporate $u_*$ in their definition
[Clarke and Hesse, 1973]. However, these scales are inappropriate since $u_*$ is our primary unknown. The geostrophic departure angle is a possible parameter for this purpose, but it was not determined with sufficient accuracy during the kytoon flights, since the kytoon had no direction sensors. The potential temperature lapse rate appears to be an indicator of equilibrium in the layer, since steady state was characterized by a well-mixed adiabatic layer. The potential stratification, as indicated by the bulk Richardson number relating temperature and velocity gradients across the layer, is invariably stable for the cases in Table 1. This was true for 90 percent of the observations taken in the March-April 1972 period, and conforms to the expected mean stratification in the area.

Such stable stratification might be expected to suppress turbulence and thereby decrease eddy viscosity to such small values that the eddy viscosity analog is no longer valid. A consequence would be a lack of turning in the Ekman layer, with turning being restricted to the near surface layer and the vicinity of the inversion. Nevertheless, eddy mixing is apparently present in these observations, since the adiabatic layer adjusts quickly to changes in surface temperature even when it is more than 100 meters deep. Examples of Ekman layer turning in a stably stratified layer are found in antarctic data taken by Riordan [1972].

An explanation for the continued presence of turbulent eddies producing an eddy viscous effect even in strong stable stratification may come from a consideration of the origin of the turbulent eddies. If one looks upon instabilities in the mean flow as local areas of turbulent generation, then only the local Richardson number, \( \text{Ri} = \frac{(g/\theta)\theta_z/U_z^2} \) need be less than 1/4 for shear instabilities to develop. Such local susceptibility to instabilities exists even in very stable bulk stratification. This locally generated turbulence could then be distributed throughout the layer by such larger-scale motions as plumes or secondary flows (rolls). A sketch of this process as related to the inherent inflection point instability associated with Ekman turning is shown in Figure 3.

The advantage of the momentum integral is that no specific eddy viscosity assumption is needed. The variation of stress with stratification
appears in the variation of $|\bar{u}|$ and $h$. The coarse assumption that the 10 m wind represents $|\bar{u}|$ produced less than 10 percent error. This assumption is expected to become less valid as velocities increase. In such cases, the velocity profiles must be measured sufficiently to use equation 1. The results shown here offer encouragement toward including this type of calculation in the forthcoming AIDJEX main experiment.
REFERENCES


ON POSSIBLE CONSTITUTIVE EQUATIONS FOR SEA ICE

by

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ABSTRACT

Constitutive equations for sea ice are derived on the basis of thermodynamic arguments. These equations have been used in simplified form by Glen and Hibler. The generalization given here may more accurately describe differential sea ice drift.

INTRODUCTION

Hibler [1973] demonstrates that differential sea ice drift may be explained quantitatively using constitutive equations for ice based on the Glen law, which describes a linear viscous fluid with bulk viscosity [Glen, 1971]. Hibler generalizes this constitutive law by assuming that hereditary effects are possible. Measurements of the divergence rate and vorticity do indicate that it is reasonable to assume linear hereditary effects. Nevertheless, more generalization may be possible and desirable in the constitutive assumptions. The present investigation deals with this generalization.

GOVERNING EQUATIONS

Thermodynamic processes are governed by the balance laws of mass, linear momentum, moment of momentum, and energy, which in local form may be written as follows:

\[ \frac{dp}{dt} + v_i \frac{\partial p}{\partial x_i} = 0 \]  

(1)
\[
\rho \frac{dv_i}{dt} = T_{ij,j} + \rho f_i \tag{2}
\]

\[
T_{ij} = T_{ji} \tag{3}
\]

\[
\rho \frac{d\varepsilon}{dt} = T_{ij}D_{ij} - q_i,i + \rho r. \tag{4}
\]

Here, \( \rho \) denotes the mass density per unit volume, \( T_{ij} \) the Cauchy stress tensor, \( \rho f_i \) the external body force per unit mass, \( \varepsilon \) the internal energy, \( q_i \) the heat flux vector, and \( r \) the energy supply. \( D_{ij} = \nu(i,j) \) is the stretching tensor and \( \nu \) the velocity. Cartesian tensor notation is used and summation over doubly repeated indices is implied. Indices in parentheses are to be symmetrized. The material time derivative following a particle is given by \( \frac{d}{dt} \).

The above equations are complemented by the second law of thermodynamics, which we write in the form of the Clausius Duhem inequality

\[
\rho \frac{d\eta}{dt} + \left(\frac{q_i}{T}\right),i \geq \rho \frac{r}{T}, \hspace{1cm} (5)
\]

where \( \eta \) is the specific entropy, \( q_i/T \) the entropy flux, and \( r/T \) the entropy supply. Following the modern trend in thermodynamics we assume (5) to be valid for all motions satisfying equations (1) through (4) [Truesdell, 1969].

Eliminating \( r \) from equations (4) and (5) gives

\[
- \rho (\dot{\psi} + \eta \dot{T}) + T_{ij}D_{ij} - \frac{q_i \dot{q}_i}{T} \geq 0 \tag{6}
\]

where \( q_i = T_{,i} \) and where the Helmholtz free energy \( \psi = \varepsilon - T\eta \) has been introduced. Inequality (6) must hold for all processes satisfying (3) and (4).

**CONSTITUTIVE EQUATIONS**

The field equations (1) to (4) must be complemented by constitutive equations. We now establish constitutive equations for \( \psi, \varepsilon, \eta, T_{ij} \) and \( q_i \). These equations depend on temperature and on variables characterizing motion.
Here we are concerned primarily with motion dependence; we treat temperature dependence only for completeness.

**Constitutive Equations of the Rate Type**

Let \( \psi \) stand for any one of the variables \( \psi, \varepsilon, \eta, T_{ij} \) or \( q_{i} \). We then define a material of the rate type to be a material with the constitutive equation

\[
\psi = \psi (\rho, x_{i}, v_{i}, v_{i,j}, v_{i,j}, \ldots, v_{i,j}, T, q_{i})
\]

(7)

In this expression, superscripts in parentheses denote \( n \)th material derivatives with respect to time. We also assume that all constitutive variables are functions of the same set of independent variables; this is called the principle of equipresence [Truesdell and Noll, 1965].

In any two motions of the continuum which differ by a time-dependent rigid rotation

\[
x_{i}'(t) = Q_{i,j}(t)x_{j}(t) + \sigma_{i}'(t),
\]

(8)

with \( Q_{ik}Q_{kj} = \delta_{ij} \), the constitutive variables transform as follows:

\[
\begin{align*}
\varepsilon' & = \varepsilon \\
\eta' & = \eta \\
\psi' & = \psi \\
T' & = T \\
q_{i}' & = Q_{i,j}q_{j} \\
T_{ik}' & = Q_{ik}T_{ij}
\end{align*}
\]

(9)

The principle of material objectivity requires that the constitutive functions have the same form in any two motions which differ by the rigid rotation given in (8). From (8) and (9) we then have

\[
\begin{align*}
T_{ik}' & = Q_{i,j}T_{ik} \\
v_{i}' & = Q_{i,j}v_{j} + \dot{q}_{i,j}x_{j} + \dot{\varepsilon}_{i}
\end{align*}
\]
and

\[ v_{i,j} = Q_{ik} Q_{jl} v_{k,l} + \Omega_{ij}, \]

where

\[ \Omega_{ij} = \dot{Q}_{ip} Q_{jp} = -\Omega_{ji}. \]

Consequently, for the constitutive equations to be objective, \( v \) cannot be an explicit independent variable of the constitutive equations and the variables \( v^{(n)}_{i,j} \) can only occur in the combinations

\[ A_{kl}^{(n)} = v_{k,l} + v_{l,k} + \sum_{\rho=0}^{n} \binom{n}{\rho} v_{p,k}^{(\rho)} v_{p,l}^{(n-\rho)}, \]

and

\[ D_{ij} = \frac{1}{2} A_{ij}^{(1)}. \]

The tensors \( A_{kl}^{(n)} \) are called the \( n \)th Rivlin-Ericksen tensors. Thus

\[ \psi = \psi (\rho, A_{kl}^{(1)}, \ldots, A_{kl}^{(n)}, T, g_k). \] (10)

(For a proof see Truesdell and Noll [1965].) Substituting

\[ \dot{\psi} = \frac{\partial \psi}{\partial \rho} \dot{\rho} + \sum_{\rho=1}^{n} \frac{\partial \psi}{\partial A_{kl}^{(\rho)}} \dot{A}_{kl}^{(\rho)} \dot{T} + \frac{\partial \psi}{\partial g_k} \dot{g}_k \]

into equation (6) gives

\[ -\rho \left( \frac{\partial \psi}{\partial T} + \eta \right) \dot{T} + T \dot{D}_{ij} - \rho \frac{\partial \psi}{\partial \rho} \dot{\rho} - \sum_{\rho=0}^{n} \rho \frac{\partial \psi}{\partial A_{kl}^{(\rho)}} \dot{A}_{kl}^{(\rho)} + \frac{\partial \psi}{\partial g_k} \dot{g}_k + \rho \frac{q_i g_j}{T} \geq 0. \] (11)

This inequality must also satisfy (1), which may be written as

\[ \dot{\rho} = -\rho D_{ij} \delta_{ij}. \]
Thus (11) becomes

\[
- \rho \left( \frac{\partial \psi}{\partial T} + \eta \right) T + \left( T_{i;j} + \rho \frac{\partial \psi}{\partial \rho} \delta_{i;j} \right) D_{i;j} - \sum_{p=1}^{n} \frac{\partial \psi}{\partial A_{k;l}^{(p)}} \dot{A}_{k;l}^{(p)} \\
+ \frac{\partial \psi}{\partial g_{k}} \dot{g}_{k} - \frac{q_{i} A_{i}^{(p)}}{T} \geq 0
\]

Inequality (12) must hold for arbitrary \( \rho, D_{i;j} = \frac{1}{2} A_{ij}^{(p)}, A_{i;j}^{(p)}, T, g_{k}, \) and their time derivatives. Since (12) is linear in \( \dot{T}, A_{k;l}^{(p)}, p > 1, \) and \( \dot{g}_{k}, \)
we conclude that

\[
\frac{\partial \psi}{\partial g_{k}} = 0,
\]

\[
\frac{\partial \psi}{\partial A_{k;l}^{(p)}} = 0 \Rightarrow \psi = \psi(\rho, T),
\]

and

\[
\eta = - \frac{\partial \psi}{\partial T}.
\]

There remains the residual inequality

\[
\gamma \equiv \left( T_{i;j} + \rho^{2} \frac{\partial \psi}{\partial \rho} \delta_{i;j} \right) D_{i;j} - \frac{q_{i} A_{i}^{(p)}}{T} \geq 0.
\]

Defining equilibrium as a process with uniform, time-independent temperature and vanishing motion, we find from (13) that \( g_{i}|_{E} = 0 \) and \( D_{i;j}|_{E} = 0, \) where \( (\cdot)|_{E} \) indicates evaluation at equilibrium. Thus \( \gamma \) assumes its minimum in equilibrium; this gives

\[
\frac{\partial \gamma}{\partial D_{i;j}|_{E}} = 0 \quad \text{and} \quad \frac{\partial \gamma}{\partial g_{i}|_{E}} = 0,
\]

implying that

\[
T_{i;j}|_{E} = - \rho^{2} \frac{\partial \psi}{\partial \rho} \delta_{i;j}, \quad q_{i}|_{E} = 0.
\]
With the decomposition
\[ T_{ij} = -\rho^2 \frac{\partial \psi}{\partial \rho} \delta_{ij} + T_{ij}^{(D)}, \] (14)
inequality (13) finally reduces to
\[ T_{ij}^{(D)} \frac{D_{ij}}{T} - \frac{q_i q_j}{T} = 0. \]
Note that \( T_{ij}^{(D)} \) depends on \( \rho \) and \( T \) only, while \( T_{ij}^{(D)} \) may depend on all variables introduced in (10).

If the material is incompressible, its behavior does not depend on the density \( \rho \). In that case (14) must be replaced by
\[ T_{ij} = -p \delta_{ij} + T_{ij}^{(D)}, \]
\[ T_{ij}^{(D)} = T_{ij}^{(D)}, \]
where \( p \) is the pressure.

A special case of the rate-type constitutive equation is the linearly viscous fluid, which is characterized by
\[ T_{ij}^{(D)} = T_{ij}^{(D)} A_{kl}^{(1)} = \frac{1}{2} \left( \xi A_{kk}^{(1)} \delta_{ij} + \zeta A_{ij}^{(1)} \right) \]
\[ = \xi v_{k,k} \delta_{ij} + \frac{1}{2} \zeta (v_{i,j} + v_{j,i}). \] (15)
Thus, for a compressible material
\[ T_{ij} = -\rho^2 \left[ \frac{\partial \psi}{\partial \rho} \right] \delta_{ij} + \xi v_{k,k} \delta_{ij} + \frac{1}{2} \zeta (v_{i,j} + v_{j,i}). \] (16)
For an incompressible material, since \( p \) is undetermined, the term \( v_{k,k} \delta_{ij} \) can be absorbed into \( p \). Thus
\[ T_{ij} = -p \delta_{ij} + \frac{1}{2} \zeta (v_{i,j} + v_{j,i}). \]
The constitutive law proposed by Glen [1971] must be considered an approximation of (16) in which $\partial \psi / \partial \rho$ is set to zero. It corresponds to a weak volume dependence.

**Constitutive Equations of the Integral Type**

In the last section we neglected history effects. Here we assume that a material's response depends on the history of its motion. The constitutive equation must then be a functional relation of the form

$$
\psi = \sum_{s=0}^{\infty} \left( t_s \rho(s), t_{A_{kl}}^{(1)}(s), \ldots, t_{A_{kl}}^{(n)}(s), t_T(s); g_k(s) \right),
$$

(17)

where

$$
t_f(s) = f(t - s)
$$

is the history of $f(s)$ up to time $t$. Constitutive equations of the form (17) are established for $\psi$, $\eta$, $\varepsilon$, $T$, and $q_i$. We find it convenient not to work with (17) but with slightly different forms. Knowing $t_f(s)$, $s \in [0, \infty)$ is equivalent to knowing $t_{r_f}(s) = t_f(s)$, $s \in (0, \infty)$ and $\dot{f} = t_f(0)$. We call $t_{r_f}(s)$ the restricted history of $f(s)$ and $\dot{f}$ its present value. Equation (17) may then be replaced by

$$
\psi = \sum_{s=0}^{\infty} \left( t_{r_s} \rho(s), t_{A_{kl}}^{(1)}(s), \ldots, t_{A_{kl}}^{(n)}(s), t_T(s), \rho, A_{kl}^{(1)}, \ldots, A_{kl}^{(n)}, T, g_k \right).
$$

(18)

Our aim now is to produce a chain rule for the functional (18). We shall not go into the mathematical details, which are given in Day [1972]. Here we only mention that $\psi$ must be continuously differentiable over a suitably defined Hilbert space whose elements are the functions

$$\{ t_{r_s} \rho(s), t_{A_{kl}}^{(1)}(s), \ldots, t_{A_{kl}}^{(n)}(s), t_T(s), \rho, A_{kl}^{(1)}, \ldots, A_{kl}^{(n)}, T, g_k \}$$

$$= \frac{\partial \psi}{\partial r_s}, \Gamma, g_k$$

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and whose inner product is given by
\[ \int_0^\infty \gamma^2(s)(\Gamma_1(s), \Gamma_2(s)) ds + g_k^1 \cdot g_k^2, \]

where \( \gamma(s) \) is a monotonically decreasing function. The time derivative of \( \psi \) can then be written as
\[
\dot{\psi} = \delta_p \psi \left( \frac{\zeta}{\rho} \right) + \sum_{j=1}^n \delta_{A_j^{(j)}} \psi \left( \frac{\zeta}{r_{k}^{A_{k_l}}} \right) + \delta_{T\psi} \left( \frac{\zeta}{r_{T}} \right) + \frac{\partial \psi(\cdot)}{\partial \rho} \dot{\rho} \\
+ \sum_{j=1}^n \frac{\partial \psi(\cdot)}{\partial A_j^{(j)}} \dot{A}_k^{(j)} + \frac{\partial \psi}{\partial T} \dot{T} + \frac{\partial \psi}{\partial g_k} \dot{g}_k.
\] (19)

In the above formula, \( \delta_p \psi \left( \frac{\zeta}{\rho} \right) \) denotes the partial Fréchet derivative with respect to \( \frac{\zeta}{\rho} \), holding all other variables fixed. By the definition of Fréchet derivatives, \( \delta_p \psi \left( \frac{\zeta}{T} \right) \) is a linear functional of \( \frac{\zeta}{T} \).

Substituting (19) into (6) one obtains
\[
- \rho \left( \frac{\partial \psi}{\partial T} + \eta \right) T + \left( T \frac{\partial}{\partial \rho} \delta_{ij} \right) D_{ij} - \sum_{j=1}^n \rho \frac{\partial \psi}{\partial A_{k_l}^{(j)}} \dot{A}_{k_l}^{(j)} \\
- \frac{\partial \psi}{\partial g_k} \dot{g}_k - \rho \frac{\partial \psi}{\partial r_{k_l}^{A_{k_l}}} \frac{\partial}{\partial T} \psi \left( \frac{\zeta}{r_{k_l}^{A_{k_l}}} \right) - \rho \sum_{j=1}^n \delta_{A_{k_l}^{(j)}} \psi \left( \frac{\zeta}{r_{k_l}^{A_{k_l}}} \right) \\
- \rho \delta_{T\psi} \left( \frac{\zeta}{r_{T}} \right) - \frac{q_{k_l} \dot{q}_{k_l}}{T} \geq 0
\] (20)

The continuity equation was used in writing this inequality. As before, this inequality must hold for arbitrary \( D_{ij}, A_{k_l}^{(j)}, T, g_k, \) and their time derivatives at time \( t \). Thus, since (20) is linear in \( T, A_{k_l}^{(j)}, \) and \( g_k, \) these terms cannot contribute to the inequality, implying that
\[
\frac{\partial \psi}{\partial A_{k_l}^{(j)}} \equiv 0, \quad \frac{\partial \psi}{\partial g_k} \equiv 0,
\]
and
\[
\eta = - \frac{\partial \psi}{\partial T}.
\]
Similarly, if we write

\[ T_{ij} = T_{ij}^{(1)} + T_{ij}^{(D)}, \]

where \( T_{ij}^{(1)} \) does not depend on \( D_{ij} \), then

\[ T_{ij} = \rho^2 \frac{\partial \psi}{\partial \rho} \delta_{ij} + T_{ij}^{(D)}. \tag{21} \]

There remains a residual inequality, but we are not interested in its form here.

If we assume an incompressible material, (21) has to be replaced by

\[ T_{ij} = -p \delta_{ij} + T_{ij}^{(D)}, \]

with \( T_{ij}^{(D)} = 0 \). As a special case we choose

\[ T_{ij}^{(D)} = \int_0^\infty \frac{1}{2} \xi(s) A_{kk}^{(1)} (t - s) \delta_{ij} ds + \int_0^\infty \frac{1}{2} \xi(s) A_{ij}^{(1)} (t - s) ds \]

\[ = \int_{-\infty}^t \frac{1}{2} \xi(t - \tau) A_{kk}^{(1)} (\tau) \delta_{ij} d\tau + \int_{-\infty}^t \frac{1}{2} \xi(t - \tau) A_{ij}^{(1)} (\tau) d\tau \tag{22} \]

This is the constitutive equation given by Hibler. Note that \( T_{ij}^{(D)} \) is not determined by the free energy functional \( \psi \).

**GENERAL LINEAR CONSTITUTIVE EQUATIONS**

Here, we treat only those constitutive equations that are not dependent on temperature and its gradient. We neglect dependence on density and consider only stress.

The most general form of the constitutive equation for \( T_{ij}^{(D)} \) that is *linear* in the Rivlin-Ericksen tensor is then, for a material of the rate type,

\[ T_{ij}^{(D)} = \sum_{\nu = 1}^n \frac{1}{2} \xi (\nu) A_{kk}^{(\nu)} \delta_{ij} + \frac{1}{2} \xi (\nu) A_{ij}^{(\nu)} \]

\[ \tag{23} \]
where $\xi^{(p)}$ and $\zeta^{(p)}$ are material constants, and for a material of the integral type

$$
T^{(D)}_{ij} = \sum_{p=1}^{n} \left[ \int_{-\infty}^{t} \xi^{(p)}(t - \tau)A^{(p)}_{kk}(\tau)\delta_{ij}d\tau + \int_{-\infty}^{t} \zeta^{(p)}(t - \tau)A^{(p)}_{ij}(\tau)d\tau \right]
$$

(24)

Note that the constitutive relations (23) and (24) are not linear in the velocity and its derivative. Hence (15) and (22) are the only constitutive equations also leading to linear dynamic equations. If it is justified dynamically to assume that $||v_1||$ is small, then the Rivlin-Ericksen tensors may be approximated by

$$
A^{(p)}_{ij} = v^{(p)}_{i,j} + v^{(p)}_{j,i}
$$

(25)

Substituting (25) into (23) and (24), then, also leads to linear dynamic equations.

Clearly, the most general linear constitutive relation for $T^{(D)}_{ij}$ is a combination of (23) and (24).

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SURFACE ATMOSPHERIC PRESSURE FIELDS AND DERIVED GEOSTROPHIC WINDS, AIDJEX 1972

by

R. A. Brown, P. Maier, and T. Fox

AIDJEX

ABSTRACT

The atmospheric freestream flow, and perhaps the oceanic flow, can be related to the surface pressure field. The basic pressure data from peripheral weather stations, ice stations, and AIDJEX buoys have been used to improve the basic Weather Service twice-daily surface analysis maps for 18-28 March and 11-16 April 1972. Discussion of the transformation of the pressure field into a geostrophic velocity field is included.

INTRODUCTION

The U.S. National Weather Service (NWS) routinely puts out twice-daily maps of surface pressure for the northern hemisphere. For the Arctic, these maps include pressure data from U.S., Canadian, and Russian weather stations, and from ice stations when they are operational. The new Russian DARMS buoys, whose accuracies satisfy WMO standards, may eventually be inserted into the system. The NWS coverage is not total: delayed communications (more than four hours past observation time) are omitted from the 0000Z and 1200Z surface pressure maps, and errors or communications difficulties occasionally interrupt the Siberian rim data.

Since practical considerations frequently limit the quantity and quality of arctic data for the NWS maps, we have developed the capability to revise those maps to include AIDJEX and any other available arctic data in local pressure maps. Our object is to receive and process the raw pressure data from the manned stations and data buoys to produce an accurate pressure field for the Arctic Basin or for a local region in this area.
The National Weather Service has established checking and smoothing routines adapted to its particular network and density of input. Because of the sparseness of our data, we have opted to use the Cressman [1959] scheme. This system of "objective weather map analysis" has proven to be comparable to the NWS method when it was used in synoptic meteorology classes at the University of Washington. The basic coding of the system was completed by V. Feltin in 1973. P. Maier has worked on the program for the past year, adding variable grid size capability, geostrophic flow calculations, and the plotting routine. Presently, data from the 1972 AIDJEX pilot study are being processed and searched for correlations between surface winds, stratification, and occasional upper wind data.

THE PRESSURE FIELD

The method of analysis devised by Cressman [1959] was designed to transform data for irregularly spaced points into a regular grid spacing. Data must be checked initially for obvious errors. Smoothing is required at appropriate intervals. The NWS surface analysis at 0000Z and 1200Z furnishes this information on its maps. The accuracy in the Arctic Basin often suffers from sparse data input and the weakness inherent in a system designed to include northern hemisphere data that range from very dense to sparse on a large grid coverage.

Cressman's scheme is convergent; that is, the more passes one makes through it, the closer the initial guess field will come to agreeing with the input data. In the case presented here, five passes were made during computation. Errors at input points were reduced to 0.1 mb or less. More passes through the scheme did not significantly reduce the error in relation to the cost. Fewer passes resulted in increased errors at most points.

To obtain an accuracy of ±0.1 mb in the main camp (Jumpsuit) area (dense data), we reduced the scale of the grid system from 200 km to 50 km to produce an expanded field. This put several grid points between stations or buoys near Jumpsuit. Errors were reduced to near zero.

An optimum grid size appears to be about one-third the distance between the closest data points to be used. For example, the distance between the
manned stations is about 150 km, making the ideal grid size about 50 km. Larger scales than this result in greater error, while smaller scales require data accuracy greater than that available. Smaller scales would show minor pressure perturbations that are probably beyond the resolution of current numerical ice models.

A geographic sketch of the region (Fig. 1) shows the approximate locations of the manned stations and buoys. Figures 2 through 20 show the computed four-hour maps for the Arctic Basin in 1972 (grid size: 200 km), from Julian day 78 (0000 GMT) to day 88 (2000 GMT) and day 102 (0000 GMT) to day 109 (2000 GMT). Figures 21 and 22 are maps for days 80 (0000 GMT) and 102 (0000 GMT) representing (a) the NWS initial field, (b) the 200 km grid corrected field, and (c) an expanded field.

The expanded field using a computed initial guess gives the most accurate results, but the cost of producing this map (or wind field) is almost twice that of an NWS initial field. The cheapest maps now cost at least 85 cents each. It remains to be seen whether the increased accuracy is worth the increased cost. Most likely, an operational procedure will be devised with the choice based on the dynamic activity of the synoptic field and the user's requirements. The capability exists to provide pressure maps on any desired field or grid size.

THE WIND FIELD

The ice motion has frequently been observed to approximately follow the surface isobars. This is to be expected from the basic dynamics, although it may not be immediately obvious. One must consider the common observations that the ice moves at 20°-45° to the right of the surface winds, that the geostrophic wind flows parallel to the isobars, and that the wind turns 0°-45° to the left as height decreases through the boundary layer. The surface pressure field can be transformed into a velocity field which fairly closely approximates the freestream velocity. The equation for atmospheric or oceanic flow is:

\[ \nabla_x + \nabla_y + \nabla_z \nabla + \frac{\nabla}{\rho} = 0 \]
The first two terms are generally much less than the last two, which in balance yield the geostrophic velocity. The flow is parallel to the isobars. Errors may arise when (1) the first term becomes significant in accelerating conditions—the isallobaric wind; (2) the second term is important when strong curvature of the flow exists and the centrifugal term must be added to the balance—the gradient wind; and (3) the pressure gradient at the surface differs significantly from that at the geostrophic level (above the boundary layer) due to strong horizontal gradients in temperature—the thermal wind. These terms have been discussed in AIDJEX Bulletin No. 20 [Brown, 1973].

THERMAL WIND

We are attempting to estimate the wind at the level where it becomes approximately geostrophic (above the arctic inversion, between 50 and 300 m) using the surface geostrophic wind. Since we have records of horizontal temperature gradients only at the surface in the 1972 AIDJEX data, we must use these to approximate the temperature gradients throughout the layer in determining the thermal wind correction. Except at low geostrophic velocities (less than 5 m/sec), the magnitude of the thermal wind correction (based on a typical inversion height of 100 m) was usually less than 10%. In the most extreme case, the correction altered the magnitude estimate based on the surface geostrophic wind by 18% and the direction by 12°. This indicates that the thermal wind may at times be significant, but can usually be ignored. A further difficulty arises when attempting a thermal wind correction. The surface temperatures can cause local anomalies so that the horizontal temperature gradient throughout the boundary layer is not truly represented. Upper air (boundary layer) temperatures are needed to check this effect. The temperature measurements on a 24 m tower will improve this calculation.

GRADIENT WIND

For values of the Coriolis parameter and of wind speed typical at the AIDJEX site, the difference between the gradient wind and the geostrophic wind becomes important when the radius of curvature of the isobars is less.
than 300 km. Relatively unsmoothed surface pressure analyses on a 50 km grid scale from the 1972 AIDJEX pilot experiment produced a number of such situations, and the gradient wind was from 10% to 25% lower than the geostrophic magnitudes.

The reliability of the curvature-effect calculation suffers from the use of isobars to approximate air trajectories, and from the inability to obtain a consistently accurate radius of isobaric curvature. When the limited data are smoothed on a 50 km grid size, the radii invariably become much greater than 300 km.

**ISALLOBARIC TERM**

The isallobaric term computed by using values averaged over the period of a typical storm yields very small corrections to the geostrophic surface winds, even in extreme cases. On shorter time scales, such as during frontal passages when large wind changes are taking place in less than five hours, the relative value of this term is often significant. Since the isallobaric effect produces a vector correction perpendicular to the first estimate of the wind vector, the wind direction is affected more than is the wind speed. Data from the 1972 AIDJEX period showed some instances of short-period wind changes which produced corrections on the order of 15° in direction and 5% in speed, and one extreme case in which the directional change was 31° and the magnitude was increased by 16% due to variation occurring in a three-hour period. Thus this term, though not usually important, may be significant for short-period variations and can be included if the temporal sensitivity of the numerical ice model merits it.

**DISCUSSION**

In general, it appears that the geostrophic wind as determined from surface pressure gradients is a good representation of the freestream flow immediately above the boundary layer. Errors become significant only when changes are taking place that are local (less than 200 km) or short-period (less than four hours). On these scales, data accuracy is
questionable; therefore, these corrections have not been routinely incorporated into the program. If information on these small scales is desired, corrections to the surface geostrophic wind will be made.

Several calculations of geostrophic wind speed at the Jumpsuit location have been plotted in Figures 23 and 24 together with the surface wind speed. The basin winds on a 200 km grid that have not been smoothed beyond the Cressman scheme correspond well to surface wind speeds, with a speed ratio of about 1.5. A local geostrophic wind calculated (to ±0.2 mb) from pressure measurements at the manned stations is also shown. The error in $G$ due to instrument error is about 10%-20% at this short separation distance. The consistently high values for the local relative to the basin calculations prompted the expanded area calculations on a 50 km grid. This expansion did result in higher values, in agreement with the locally calculated magnitudes; but their correspondence with surface winds was significantly less than the basin calculations.

SUMMARY

The 0000Z-1200Z surface pressure maps from the Weather Service have been used as initial input to a basic calculation of the Arctic Basin pressure field on a 200 km grid. The data are processed by computer to interpolate data, remove erroneous data, and smooth the pressure field.

The corrected maps are qualitatively similar to NWS pressure fields, but they usually show significantly modified gradients. In particular, when a perturbation pressure (e.g., a front) is traversing the ocean, the corrected maps delineate its history more accurately. Since wind fields depend on pressure gradients, the derived wind fields are likely to be much better with local corrections. In addition to the implied surface wind direction (approximately parallel to the isobars) and magnitude (proportional to isobar density), the large-scale pressure gradients, with a persistent north-south orientation, are evident on these maps.

The surface geostrophic wind shows excellent correlation with the surface wind observed during the experiment. This lends credence to the use
of geostrophic drag coefficients. The correlation existed despite the continual presence of a strong inversion. This somewhat allays the fear that the inversion would decouple the surface wind from the geostrophic wind.

The pressure maps for any period during the 1972 AIDJEX pilot study can be calculated upon request to the AIDJEX office.

REFERENCES


Fig. 1. Map of the Alaskan quadrant, showing the approximate locations of AIDJEX manned stations and buoys during 1972 pilot study.
Fig. 2. Surface isobars for the Beaufort Sea quadrant, Julian day 78. The last two digits of the pressure in millibars are shown. The axes are labeled in arbitrary grid units equal to 200 km per increment.
Fig. 3. Surface isobars for the Beaufort Sea quadrant, Julian day 79. The last two digits of the pressure in millibars are shown. The axes are labeled in arbitrary grid units equal to 200 km per increment.
Fig. 4. Surface isobars for the Beaufort Sea quadrant, Julian day 80. The last two digits of the pressure in millibars are shown. The axes are labeled in arbitrary grid units equal to 200 km per increment.
Fig. 5. Surface isobars for the Beaufort Sea quadrant, Julian day 81. The last two digits of the pressure in millibars are shown. The axes are labeled in arbitrary grid units equal to 200 km per increment.
Fig. 6. Surface isobars for the Beaufort Sea quadrant, Julian day 82. The last two digits of the pressure in millibars are shown. The axes are labeled in arbitrary grid units equal to 200 km per increment.
Fig. 7. Surface isobars for the Beaufort Sea quadrant, Julian day 83. The last two digits of the pressure in millibars are shown. The aces are labeled in arbitrary grid units equal to 200 km per increment.
Fig. 8. Surface isobars for the Beaufort Sea quadrant, Julian day 84. The last two digits of the pressure in millibars are shown. The axes are labeled in arbitrary grid units equal to 200 km per increment.
Fig. 9. Surface isobars for the Beaufort Sea quadrant, Julian day 85. The last two digits of the pressure in millibars are shown. The axes are labeled in arbitrary grid units equal to 200 km per increment.
Fig. 10. Surface isobars for the Beaufort Sea quadrant, Julian day 86. The last two digits of the pressure in millibars are shown. The axes are labeled in arbitrary grid units equal to 200 km per increment.
Fig. 11. Surface isobars for the Beaufort Sea quadrant, Julian day 87. The last two digits of the pressure in millibars are shown. The axes are labeled in arbitrary grid units equal to 200 km per increment.
Fig. 12. Surface isobars for the Beaufort Sea quadrant, Julian day 88. The last two digits of the pressure in millibars are shown. The axes are labeled in arbitrary grid units equal to 200 km per increment.
Fig. 13. Surface isobars (at 3 mb intervals) for the Beaufort Sea quadrant, Julian day 102. The last two digits of the pressure in millibars are shown. The axes are labeled in arbitrary grid units equal to 200 km per increment. The following code indicates manned stations and buoys whose pressures are used: Jumpsuit - 0; Blue Dog - 1; Brass Monkey - 2; buoys - 3 to 8.
Fig. 14. Surface isobars (at 3 mb intervals) for the Beaufort Sea quadrant, Julian day 103. The last two digits of the pressure in millibars are shown. The axes are labeled in arbitrary grid units equal to 200 km per increment. The following code indicates manned stations and buoys whose pressures are used: Jumpsuit - 0; Blue Dog - 1; Brass Monkey - 2; buoys - 3 to 8.
Fig. 15. Surface isobars (at 3 mb intervals) for the Beaufort Sea quadrant, Julian day 104. The last two digits of the pressure in millibars are shown. The axes are labeled in arbitrary grid units equal to 200 km per increment. The following code indicates manned stations and buoys whose pressures are used: Jumpsuit – 0; Blue Dog – 1; Brass Monkey – 2; buoys – 3 to 8.
Fig. 16. Surface isobars (at 3 mb intervals) for the Beaufort Sea quadrant, Julian day 105. The last two digits of the pressure in millibars are shown. The axes are labeled in arbitrary grid units equal to 200 km per increment. The following code indicates manned stations and buoys whose pressures are used: Jumpsuit - 0; Blue Dog - 1; Brass Monkey - 2; buoys - 3 to 8.
Fig. 17. Surface isobars (at 3 mb intervals) for the Beaufort Sea quadrant, Julian day 106. The last two digits of the pressure in millibars are shown. The axes are labeled in arbitrary grid units equal to 200 km per increment. The following code indicates manned stations and buoys whose pressures are used: Jumpsuit – 0; Blue Dog – 1; Brass Monkey – 2; buoys – 3 to 8.
Fig. 18. Surface isobars (at 3 mb intervals) for the Beaufort Sea quadrant, Julian day 107. The last two digits of the pressure in millibars are shown. The axes are labeled in arbitrary grid units equal to 200 km per increment. The following code indicates manned stations and buoys whose pressures are used: Jumpsuit - 0; Blue Dog - 1; Brass Monkey - 2; buoys - 3 to 8.
Fig. 19. Surface isobars (at 3 mb intervals) for the Beaufort Sea quadrant, Julian day 108. The last two digits of the pressure in millibars are shown. The axes are labeled in arbitrary grid units equal to 200 km per increment. The following code indicates manned stations and buoys whose pressures are used: Jumpsuit - 0; Blue Dog - 1; Brass Monkey - 2; buoys - 3 to 8.
Fig. 20. Surface isobars (at 3 mb intervals) for the Beaufort Sea quadrant, Julian day 109. The last two digits of the pressure in millibars are shown. The axes are labeled in arbitrary grid units equal to 200 km per increment. The following code indicates manned stations and buoys whose pressures are used: Jumpsuit - 0; Blue Dog - 1; Brass Monkey - 2; buoys - 3 to 8.
Fig. 21. Surface isobars for the Beaufort Sea quadrant, Julian day 80. The last two digits of the pressure in millibars are shown. The maps represent (a) the NWS initial field, (b) the 200 km grid corrected field, and (c) an expanded field.
Fig. 22. Surface isobars for the Beaufort Sea quadrant, Julian day 102. The last two digits of the pressure in millibars are shown. The maps represent (a) the NWS initial field, (b) the 200 km grid corrected field, and (c) an expanded field.
Fig. 23. Measured surface winds and calculated geostrophic wind speed at Jumpsuit for Julian days 78-88. Geostrophic winds calculated using a basin grid of 200 km, a reduced grid of 50 km, and local gradients from manned camps.
Fig. 24. Measured surface winds and calculated geostrophic wind speed at Jumpsuit for Julian days 102-109. Geostrophic winds calculated using a basin grid of 200 km and a reduced grid of 100 km.