AIDJEX BULLETIN

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Front cover: Onan, his tracks and shadow, at Blue Fox. Photo by Bill Meyers.

Back cover: The AIDJEX model undergoing trials. In this run, which was only partially successful, the driving forces were seriously overestimated, but the strain is clearly visible. Photo by Bill Myers.
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The AIDJEX Bulletin aims to provide both a forum for discussing AIDJEX problems and a source of information pertinent to all AIDJEX participants. Issues—numbered, dated, and sometimes subtitled—contain technical material closely related to AIDJEX, informal reports on theoretical and field work, translations of relevant scientific reports, and discussions of interim AIDJEX results.

The first article in Bulletin 33 appeared in December 1975 as Technical Report No. 36 from the Beaufort Sea Project, Department of the Environment, Canada. We thank Alan Milne and Doris Aanhout from that project, not only for allowing us to print the report here, but also for lending figure plates and assisting in other details.

Every Bulletin has its small rewards for the editor, and this Bulletin has three: the vividness of Peter Wadhams's description, on page 46, of a strip of oiled ice as "sinuous and Gordian"; the charm of John Nye's examples of patterns, on page 119; and the straightforwardness of Eric Becker's concluding remarks (page 147) on the value of the finite element method. Now can anyone tell us who is the beauty of Kashmir in the line "The beauty of Kashmir lay drooping his head"?

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SEA ICE TOPOGRAPHY IN THE BEAUFORT SEA
AND ITS EFFECT ON OIL CONTAINMENT

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ABSTRACT

The topography of the Beaufort Sea ice cover has been examined from airborne laser profiles obtained in September and October 1974 by the Atmospheric Environment Service, Environment Canada, and in April 1975 by the Canadian Maritime Command, Department of National Defence. Mean ridge heights and spacings were deduced for the elements of a grid covering much of the Beaufort Sea. In summer the mean ridge height increased linearly with the ridge frequency. For higher ridges the distributions of ridge heights in both seasons followed an identical empirical law of form $P(h) = A \exp(-Bh)$. This law was used together with ice drift information to predict extreme values of ridge height for different time intervals and spatial areas. Tentative predictions of extreme keel draft were made using reasonable factors for freeboard to draft conversion, and compared with depths at which scouring is found on the Beaufort Sea Shelf. A longitudinal profile of a shear ridge obtained in 1972 by an unmanned Arctic research submersible (UARS) of the University of Washington has been analyzed in an attempt to predict the minimum and maximum depths to be expected in a given keel linkage of known mean depth. On the basis of these and other studies of the Beaufort Sea Project, a discussion is given of the extent to which sea ice deformation features may govern the long-term spread of oil under ice.

1. INTRODUCTION

This report attempts to give a quantitative description of pressure ridge and keel distributions in the Beaufort Sea. The distributions are used as a basis for speculation on how the spread of oil after an under-ice blowout may be hindered (or helped) by the ice topography.
A general description of the ice cover in the Beaufort Sea has been given by Kovacs and Mellor [1974]. It suffices here to note that the winter ice cover consists of three zones:

**Fast ice zone.** A continuous sheet of normally smooth ice, stretching from the shore to anchoring points on grounded pressure ridges or ice island fragments. Its outer edge generally coincides with the 18-20 m depth contour, and its outer portions may include heavy ridging or rubble fields generated by early winter storms and subsequently "frozen in place." Detailed descriptions of this zone are given by Cooper [1974] and Stringer [1974].

**Shear zone,** or seasonal ice zone, found from the seaward edge of the fast ice to roughly the edge of the continental shelf. It is a zone of rapidly deforming, heavily ridged, and highly irregular ice acting as a boundary between the circulating ice of the Beaufort Gyre and the fast ice. First-year ice predominates but with some multiyear floes and ice island fragments; this has been termed the "offshore province" by Weeks et al. [1971].

**Polar pack ice zone,** extending onward into the Arctic Basin, is in winter composed of multiyear floes with first-year ice growing and sandwiched between them. Its long-term average motion is a clockwise gyral circulation, but on a time scale of days the motion is very complex and irregular, being governed by the wind stress field.

In summer the fast ice breaks up and disperses, and an open water zone may extend up to 200 km from the coast, although it is always subject to closure when storms drive the polar pack southwards. The former shear zone is now a marginal ice zone for the polar pack and opens up considerably with many leads and polynyi.

2. **RESULTS OF AES LASER PROFILING, SUMMER 1974**

2.1 **Technique**

Laser profiles were obtained in the Beaufort Sea during the period 4 September to 7 October 1974, by Lockheed Electra aircraft CF-NAY and CF-NAZ of the Atmospheric Environment Service (AES), Environment Canada. The instrument employed was a Spectra-Physics Geodolite 3A laser profilometer [Ketchum, 1971], in which the phase shift between amplitude-modulated beams from a CW He-Ne laser in transmission and reflection gives the range to the reflecting target. A continuous terrain profile is obtained, which was displayed as a galvanometer light trace on ultraviolet sensitive paper. Such a recording system is not a good one from the point of view of quantitative analysis, but it permits rapid assessment of ice roughness and is therefore relevant to the operational needs of AES. The chart record was calibrated in feet, full scale deflection corresponding to 100 feet of elevation. An example of a raw profile is shown in Figure 1, the long-period amplitude changes being due to vertical motion of the aircraft.

The records were analyzed manually in the following way. A profile of aircraft porpoising, connecting smooth ice surfaces in a continuous curve, was
drawn on to the record by hand, and the elevation of each ridge was measured relative to this profile. Ridges were assigned to 0.5 ft. height intervals, a cut-off height of 3 feet being chosen as the minimum ridge height which can always be easily resolved against the background roughness. A broad pressure ridge, especially if traversed obliquely, may display several peaks and a consistent criterion for defining an "independent ridge" is necessary to avoid a serious overestimate of ridge frequency.

A criterion was devised by this author in 1972 whereby a ridge is said to be "independent" when its maximum elevation is not less than twice that of the shallowest troughs on either side of it. This is an adaptation of the Rayleigh criterion in optics, and has been employed in the analyses of under-ice sonar profiles by Williams et al. [1975] and of Arctic Ocean laser profiles by Lowry [1974]. It is to be hoped that this or a similar criterion can be universally adopted; this would greatly facilitate the comparison of data obtained by various authors [see also Westhall and Li, 1976].

Navigational fixes were obtained from a Bendix doppler navigation system coupled to a MINAC 5 latitude/longitude computer [Archibald, 1972], and were recorded in the flight log every 2-5 minutes. From timing marks on the profilometer record it was then possible to identify the position of any section of record to an accuracy of approximately ±3 km. The Beaufort Sea area was divided into a grid of spacing 0°30' in latitude and 2°00' in longitude and the results of analyzing all sections of profile that fell within a given grid division were pooled. The grid is a fairly coarse one, but it was necessary to obtain a sufficient track length in each grid division for a valid statistical analysis.

Figure 2 shows the track sections that were analyzed, together with the intensity of coverage (number of kilometers of track in each grid division). In all, 2028 km of laser profile were analyzed; in addition, several hundred kilometers of inshore flight data were examined but found to consist mainly of open water.

![Fig. 1. Laser profile of Arctic sea ice.](image-url)
Fig. 2. AES flights, summer 1974: (a) flight tracks; (b) key to grid divisions and kilometers of laser profile analyzed in each.
2.2 Open Water

It was possible to identify stretches of open water and young ice (ice only a few centimeters thick) because the output of a precision radiation thermometer (PRT) was displayed on the same chart record as the laser profile. A PRT measures the emission from a spot of 1° arc directly beneath the aircraft. The radiation temperature of open water was commonly 1°C above that of young ice and 2°C above that of the thicker first- and multiyear ice. This was a considerable advantage because from the laser trace the profile of waves from a wide polynya can be easily confused with the rough profile of a multiyear floe.

Figure 3 shows the distribution of "open water" (defined as true open water plus young ice less than 1°C cooler than open water) over the area surveyed. The figure actually shows the percentage of track length along which open water was encountered, but if the leads and polynyas are assumed to have random orientations [Mock et al., 1972] this is an unbiased estimate of the percentage area of open water. Since the results of flights made over a five-week period were pooled, Figure 3 is not an instantaneous picture of the extent of the ice cover, but rather it is a smoothed composite of late summer conditions. The values for the southern Beaufort Sea are therefore not very meaningful, because the percentage of open water in any grid division could fluctuate greatly as the ice margin made excursions in response to the wind. However, the values for grid divisions farther from shore, and particularly along the west coast of Banks Island, are of considerable interest. This represents the seasonal pack ice zone, the summer equivalent of the shear zone. In Figure 3, 13 grid divisions have been assigned, rather arbitrarily, to this seasonal zone, and it can be seen that, with the exception of two divisions to the north of Alaska where the ice presses in toward the land, the percentage of open water in the zone is anywhere from 5 to 17. The mean figure for the 13 divisions is \(8.7 \pm 1.4\)% , or, if the two Alaskan areas are excluded, \(10.2 \pm 1.3\)% . In contrast, Wittmann and Schule [1966] found less than 5% open water in winter, showing that the seasonal ice zone opens up considerably during the summer.

2.3 Frequency of Ridges

Figures 4 and 5 show the mean ridge frequency in each grid division expressed as ridges encountered per 100 km (Fig. 4) of total laser track (Fig. 5). In each division the large number expresses the ridges per 100 km, while the small number \(\varepsilon_1\) is the probable range of accuracy, calculated from the approximate formula

\[
\varepsilon_1 = \sqrt{n} \times \frac{100}{s}
\]

(1)

where \(\varepsilon_1\) = standard error for number per 100 km, 
\(n\) = number of ridges counted along track, 
\(s\) = length of track in kilometers.
Thus, from the point of view of oil spilled under a large floe, Figure 4 shows the frequency with which ridges will be encountered within the boundaries of that floe before any leads are reached, while Figure 5 gives the larger-scale picture of ridge frequency over an average line segment of pack ice which includes leads and polynyi. Where the ice concentration is almost 100% the two figures are obviously almost the same. We find a very wide range of frequencies, but once again a clear pattern emerges. Over an area of the eastern Beaufort Sea roughly corresponding to the "seasonal zone" of Figure 3, the ridge frequency is high, varying from 6 to more than 12 per km of ice cover. Over the southeastern Beaufort Sea where the percentage of open water is high, the ridge frequency is low, about 2-3 per km even when corrected for open water, while in the southwestern Beaufort Sea the frequency is similarly low despite the high concentration of ice cover.

It is instructive to compare these results with those of Hibler et al. [1974], where a table of ridging intensities is presented for various parts of the Arctic Ocean and times of year. Five of the regions considered by Hibler et al. fall within the Beaufort Sea area, and these have been marked on Figures 4 and 5 with the identification numbers used by Hibler. The ridge frequencies per km (presumably of total track) are shown in Table 1. We expect these figures to be lower than our own because the cutoff height was 4 feet rather than 3. Regions 11 and 12 lie within the polar pack of the Beaufort Gyre, outside our area of measurement, and show a variable but generally low ridging intensity, the solitary summer measurement being
Fig. 4. AES, summer 1974. Ridges per 100 km ice cover; circled numbers identify areas profiled by Hibler et al. [1974].

Fig. 5. AES, summer 1974. Ridges per 100 km of profiling track.
lower still. Region 16 corresponds to our grid division T, and the
frequencies are in good agreement, suggesting that winter ridge frequencies
can be estimated by taking the summer values for ice cover only (i.e., in
winter the pack closes up but does not increase radically in ridging
intensity).

The same data were analyzed independently by Tucker and Westhall [1973],
using as a criterion for an independent ridge that the troughs on either
side should be at least 2 feet lower than the peak. This is the same
criterion as Hibler's, although ridge frequencies differed by up to 20%
from those given in Table 1. From their data Tucker and Westhall were
also able to construct approximate contours of ridging intensity over
the Arctic Ocean for different seasons. They show that the ridge
frequency is lowest in the southern Beaufort Sea (generally less than
2 per km for ridges higher than 4 feet) and that there is not signifi-
cant variation in this area with season. The highest frequency of all
(8 or more per km) is found in the shear zone along the northwest border
of the Canadian Archipelago, Ellesmere Island, and north Greenland, and
here the frequency is significantly greater in winter. The increase in
frequency begins at the latitude of northern Banks Island, which is in
agreement with the AES results.

2.4 Mean Height of Ridges

We now consider ridge heights. Figure 6 shows the mean ridge height in
each grid division in meters, relative to a cutoff height of 0.91 m (3 ft).
Again, the small numbers ε2 are the standard errors in each estimate, calcu-
lated by assuming a possible error of ±0.5 ft (0.152 m) in an individual
ridge height measurement. We find a wide range of mean elevations, from

<table>
<thead>
<tr>
<th>Region</th>
<th>12-18 Jan</th>
<th>18-23 Mar</th>
<th>4-6 Oct</th>
<th>12-27 Mar</th>
<th>6-9 Feb</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. 11</td>
<td>1.2</td>
<td>5.4</td>
<td>1.6</td>
<td>1.4</td>
<td></td>
</tr>
<tr>
<td>No. 12</td>
<td>1.4</td>
<td>2.2</td>
<td>1.0</td>
<td>1.3</td>
<td>2.0</td>
</tr>
<tr>
<td>No. 16</td>
<td>0.9</td>
<td>7.6</td>
<td>7.8</td>
<td>3.3</td>
<td></td>
</tr>
</tbody>
</table>

Note: cut-off height is 4 ft.
Fig. 6. AES, summer 1974. Mean ridge height in meters with standard error.

less than 1.2 m to almost 1.7 m, but with a concentration between 1.3 and 1.5 m, the higher values being generally found in the northernmost divisions.

Gonin [1960] found a linear positive correlation between average hummock height (for hummocks less than 0.6 m high, measured from stereo air photographs) and percentage cover of deformed ice. Hibler et al. [1972] found a similar correlation between mean keel depth and number of keels per km for under-ice sonar profiles.

In Figure 7 the mean ridge height is plotted against the number of ridges per km of ice cover. The results for the northern group of grid divisions (north of 71°30') appear to fit a linear positive correlation, and a large number of the southern divisions appear to lie on the same regression line. However, a group of seven southern divisions (J, N, R, S, T, V, and Z) have anomalously large mean heights. These all lie in the southeastern Beaufort Sea in a region of low percentage ice cover, and this suggests a possible reason for the anomaly. The ice in this region is very broken and scattered, but in origin it is a mixture of broken-out fast ice and ice from the first and most heavily ridged part of the shear zone. The very edge of the winter shear zone [Kovacs and Mellor, 1974] consists of a narrow band of exceptionally heavy ridging with a very large mean ridge height [Klimovitch,
Fig. 7. Mean ridge height plotted against ridge frequency. Control limits are one standard deviation from regression line.

1972]. A mixture of two such ice types would have a moderate ridge frequency (the diluting effect of the smooth fast ice) coupled with an anomalously high mean ridge height (from the shear zone ice). Such ice conditions are therefore atypical of the Beaufort Sea in general.

Leaving the anomalous values aside, the correlation coefficient between \( \bar{h} \) and \( N \) is 0.86, which is significant at the 0.1% level on an analysis of variance test. It is physically reasonable to expect the stresses which generate a high concentration of ridges to also generate ridges with a greater mean elevation. A linear regression of \( \bar{h} \) (mean ridge height in meters) on \( N \) (number of ridges per km of ice cover) yields

\[
\bar{h} = 0.0201N + 1.201
\]  

The standard error of estimate is 0.040 m, and in Figure 7 the regression line has been drawn in together with control limits placed one standard error of estimate on either side. It is thus feasible to estimate \( \bar{h} \) for summer ridges in the Beaufort Sea when given \( N \), which itself can be measured by cheaper means than laser profiling (e.g., analysis of aerial photographs). From these present results the order of accuracy would not be great—an estimate would have a 95% probability of being within ±0.08 m of the value given by the regression line— but it is reasonable to hope that with more data the control limits can be narrowed. This will greatly ease the routine gathering of sea ice morphology data in the future.
2.5 Distribution of Ridge Heights

Table 2 is a complete record of the ridges that were counted in each grid division. It is clear that the manual technique of measuring heights from a chart record introduces a bias to the results since the overall totals for integral numbers of feet greatly exceed those for half-foot intervals. The graticule used to estimate the heights was ruled in feet and a gestalt bias creeps in whereby the eye tends to take the ridge to the nearest whole number of feet instead of ascribing it to a half-foot interval. If the bias is a constant factor it means that, say, the 4 ft height class, instead of consisting of ridges between 3.75 ft and 4.25 ft, actually consists of ridges between (3.75 - δ) ft and (4.25 + δ) ft, where δ is a small constant of magnitude about 0.05 ft. Similarly, the 4.5 ft class consists of ridges only in the range (4.25 + δ) ft to (4.75 - δ) ft. If hand analysis is to be used at all, it is clearly necessary for a single analyst to carry out all the measurements, so that the personal bias is fairly uniform throughout the data.

Without knowing the shape of the distribution in advance there is no valid way of redistributing the data even if a value for δ could be estimated. Therefore, in attempting to compare the data with theoretical distributions we have two approaches: the "whole-foot" and "half-foot" intervals can be treated as two separate populations; or the data from two adjacent intervals can be pooled, producing a histogram with foot-wide height intervals. The second approach is easier, but has the disadvantage that the width of the interval becomes quite large in relation to the rapid decrease of ridge frequencies.

The only theoretical ridge height distribution proposed so far is that of Hibler et al. [1972]:

\[ P(h) \, dh = 2\lambda h e^{\lambda h^2} e^{-\lambda h^2} \, dh \]  \hspace{1cm} (3)

where \( P(h) \) is the probability density function for ridge height; 
\( \bar{h} \) is the mean ridge height, estimated from the data; 
\( h_0 \) is the low-value height cutoff; 
\( \lambda \) is a parameter which must be derived by iteration from the relationship

\[ \exp (-\lambda h_0^2) = \bar{h}(\lambda \pi)^{1/2} \text{ erfc} (\frac{1}{2} h_0) \]  \hspace{1cm} (4)

It should be noted that (3) is not an analytical expression, since \( \bar{h} \) should be the mean of the distribution whereas in fact it is a sample mean. Hibler et al. found that this distribution offered a good fit to large samples of ridge and keel data from various parts of the Arctic Basin.

Figure 8 shows the results of applying this distribution to our data. In (a) the overall totals are plotted in foot-wide height intervals (except for the 3 ft height class, which is kept separate) and the values
## TABLE 2

**DISTRIBUTION OF RIDGE HEIGTHS IN AES SUMMER DATA**

<table>
<thead>
<tr>
<th>SW Corner of Grid</th>
<th>3</th>
<th>3%</th>
<th>4%</th>
<th>5%</th>
<th>6%</th>
<th>7%</th>
<th>8%</th>
<th>9%</th>
<th>10%</th>
<th>11%</th>
<th>12%</th>
<th>13%</th>
<th>14%</th>
<th>15%</th>
<th>16%</th>
<th>17%</th>
<th>18%</th>
<th>19%</th>
<th>20%</th>
<th>21 TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>44° N 136° W</td>
<td>8</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
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<td>1</td>
<td>1</td>
</tr>
<tr>
<td>70° N 136° W</td>
<td>10</td>
<td>6</td>
<td>20</td>
<td>13</td>
<td>17</td>
<td>6</td>
<td>12</td>
<td>5</td>
<td>7</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>3</td>
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<td>1070</td>
</tr>
<tr>
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<td>24</td>
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<td>22</td>
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<td>20</td>
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<td>10</td>
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<td>6</td>
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<tr>
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</tr>
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<td>11</td>
<td>21</td>
<td>15</td>
<td>21</td>
<td>2</td>
<td>13</td>
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<td>2</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>70° N 137° W</td>
<td>72</td>
<td>26</td>
<td>33</td>
<td>17</td>
<td>25</td>
<td>5</td>
<td>8</td>
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<td>129</td>
<td>151</td>
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predicted from (3) are shown in dotted lines. In this case \( h_0 \) was taken as 2.75 ft, i.e., \( \delta \) was assumed to be zero, and the predictions were obtained by calculating

\[
\int_{h_1}^{h_2} P(h) \, dh
\]

where, for instance, the 3.5-4 ft pooled class would have limits \( h_1 = 3.25 \) ft, \( h_2 = 4.25 \) ft. The fit is good only in the moderate height classes. We might expect a poor fit for the 3 ft class, since these ridges are difficult to resolve against the background roughness, but in fact the fit is poorest in the highest classes. This is made more clear in part A of Table 3, where experimental and theoretical values are compared directly. Here we have tested various values of \( \delta \), in each case taking \( h_0 \) as \( (2.75 - \delta) \) ft, \( h_1 \) as \( (n + 0.25 + \delta) \) ft, and \( h_2 \) as \( (n + 1.25 + \delta) \) ft. When \( \delta = 0.1 \) ft, the fit is generally improved up to a ridge height of 8 ft, but above this height all the predictions are similar and far too low. None of the predictions offers a significant likelihood of a ridge higher than 13 ft, whereas in fact the heights go up to 21 ft (for which theory predicts only \( 10^{-6} \) ridges!). It is apparent that the occurrence of high ridges in the coastal zone of the Beaufort Sea far exceeds the predictions of the Hibler theory. Referring to Table 2, we find that of the 14 ridges observed with a height of 16 ft or more, no fewer than 12 occur north of 71°30'N, so that the area west of Banks Island is the chief source of these very high ridges. The highest free-floating ridge yet observed in the Arctic Ocean, with an elevation of 12.8 m (39 ft), occurred slightly west of this area, at 74°N, 130°W [Kovacs et al., 1973].

In (b) and (c) of Figure 8 we consider the whole-foot and half-foot classes, respectively, as separate populations; in evaluating the theoretical distribution the integral in (5) is carried over a one-foot interval centred on the height class in question. Again we find a very poor fit with theory for the higher ridges. The objection may be raised that the overall ridge counts include ice from different areas with widely varying characteristics, so that a theory which holds true for a homogeneous profile may not hold true for the sum of many profiles. To test this, (d) and (e) of Figure 8 compare theory and experiment for the two grid divisions with the highest ridge counts. Again the fit is poor, and in part B of Table 3 we find that introducing various values for \( \delta \) makes very little difference to the predictions for high ridges in the case of grid division E. A second possible objection is that our estimates of \( \bar{h} \) are too low; in calculating \( \bar{h} \) we consider each class as centred on its mean height value. For instance, all ridges in the 4 ft class are taken as being 4 ft high, whereas in fact there are a greater number between 3.75 and 4 ft than between 4 and 4.25 ft. However, the effect of reducing \( \bar{h} \) while keeping other parameters unchanged is to increase \( \lambda \), and from (3) this can be seen to produce an even more rapid fall-off in ridge frequency with increasing height. The theoretical predictions will thus become even worse at the high end of the distribution.
Fig. 8. Theoretical and observed ridge height distributions. (a) Overall results, summer 1974. (b) Results for height classes centred on whole numbers of feet. (c) "Half-foot" categories. (d) Grid division P. (e) Grid division E.
TABLE 3

COMPARISON OF OBSERVED RIDGE COUNTS WITH THEORY OF HIBLER et al. [1972], WITH VARIOUS VALUES FOR BIAS FACTOR $\delta$.

A. CUMULATIVE AES DATA FOR ALL GRID DIVISIONS

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B. AES DATA FOR ONLY GRID DIVISION E (SW corner: 73°30'N, 128°W)

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Hibler et al. derived their distribution by making two assumptions: first, that all ridges are similar in geometrical cross section (e.g., if the cross section is idealized as an isosceles triangle, the angle of the triangle will be the same for all ridges); second, that all possible ridge height arrangements that yield the same net deformation of the ice cover are equally likely. The second assumption allows variational techniques to be used, as in statistical mechanics, to derive the ridge height distribution; of the ensemble of all possible icefields with a given quantity of deformed ice, one ridge height distribution can be shown to occur in the maximum number of possible ways, and this is the distribution that will be found in nature. Now Hibler's first assumption can be questioned because the slope angles of ridges are in fact quite variable, averaging about 19° for multiyear ridges [Kovacs et al., 1973] and 24° for first-year ridges [Kovacs, 1972]. Let us suppose that ridges tend towards the same mean angle regardless of height, but that the angles are distributed narrowly about this mean. The variational technique is still approximately valid, yielding the same distribution as before. But our new assumption will then cause this distribution to become "flattened out." In other words, a given height class will spill its shallower ridges down into lower height classes and its steeper ridges up into higher classes. Since the distribution is monotonically decreasing, the result will be that the very highest classes receive more ridges from the lower classes than they lose to them. This explains the greater prevalence of very high ridges combined with a generally good fit to Hibler's distribution in the more moderate size classes. Thus distribution (3) cannot be used as a prediction system for high ridges.

3. RESULTS OF DND FLIGHTS, APRIL 1975

3.1 Technique

Two flights over the Beaufort Sea were made in April 1975 by a Canadair Argus aircraft of Maritime Proving and Evaluation Unit, Department of National Defence, CFB Summerside, PEI. Both flights began and ended at Inuvik. The first, on 20 April, followed the 135th meridian up to 76°30'N and then ran westward as far as the AIDJEX camp at about 145°W, returning by the same route. The second, on 26 April, followed the same route northward, but returned along the 139th meridian and carried out an inshore survey before returning to Inuvik. Laser profiles were obtained throughout the first flight using a Geodolite profilometer, but this failed to operate during the second flight. In addition, side-looking airborne radar (SLAR) imagery was taken along the flight lines and in low-level passes over the AIDJEX camp in support of a separate project carried out by R.O. Ramseier. Figure 9 shows the track of the laser profiling flight of 20 April.

The laser terrain profile was recorded in analogue form on two magnetic tape systems—a 4-track recorder and a 14-track FM recorder—and the output of the profilometer was also monitored on an oscilloscope. This system permits a full quantitative analysis of the data to be carried out. The first step was digitization of the record on a 14-bit analog-to-digital converter at 240 Hz using a 0-120 Hz low pass analogue filter to remove noise down to the Nyquist frequency. A high digitization rate (corresponding to 3 points per meter of profile) was necessary to resolve properly the structure of each ridge, especially across the peak, so that the true
height is recorded. The second step is removal of aircraft altitude variation, noise spikes, and 360° phase shifts (where the profilometer trace returns to zero at the beginning of a new 100 ft interval). This is best done using a computer technique developed at the Defence Research Establishment, Ottawa [Brochu and Lowry, 1975; Lowry and Brochu, 1975]. The profile is displayed in sections of approximately 2 km on an interactive terminal, where spikes and phase shifts are removed manually. The operator then draws in a profile of aircraft motion by joining adjacent minima with straight line sections. The piecewise straight line profile is filtered to generate a smooth curve of aircraft motion, which is subtracted from the raw profile. The "clean" profile of pure terrain topography is returned to store and is then ready for computer analysis of ridge height and spacing distributions.

Because of time restrictions, we present here only a preliminary analysis which was carried out by plotting the analogue record on a high-speed chart recorder and analysing by hand in the same way as for the 1974 AES profiles.
3.2 Results of Laser Profiles

The area covered by the laser profile was analyzed in terms of a grid system similar to that used for the AES profiles. Figure 9 shows the arrangement of grid divisions; up to 73°N the size is 0°30' of latitude by 2° of longitude, while farther north where conditions change more slowly a coarser grid of 1° by 2° or 0°30' by 4° is used. Figure 9 also shows the number of km of record analyzed in each division; the total track length treated was 889 km. The Argus is equipped with an inertial navigation system, and position fixing on the laser record was of similar accuracy to the AES profiles.

Ridge heights were assigned to categories of width 1.28 feet (0.39 m), corresponding to the scale graticule on the chart recorder. The noise level on the record was higher than for the AES records, and it was found that reliable ridge counts could be made only above a cut-off of 3 scale divisions (3.84 feet, 1.17 m). Assuming absence of bias in the analysis, the lowest height category therefore contains ridges with a minimum height of 2.5 divisions (3.2 feet, 0.98 m) and this is then $h_0$ as defined in (3). The higher cut-off means that ridge frequencies can be compared directly with the AES results only if the 3 ft. category in the AES data is ignored. Under these circumstances the AES cut-off is 3.25 feet (assuming $\delta = 0$), or 0.99 m, so the two sets of data are directly comparable, with almost identical $h_0$ values of approximately one meter.

Figure 10 shows the mean ridge frequency in each grid division expressed as ridges per 100 km of track with a standard error as defined in (1). Except in the vicinity of the wide lead at the edge of the shorefast ice (see Section 3.3) the ice cover was virtually 100%, broken only by narrow leads, and so the ridge frequency is also per 100 km ice cover. In the south the ridge frequency is low, corresponding to the fast ice zone (to 70°N) and the shear zone, which does not appear to be heavily ridged at this longitude. Farther north the ridge frequency appears to increase slowly with latitude, an effect which is demonstrated in Figure 11.

In only three grid divisions is a direct comparison possible with the AES data, together with one area in the polar pack for which Hibler presents data. The comparison is given in Table 4.

There are no clear trends in these comparisons, mainly because the three southerly grid divisions are in the very variable coastal zone of the Beaufort Sea. The ice there in summer is a mixture of seasonal ice and broken-out fast ice, and in April consists of the outer fast ice zone and then the edge of the shear zone which has already begun to open up. Therefore, we can draw few conclusions except to note the low ridge frequency in April. The Hibler station shows a general constancy of ridge frequency except for an anomalously high value in March 1971. We note also from Figure 11 a general clustering of frequencies in the polar pack at 1.5 to 2.1 per km except for a single high value of 3.1. Those familiar with the polar pack in the Beaufort Gyre (e.g., Herbert, 1970) report occasional large fields of pressured ice, sometimes 30 km in diameter, with very heavy ridging. These are thought to be the result of pressuring of a large frozen polynya. Such a field would give
Fig. 10. DND April 1975. Ridges per 100 km profiling track.

Fig. 11. Variation of ridge frequency with latitude. Longitude 135°W unless otherwise stated.
Table 4: Ridge Data Available From More Than One Source for Beaufort Sea Regions

<table>
<thead>
<tr>
<th>Region</th>
<th>Hibler et al. [1974, Table 4]</th>
<th>AES, Summer 1974</th>
<th>DND, April 1975</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ridge per 100 km Flight Track</td>
<td>Ridges per 100 km Flight Track</td>
<td>Ridges per 100 km Flight Track</td>
</tr>
<tr>
<td></td>
<td>( h_0 = 1.22 \text{ m} )</td>
<td>Ridges per 100 km</td>
<td>Ice Cover</td>
</tr>
<tr>
<td></td>
<td>Flight Track</td>
<td>Ice Cover</td>
<td>( h )</td>
</tr>
<tr>
<td>Z (Fig. 2b)</td>
<td>--</td>
<td>31 ± 7</td>
<td>92 ± 20</td>
</tr>
<tr>
<td>U (Fig. 2b)</td>
<td>--</td>
<td>46 ± 5</td>
<td>105 ± 12</td>
</tr>
<tr>
<td>Q (Fig. 2b)</td>
<td>--</td>
<td>107 ± 13</td>
<td>118 ± 14</td>
</tr>
<tr>
<td>11 (Fig. 4)</td>
<td>129 (Jan 1971)</td>
<td>160 (Mar 1972)</td>
<td>140 (Feb 1973)</td>
</tr>
</tbody>
</table>

An anomalously high ridge frequency to the area in question so we suggest that for winter ice in the Beaufort Sea Gyre the range 1.5 to 2.1 per km represents the normal ridge frequency. This is very much lower than the summer values found in the shear zone off Banks Island.

Figure 12 shows the mean ridge height in meters relative to the 0.99 m cut-off, together with the error. If we plot mean ridge height against ridge frequency we obtain Figure 13, which is not a simple straight line as in Figure 7. Three regions can be distinguished clearly: the area south of 71°30'N, incorporating the shear zone and outer fast ice, which has a low ridge frequency and a low mean height; the polar pack as far north as 75°30' at 136°W, with a moderate ridge frequency and a large mean height; and the area north of 75°30' and west to 146°W, which has a lower mean height and a high ridge frequency. The last-named area corresponds roughly to the centre of the Beaufort Gyre, which has been located by Coachman [1969] at 80°N 140°W. The track at 136°W corresponds to the eastern part of the Beaufort Gyre, with a southward mean ice drift. Each region within itself shows a positive linear correlation between frequency and mean height, but the simple relationship found in the summer for the coastal Beaufort Sea does not occur in the winter polar pack.

The overall ridge height distribution is again quite different from the predictions of Hibler. Figure 14 compares theory and observation, and once again the observed frequency of high ridges is much greater than the prediction. In Section 4 we shall show that there is a remarkable agreement between the observed occurrences of high ridges in the AES and DND data, and that this can be used as the basis for an empirical prediction of extreme ridge heights.
Fig. 12. DND April 1975. Mean ridge height in meters, relative to 0.99 m cutoff.

3.3 **Open Leads**

During the flight of 26 April the author kept a count of open leads from the nose bubble of the aircraft during a 5½ hour period. Only leads which passed directly under the aircraft's flight track were counted, and an "open" lead was defined as one possessing a continuous strip of open water or of very recently formed ice. From the air open water appeared black and new ice various shades of grey, getting lighter as the ice grew thicker. Only the black of open water or the very dark grey of ice only a day or two old were counted. Leads seemed to fall into two classes: new leads, which were narrow cracks with clean edges snaking across the icefield and cutting level ice and pressure ridges alike; and old, wide leads which showed signs of frequent opening and closing and which were usually frozen over with thick ice except for a narrow active portion. As the aircraft flew south and entered the shear zone, the frequency of leads increased dramatically and the open leads themselves were wider, indicating a divergence in the stress on the pack. Finally a very wide lead (of estimated width 2 km) was crossed, marking
Fig. 13. Mean ridge height plotted against ridge frequency.

Fig. 14. Theoretical and observed ridge height distributions.
the boundary of the shorefast ice. The outer part of the shorefast ice showed occasional cracks, indicating the earliest stages of break-up, but the inner zone was a continuous sheet.

Figure 15 shows the track of the aircraft. After reaching Herschel Island, the aircraft flew a pattern over Mackenzie Bay and eastward to Cape Dalhousie. The wide lead was crossed several times and its path could be traced; it followed the approximate line of the 18-20 m depth contour, in agreement with the observations of Stringer [1974] for the north Alaskan coast. Figure 15 also shows the results of lead counts during successive 5-minute periods. Each of these counts was converted into a lead frequency per km using ground speed information from the aircraft flight log, and the results are shown in Figure 16. It can be seen that a low lead frequency in the central polar pack gives way quite suddenly to a high frequency in the seasonal zone. It appears that the moving pack in the seasonal zone was beginning to open out and retreat prior to the spring break-up of ice in the fast ice zone. The data in Figure 16 are summarized by the mean values in Table 5 and compared with data obtained by Wittmann [Weeks et al., 1971] in BIRDSEYE flights up to 1966. It can be seen that the lead frequency is about one order of magnitude less than the frequency of ridging.

TABLE 5
COMPARISON OF LEAD FREQUENCIES FROM BIRDSEYE FLIGHTS AND 1975 DND FLIGHT.

<table>
<thead>
<tr>
<th>Data Source</th>
<th>Mean Number of Leads per km</th>
<th>Standard Deviation Leads per km</th>
</tr>
</thead>
<tbody>
<tr>
<td>DND April 1975</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Polar pack</td>
<td>0.18</td>
<td>0.03</td>
</tr>
<tr>
<td>Shear zone</td>
<td>0.72</td>
<td>0.07</td>
</tr>
<tr>
<td>Outer fast ice</td>
<td>~0.1</td>
<td>-</td>
</tr>
<tr>
<td>Inner fast ice</td>
<td>nil</td>
<td>-</td>
</tr>
<tr>
<td>BIRDSEYE to 1966</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shear zone (winter)</td>
<td>0.41 in openings &gt; 30 m across</td>
<td></td>
</tr>
<tr>
<td>Seasonal zone (summer)</td>
<td>0.18 in openings &gt; 30 m across</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.72 in openings &lt; 30 m across</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 15. Track of DND flight, 26 April 1975, and numbers of open leads counted in 5-min. periods.

Fig. 16. Distribution of open leads, 26 April 1975.
4. PREDICTION OF MAXIMUM RIDGE HEIGHT

To assess the safety of sea bed operations on the shelf it is vitally important to know the maximum depth of keel that can be expected in a given linear stretch of track or a given area of icefield. In the absence of long underwater sonar profiles of the ice bottom to match the long laser profiles of the ice surface we are forced to approach the problem in two stages:

1) Estimation of maximum ridge height along tracks or in areas.
2) Estimation of a conversion factor to obtain keel depth from ridge height.

Stage 2 is dealt with in Section 5 of this report.

We have shown that the distribution (3) is invalid for the prediction of high ridges in the area of interest. Fortunately our data appear to fit a simpler relationship, which can be used at least as a rule of thumb for this purpose. For the AES data Figure 17a shows the pooled probability density per foot (i.e., number of ridges in two adjacent classes divided by total number counted) plotted on a log scale against the ridge height in feet. The results for different grid divisions all appear to lie on a single straight line; the divisions used each have enough ridges in them to test the relationship up to a height of 12 ft, and then the overall data permits the relationship to be tested up to 16 ft. The line of best fit has a gradient of about one decade per 5 ft ridge height, and passes almost exactly through the origin. There is a slight positive deviation from the line for the lowest (4 and 5 ft) classes, and there would be an even greater positive deviation for the 3 ft class if it were plotted, so the relationship is valid only for higher ridges.

In Figure 17b it is shown that the relationship holds for the DND data as well; in fact the pooled data from the AES and DND profiles fit an identical line. This is a remarkable result which suggests a general law governing ridge heights. To construct Figure 17b the pooled AES data were adjusted to a cut-off of 0.98 m and compared with the pooled DND data which have a 0.99 m cut-off.

A linear regression of \( \log P_f \) (where \( P_f \) is probability density per foot) on \( h_f \) (ridge height in feet) yields

\[
\log_{10} P_f = -0.2121 h_f + 0.372
\]  

with a standard error of estimate of 0.0572. The line of best fit together with control limits of two standard deviations are shown in Figure 17b. We convert now to meters and express (6) as a decaying exponential. Thus if

\[ P(h) \, dh = \text{probability that a ridge encountered at random will have a height in the range } h \text{ to } (h + dh) \text{ meters, given that its height is greater than one meter,} \]

then (6) becomes

\[
P(h) \, dh = 7.7269 \exp(-1.6026 h) \, dh
\]
Fig. 17(a). AES summer 1974. Log-lin plot of observed ridge height distributions.

Fig. 17(b). Log-lin plot of observed ridge height distributions for AES data, summer 1974 (open circles) and DND data, April 1975 (closed circles). Control limits two standard deviations from regression line.
If we now make the tentative assumption that the relationship which holds over the range of observation continues to hold for higher ridges, we are in a position to estimate maximum ridge heights. Equation 7 can be used to predict the probability densities of very high ridges, and we can also define a cumulative probability density function:

\[ P_c(h) = \int_h^\infty P(h) \, dh \tag{8} \]

\( P_c(h) \) is the probability that a ridge encountered at random will have a height of at least \( h \) meters. Equations (7) and (8) give

\[ P_c(h) = 0.6240 \, P(h) \tag{9} \]

Table 6 shows some probabilities for ridge heights up to 14 m. The standard error of estimate in equation (6) is equivalent to an error of \( \pm 14\% \) in \( P(h) \) and \( P_c(h) \).

**TABLE 6**

**PREDICTION TABLE FOR OCCURRENCE OF HIGH RIDGES**

<table>
<thead>
<tr>
<th>Ridge height (m)</th>
<th>( P(h) ) Probability density</th>
<th>( P_c(h) ) Probability that ridge height is not less than ( h )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>( 2.6 \times 10^{-3} , \text{m}^{-1} )</td>
<td>( 1.6 \times 10^{-3} )</td>
</tr>
<tr>
<td>6</td>
<td>( 5.2 \times 10^{-4} , \text{m}^{-1} )</td>
<td>( 3.2 \times 10^{-4} )</td>
</tr>
<tr>
<td>7</td>
<td>( 1.0 \times 10^{-4} , \text{m}^{-1} )</td>
<td>( 6.5 \times 10^{-5} )</td>
</tr>
<tr>
<td>8</td>
<td>( 2.1 \times 10^{-5} , \text{m}^{-1} )</td>
<td>( 1.3 \times 10^{-5} )</td>
</tr>
<tr>
<td>9</td>
<td>( 4.2 \times 10^{-6} , \text{m}^{-1} )</td>
<td>( 2.6 \times 10^{-6} )</td>
</tr>
<tr>
<td>10</td>
<td>( 8.5 \times 10^{-7} , \text{m}^{-1} )</td>
<td>( 5.3 \times 10^{-7} )</td>
</tr>
<tr>
<td>11</td>
<td>( 1.7 \times 10^{-7} , \text{m}^{-1} )</td>
<td>( 1.1 \times 10^{-7} )</td>
</tr>
<tr>
<td>12</td>
<td>( 3.4 \times 10^{-8} , \text{m}^{-1} )</td>
<td>( 2.1 \times 10^{-8} )</td>
</tr>
<tr>
<td>13</td>
<td>( 6.9 \times 10^{-9} , \text{m}^{-1} )</td>
<td>( 4.3 \times 10^{-9} )</td>
</tr>
<tr>
<td>14</td>
<td>( 1.4 \times 10^{-9} , \text{m}^{-1} )</td>
<td>( 8.7 \times 10^{-10} )</td>
</tr>
</tbody>
</table>

Use of the table is simplest if we wish to estimate maximum ridge height along a linear track. Suppose we consider a line of \( L \) km in a region where the mean ridge frequency is \( \mu \) per km. Then we simply look up \((1/L\mu)\) in the prediction table for \( P_c(h) \) and this gives us the likely maximum ridge height along this line. For instance, a problem of great practical importance is to estimate a safe minimum depth for a sea bed operation (such as a wellhead or an instrument mooring) such that it will not be
disturbed by a pressure ridge keel over a period of, say, a year. If the position of the operation is beyond the fast ice zone and within the moving ice of the Beaufort Gyre, the ice passing over this fixed point is equivalent to a track being profiled over stationary ice. In the seasonal ice zone the average winter ice drift is about 2.2 to 2.6 km/day [Coachman and Barnes, 1969], which is of the order of $10^3$ km/yr. Assuming heavy ridging of, say, 10 ridges/km, we have $L_\mu = 10^4$ ridges. From the prediction table we find that the likely maximum ridge height in such a sample is about 6.7 m (22 feet). Over ten years, with $L_\mu = 10^5$, the likely maximum height encountered is 8.2 m (27 feet). On a more modest estimate of $L_\mu = 10^3$, implying light ridging of 2 per km and a low mean annual drift of 500 km, we obtain a maximum ridge height of 5.3 m (17 feet). The final step, conversion to a maximum keel depth, will be considered in Section 5.

A rather more difficult problem is to estimate the maximum ridge height occurring at a given instant over a certain area of icefield. This may arise when we ask such questions as, how many grounded ridges are there at a given moment in the outer fast ice zone? Questions concerning the possibility of ice scour may also be cast in this form, although if we are asking whether ice scour could have taken place in a particular location over a certain historical period (an important question in the discussion of the true age of ice scours) we are asking a question of the linear type, concerning the number of ridges that have drifted over the point in the time allowed.

An approach to the area problem can be made using the concept of ridge density, $R_0$, developed by Mock et al. [1972], this being the total length of ridging per unit area of icefield. The data of Mock et al. from three areas of the Beaufort Sea give a roughly constant ratio of 1.6 between $R_0$ (estimated from aerial photographs) and ridge frequency $\mu$. Now if $1/\mu$ is the mean spacing between ridges encountered on a straight line track, it is also the mean length that a ridge extends before being crossed by another ridge. Thus we can imagine the density $R_0$ in one square km being divided into ridge linkages each of length $1/\mu$; there will be $R_0\mu$ of these per square kilometer. If it is valid to think of each ridge linkage as a separate ridge, then $R_0\mu = (8/5) \mu^2$ is the "number of ridges per unit area," $N_a$. Then, to estimate maximum ridge height in an area $A$ km$^2$, we look up $1/A N_a$ in the prediction table. As an example, let us estimate the maximum ridge height to be found at any given moment in the Beaufort Sea. The area of the Beaufort Sea continental shelf is $2 \times 10^5$ km$^2$, and of the whole Beaufort Sea (within a line from Prince Patrick Island to Point Barrow) $= 4 \times 10^5$ km$^2$. Taking a value of about 8 ridges per km, we have $N_a = 10^2$ km$^{-2}$, and thus

$$1/A N_a = 5 \times 10^{-8} \quad \text{(shelf)}$$
$$2.5 \times 10^{-8} \quad \text{(total)}.$$  

From the prediction table the likely maximum ridge height is about 11.5 m (38 feet) for the shelf and 11.9 m (39 feet) for the whole Beaufort Sea. The good agreement with the 12.8 m ridge found by Kovacs et al. [1973] shows that this approach may have some quantitative validity. We must emphasize its speculative nature, however; a great deal more data are required before these prediction tables can be regarded as reliable.
5. MORPHOLOGY OF ICE KEELS

5.1 Conversion of Ridge Height to Keel Depth

No valid deterministic or even statistical means of converting ridge height to keel depth has been established. To do so we require long simultaneous profiles of top and bottom surfaces along approximately the same tracks, as may be obtained by a submarine acting in cooperation with an aircraft employing a laser. Estimates of the conversion factor are mainly averages based on a relatively small number of ridges that have been laboriously drilled through or profiled from the side using sonar. We do not even expect the conversion factor to be a constant, since the overall isostatic distribution of mass above and below sea level depends on the shape of the ridge, its age (i.e., snow and ice densities within it) and mode of formation.

This was recognized by Ackley et al. [1974], who devised two theoretical models for keel depth conversion. The first was point isostatic (i.e., each point on a ridge generates a point directly below it on the keel), but took account of the variation of ice density with ridge thickness. The second allowed for the fact that the mass of a ridge is distributed over a larger area in the keel, and the conversion is done by a linear sawtooth filter which acts on the surface ridge profile. Thus the second model, while more realistic physically, does not generate a constant conversion factor.

Both models use an empirical relationship between the mean specific gravity $\rho_1$ of an ice column and its freeboard $f_1$ in meters:

$$\rho_1 = 0.974 - 0.194 f_1 \quad f_1 < 1.077$$

$$= 0.765 \quad f_1 \geq 1.077$$

(10)

This is the mean of a large number of measurements on multiyear floes and ridges. For a ridge higher than 1.08 m, equation (10) implies a ratio of 3.0:1 for draft:freeboard on a point isostatic model. The draft:freeboard ratio $R$ is simply given by

$$R = \rho_1/(\rho_w - \rho_1)$$

(11)

where $\rho_w$, the specific gravity of displaced water, is taken by Ackley et al. to be 1.0203. For level ice, with $f_1 \ll 1.08$, the ratio is much higher than 3. The distributed model gives a lower $R$ than the point isostatic for any idealized ridge shape, because the point on the keel directly beneath the ridge crest is generated by a convolution over a range of ridge freeboards which are all less than that of the crest. A 1975 Gulf Oil observational study in the coastal Beaufort Sea found close agreement with the point isostatic ratio for free-floating multiyear ridges. First-year ridges appear to have a higher ratio, however, and Kovacs [1972] found a mean value of 4.2:1.
The conversion of data from laser records is complicated by the fact that the laser measures ridge height above the level ice surface rather than ridge freeboard. If the freeboard of ambient level ice is \( f_0 \), a ridge of height \( h \) has a freeboard \( (f_0 + h) \) and a correspondingly increased draft.

There are now three ratios of interest:

\[
\begin{align*}
R & \quad \text{(keel draft)} \quad : \quad \text{(ridge freeboard)} \\
R_2 & \quad \text{(keel draft)} \quad : \quad \text{(ridge height) given by} \\
& \quad \quad \quad \quad R_2 = R(f_0 + h)/h \quad (12) \\
R_3 & \quad \text{(keel depth below level ice bottom)} \quad : \quad \\
& \quad \quad \quad \quad \text{(ridge height), given by} \\
& \quad \quad \quad \quad R_3 = R(h + 2f_0 - t)/h \quad (13)
\end{align*}
\]

where \( t \) is the level ice thickness and is related to \( f_0 \), according to equation (10), by

\[
t = (0.974 - 0.194 f_0) = (t - f_0) \rho_w \quad (14)
\]

\( R_2 \) is the ratio of importance in considerations of ice grounding and safe depths of water, while \( R_3 \) is of interest in considering the geometrical barrier offered by a keel to spreading oil; \( R_3 < R < R_2 \).

According to Maykut and Untersteiner [1969] the mean value of \( t \) in the Arctic Basin is 3.0 m in April and 2.7 m in September, and Koerner [1970] found a mean thickness of 2.8 m in multi-year floes over a year. These give values for \( f_0 \) of 0.317 m, 0.252 m, 0.272 m, respectively, using equation (10).

To summarize, the draft/height ratio is greater than \( R \) by a factor which diminishes with increasing ridge height, and which is about 30% for a 1 m ridge. To offset this, the more realistic distributed isostatic model implies a lower value for \( R \) at a ridge crest. For simplicity, we shall take the draft/height ratio \( R_2 \) to be 3.0:1 for ice in the polar pack (which is mainly multiyear) and 4.0:1 for ice in the shear zone or offshore province (which is mainly first-year). Depth predictions will be made on this basis.

5.2 Maximum Keel Drafts

Table 7 is the result of converting Table 6 into a keel draft prediction table using two possible values for \( R_2 \). We now have a probability density \( P^K(h) \), which is the probability per meter that a randomly encountered keel will have a draft \( h \) meters; and a probability \( P^C(h) \), which is the probability that a randomly encountered keel will have a draft of at least \( h \) meters. These are related to \( P(h) \) and \( P_C(h) \) by

\[
P^K(h) \, dh = \frac{P(h/R_2)}{R_2} \, dh \quad (15)
\]
\[ P_C^k(h) = 0.624 R_2 P_C(h) \] \hspace{1cm} (16)

In Section 4 various ridge height maxima were estimated which can now be converted to keel drafts. For ridges crossing a given point in the shear zone the 5.3 m figure for a year's maximum based on light ridging and slow drift becomes 21.2 m in draft; 6.7 m for a yearly maximum based on heavier ridging and faster drift becomes 26.8 m in draft; and 8.2 m for a ten-year maximum becomes 32.8 m. These estimates are all subject to the wide uncertainty in \( R_2 \), but we can compare them with various phenomena found on the Shelf.

First, 20 m is the normal maximum water depth for shorefast ice, corresponding to the maximum depth at which significant numbers of firmly grounded pressure ridges are found. A keel would have to be rather deeper than 20 m while freely floating in order to drive itself aground at that depth.

**TABLE 7**

PREDICTION TABLE FOR OCCURRENCE OF DEEP KEELS

<table>
<thead>
<tr>
<th>Keel Draft (m)</th>
<th>Probability density ((m^{-1}))</th>
<th>Probability that keel draft is at least ( h )</th>
<th>Probability density ((m^{-1}))</th>
<th>Probability that keel draft is at least ( h )</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>(6.4 \times 10^{-6})</td>
<td>(1.6 \times 10^{-3})</td>
<td>(5.9 \times 10^{-5})</td>
<td>(1.1 \times 10^{-4})</td>
</tr>
<tr>
<td>22</td>
<td>(2.9 \times 10^{-6})</td>
<td>(7.2 \times 10^{-4})</td>
<td>(2.0 \times 10^{-5})</td>
<td>(3.8 \times 10^{-5})</td>
</tr>
<tr>
<td>24</td>
<td>(1.3 \times 10^{-6})</td>
<td>(3.2 \times 10^{-4})</td>
<td>(7.0 \times 10^{-6})</td>
<td>(1.3 \times 10^{-5})</td>
</tr>
<tr>
<td>26</td>
<td>(5.8 \times 10^{-5})</td>
<td>(1.4 \times 10^{-4})</td>
<td>(2.4 \times 10^{-6})</td>
<td>(4.5 \times 10^{-6})</td>
</tr>
<tr>
<td>28</td>
<td>(2.6 \times 10^{-5})</td>
<td>(6.5 \times 10^{-5})</td>
<td>(8.2 \times 10^{-7})</td>
<td>(1.5 \times 10^{-6})</td>
</tr>
<tr>
<td>30</td>
<td>(1.2 \times 10^{-5})</td>
<td>(2.9 \times 10^{-5})</td>
<td>(2.8 \times 10^{-7})</td>
<td>(5.3 \times 10^{-7})</td>
</tr>
<tr>
<td>32</td>
<td>(5.2 \times 10^{-6})</td>
<td>(1.3 \times 10^{-5})</td>
<td>(9.7 \times 10^{-8})</td>
<td>(1.8 \times 10^{-7})</td>
</tr>
<tr>
<td>34</td>
<td>(2.3 \times 10^{-6})</td>
<td>(5.8 \times 10^{-6})</td>
<td>(3.3 \times 10^{-8})</td>
<td>(6.2 \times 10^{-8})</td>
</tr>
<tr>
<td>36</td>
<td>(1.1 \times 10^{-6})</td>
<td>(2.6 \times 10^{-6})</td>
<td>(1.1 \times 10^{-8})</td>
<td>(2.1 \times 10^{-8})</td>
</tr>
<tr>
<td>38</td>
<td>(4.7 \times 10^{-7})</td>
<td>(1.2 \times 10^{-6})</td>
<td>(3.9 \times 10^{-9})</td>
<td>(7.4 \times 10^{-9})</td>
</tr>
<tr>
<td>40</td>
<td>(2.1 \times 10^{-7})</td>
<td>(5.3 \times 10^{-7})</td>
<td>(1.4 \times 10^{-9})</td>
<td>(2.5 \times 10^{-9})</td>
</tr>
<tr>
<td>42</td>
<td>(9.5 \times 10^{-8})</td>
<td>(2.4 \times 10^{-7})</td>
<td>(4.6 \times 10^{-10})</td>
<td>(8.7 \times 10^{-10})</td>
</tr>
<tr>
<td>44</td>
<td>(4.3 \times 10^{-8})</td>
<td>(1.1 \times 10^{-7})</td>
<td>(1.6 \times 10^{-10})</td>
<td>(3.0 \times 10^{-10})</td>
</tr>
<tr>
<td>46</td>
<td>(1.9 \times 10^{-8})</td>
<td>(4.8 \times 10^{-8})</td>
<td>(5.5 \times 10^{-11})</td>
<td>(1.0 \times 10^{-10})</td>
</tr>
<tr>
<td>48</td>
<td>(8.6 \times 10^{-9})</td>
<td>(2.1 \times 10^{-8})</td>
<td>(1.9 \times 10^{-11})</td>
<td>(3.5 \times 10^{-11})</td>
</tr>
<tr>
<td>50</td>
<td>(3.8 \times 10^{-9})</td>
<td>(9.6 \times 10^{-9})</td>
<td>(6.5 \times 10^{-12})</td>
<td>(1.2 \times 10^{-11})</td>
</tr>
</tbody>
</table>
Ice scouring of the sea bed has been extensively mapped in recent years, both off Alaska [Reimnitz and Barnes, 1974] and in Mackenzie Bay [Pelletier and Shearer, 1972; Shearer and Blasco, 1975; Lewis, 1975]. It is found that scouring is common between water depths of 10 m and 50 m, with a scour frequency of about 10 per km which shows a peak at around 30 m depth. About 50 m depth the frequency falls sharply, with no scours beyond 75 m. The scours in deeper water are partially filled with sediments, suggesting that they are relics of a period when sea level was lower; the sedimentation rate is only about 1 m per 1000 years. The broad scour shapes made by ice island fragments can be distinguished from the parallel narrow tracks made by the deepest blocks of a single pressure ridge keel, and pressure ridge scours are found throughout the depth range. Using present sea level we can estimate the maximum depth at which scouring is common by supposing that a scour preserves its identity for approximately 1000 years, and that the criterion for "frequent scouring in a given area" is that a given point should enter the prediction table at one scour per 1000 years. For 1000 years we have $(L\mu) = 10^7$, but this should possibly be reduced by a factor of 2 to take account of the fact that ice drift is slower in the inner portion of the shear zone nearest the shorefast ice, since once a ridge has grounded it may hold up the motion of a large area of ice around it for a considerable period. Thus with $(1/L\mu) = 2 \times 10^{-7}$ the prediction table gives about 42 m as the likely maximum depth. At this depth every point on the bottom has the expectation of being struck at least once per 1000 years by a ridge, so scouring out to a depth of at least 40 m can be accounted for by pressure ridge grounding at present sea level.

During a submarine transit of about 1200 km in the Trans-Polar Drift Stream, Swithinbank [1972] found a maximum draft of 30 m. The ice in this area is mainly first-year, and assuming heavy ridging of 8 per km we have $L\mu = 10^4$ and an expected maximum draft of 26.8 m, somewhat less than observed.

Finally, the figures for maximum ridge heights at any instant on the Beaufort Sea or Shelf—11.5 and 11.9 m—convert to drafts of 46.0 and 47.6 m, respectively. The entire Arctic Ocean ice cover has an area of $1.5 \times 10^7$ km², and we can use the ridge density model developed in Section 4 to estimate the maximum draft at any instant. We estimate the overall mean ridge frequency at between 3 and 5 per km, and we use a composite figure of 3.5 for $R_2$ on the assumption that the Arctic cover is 50% multi-year ice. These values yield estimated maxima of 45 m and 48 m, respectively, for the 3- and 5-ridge cases. The similarity of these estimates to those for the much smaller area of the Beaufort Sea is mainly due to the sensitivity of the keel prediction table to variations in $R_2$. The first conclusion is that since $R_2$ is not yet known to any accuracy the keel draft predictions must be regarded as very approximate. The second conclusion is that the coastal areas of the Arctic, such as the Beaufort Sea, are probably the site of the deepest keels in the Arctic Ocean, since they have a combination of high ridge frequencies and high $R_2$ (on account of the preponderance of first-year ice). The deepest keel yet measured in the Arctic Ocean was in an unspecified location and had a draft of 47 m [Lyon, 1967].

32
5.3 Depth Fluctuations along a Solitary Keel

The effective geometrical depth which a keel presents to an approaching oil slick is not its mean depth, but the minimum depth relative to the level ice bottom in the keel linkage considered. Similarly, when scouring occurs the first part of a keel to take the ground is that point on the crest which has a maximum draft. The jumbled block structure of a keel results in a random crest profile with significant fluctuations about the mean depth. An opportunity to study these arose when a sonar profile was obtained along a keel crest by an unmanned arctic research submersible (UARS) of the Applied Physics Laboratory, University of Washington [Francois and Nodland, 1972, 1973]. The original sonar profile data were made available to the author by R.E. Francois.

The profile was taken on 9 May 1972 from ice island T-3, which was then very far north in the Beaufort Gyre at 84°N, 84°W. The ridge concerned was a shear ridge, estimated to be 6 to 8 years old, which lay in the sea ice of Colby Bay, an indent in the "coastline" of T-3. The remote-controlled UARS ran a star-shaped pattern at a depth of 46 m, and in one of its runs successfully profiled some 330 m of the ridge keel. Three sonar profiles were obtained simultaneously by narrow-beam (1°) transducers, one of which looked directly upwards while the other two looked sideways at 6° to the vertical. Figure 18(a) shows the output \( y(x) \) of profiler 2 (vertical); profiler 1 looked to the left and thereby obtained a parallel profile offset laterally by 4.4 m, while profiler 3 failed to pick up the keel. The local draft of level ice (2.576 m) has been removed from Figure 18(a) leaving only the relief of the keel.

It is apparent that the variance consists of a short-wavelength scale due to the block structure of the keel and a long-wavelength scale due either to a real variation in mean keel depth or, more likely, to a meander in the line of the keel so that the rectilinear track of the UARS did not remain directly beneath the crest. To estimate the fluctuations due to the block structure alone it is necessary to remove the long-wavelength variations. The profile was digitized at 0.325 m intervals, giving 1024 points (overall length 332.1 m), and a low-pass profile was generated using a 101-point running mean with reflections at both end points. Figure 18(b) shows the low-pass profile \( y_0(x) \), whose depth varies from 0.321 m to 3.377 m. The greatest depth probably corresponds to the true mean keel depth, while all lesser depths refer to side slopes of the keel. It is thus a shallow keel, with an overall draft of 5.95 m; the surface ridge has a height of about 1.7 m [Francois and Nodland, 1972].

We wish to estimate the minimum and maximum keel reliefs \( y_{\text{min}}(x) \) and \( y_{\text{max}}(x) \) in a given keel linkage \( \lambda \). Ideally we would like a means of predicting such values for any \( \lambda \) and any mean depth \( \bar{y} \). For a profile which is a Gaussian random function, Cartwright and Longuet-Higgins [1956] showed that the elevations of the highest peak and lowest trough in a given linkage can be predicted in terms of the mean number of peaks per unit distance and a spectral width parameter derived from the power spectrum of the profile. To test for the Gaussian nature of this profile, a series of local standard deviations \( \sigma(y_0) \) was calculated for 0.5 m-wide categories of depth \( y_0 \). The deviation \( [y(x) - y_0(x)]/\sigma(y_0) \) was then calculated for each value \( y(x) \) with the results shown in Figure 19. There is a clear
Fig. 18. UARS sonar profile. (a) Output of profiler 2, (b) low-pass profile, (c) normalized profile.

Fig. 19. Profiler 2, distribution of depth relative to local mean.
departure from a Gaussian, with an excess of high peaks and a
deficit of deep troughs. This is partly because no point is likely to
have \( y < 0 \), so that troughs deeper than a certain multiple of \( \sigma(y_0) \) are
impossible; this tends to generate a Rayleigh distribution. Also, in
a keel composed of angular random blocks, a steep-sided trough tends to
be shadowed by neighbouring blocks or even concealed by a block protrud-
ing under it, whereas steep peaks are free to stand clear. Predictions
of trough depths attempted on the basis of Cartwright and Longuet-Higgins
were found to give serious overestimates.

Figure 20 shows that the local standard deviation \( \sigma(y_0) \), for points \( y(x) \)
having \( y_0 \) in the ranges \( 0 - 0.5 \) m, \( 0.5 - 1.0 \) m, etc., shows an increasing
trend when plotted against the mean depth of the points in each category.
This suggests that we can "normalize" the profile by transforming the
depths to

\[
u(x) = \frac{y(x)}{y_0(x)}
\]

and we can then make predictions from the normalized depth profile which
will be approximately valid for keels of any mean depth. This procedure
is rigorous only if \( \sigma(y_0) \propto y \) and (frequency of peaks) \( \propto y_0 \). Neither
condition is fulfilled, but we still expect that the results will be of
value over a restricted range of ridge types. Figure 18(c) shows \( u(x) \)
for profile 2.

![Graph showing local standard deviation against local mean depth.](image)

**Fig. 20.** UARS profiler 2, local standard deviation against local
mean depth.
The quantities $\hat{u}_{\text{max}}(\ell)$, $\hat{u}_{\text{min}}(\ell)$ were computed for all possible linkages $\ell$, where

$$\hat{u}_{\text{max}}(\ell) = \frac{1}{N} \sum_{n_1=1}^{N} \left| u_{\text{max}} \right| n_1, \ell \quad (18)$$

$$\hat{u}_{\text{min}}(\ell) = \frac{1}{N} \sum_{n_1=1}^{N} \left| u_{\text{min}} \right| n_1, \ell \quad (19)$$

$$\ell = n_2 s \quad n_2 = 1, 2 \ldots (N - 1) \quad (20)$$

In these equations

- $\ell$ = keel linkage over which maximum or minimum is estimated
- $N$ = total number of data points (1024)
- $s$ = interval between contiguous points (0.325 m)
- $\left| u_{\text{min}} \right| n_1, \ell$ = minimum value of $u$ in a linkage $\ell$ beginning at the $n_1$-th point
- $\hat{u}_{\text{min}}(\ell)$ = expectation of minimum value of $u$ in a linkage $\ell$ (maximum values defined similarly).

A linkage such that $(N - n_1) < n_2$ was computed by repeating the record from the start; in this way every data point was given equal weight in the computation of equations (18) and (19).

Figure 21 shows the results for both profiles. For small linkages there is symmetry between $\hat{u}_{\text{max}}$ and $\hat{u}_{\text{min}}$, which is to be expected if the record really were a Gaussian random process. On this scale we are looking at the depth variations of a single ice block in the keel crest. At a linkage of about 13 m the $\hat{u}_{\text{min}}$ curve begins to flatten out more rapidly than the $\hat{u}_{\text{max}}$ curve; this is shown clearly in the magnified Figure 21(b) covering the first 26 m. Also at 13 m each curve (Figure 21(a)) shows a distinct knee, where the initial rapid deviation from unity turns into a gentler progression (the negative values of $\hat{u}_{\text{min}}$ for profile 1 at high $\ell$ are caused by a single trough whose draft is less than the mean of the level ice). It is clear, then, that 13 m represents a horizontal length scale for the size of ice blocks composing the keel, a scale which can be confirmed visually by inspection of Figure 18(a). Thus, if a submerged observer travels 13 m along the keel crest he will crawl under a single ice block and will encounter depths that vary between about 0.4 and 1.6 of the mean depth. If he is looking for a further comparable increase in the range of depth, to 0.0 and 2.4, he must travel at least 300 m, which is a typical survival distance for a ridge before it is crossed by another ridge.
Fig. 21(a). Expectation of extreme depths against keel linkage.

Fig. 21(b). Expectation of extreme depths at low keel linkages.
The $\hat{\gamma}_{\text{min}}$ curve in Figure 21 should be treated with reserve. Figure 22 shows profiles 1 and 2 plotted together, and it can be seen that a trough in one profile is often covered by a peak in the other. This may imply the existence of a channel slanting through the keel, which would be permeable to oil, or there may be a lateral as well as a longitudinal structure of blocks, so that troughs shown in a single profile do not penetrate the whole width of the keel. The keel that we have examined has a relief of about 3 m and a block diameter of 13 m. It is also a shear ridge, whose formation typically does not involve the piling up of a great heap of blocks. Therefore it is reasonable to assume that the keel relief is one of individual blocks, lying at random angles, but strung out more or less in single file. A trough shown in one profile probably penetrates the whole keel, albeit at an angle so that a profile offset by 4.4 m does not show the trough in the same place. Thus, the "universal" normalized curve of Figure 21 can be used to predict minimum depths for shallow shear ridges not more than, say, 8 m in total draft, and it shows that such ridges are probably quite permeable to oil. The results may also apply to shallow pressure ridges built of single blocks, but deep pressure ridges must have far lower relative depth fluctuations, although the absolute fluctuations are probably of similar, or greater, magnitude. As a rough guide, then, we can expect a very deep keel to show fluctuations of at least $\pm 2$ m about the mean draft, the deeper spurs being the first to gouge the sea bed when the keel moves into shallow water.

Fig. 22. UARS profiles 1 and 2.

6. EFFECT OF MORPHOLOGY IN OIL CONTAINMENT

6.1 Volume and Nature of Oil Release

The most severe type of accident that could occur would be the blowout of an exploratory well near the end of the summer drilling season, such that it could not be closed off until the following year. According to oil industry sources, the type of oil-bearing structure found in the Beaufort Sea could be expected to give an initial flow rate of some 2500 bbl/day$^{-1}$ ($400$ m$^3$ day$^{-1}$), reducing after a month to a steady rate of 1000 bbl/day$^{-1}$ ($160$ m$^3$ day$^{-1}$). Integrated over a year this yields $4 \times 10^5$ bbl of oil ($6.4 \times 10^8$ m$^3$), which we may take as a standard.
"blowout scenario." Actual exploratory drilling may, of course, show flow rates that are quite different from this. It should be noted that the Torrey Canyon disaster released about $7 \times 10^5$ bbl of oil. An estimated 800 ft$^3$ (23 m$^3$) of free gas per barrel of oil is also expected from a blowout, and it is shown in Topham [1975] how this will generate a forced convection of water which will melt a large pool over the site of the blowout.

In addition to a severe, but unlikely, accident of this sort we can expect small accidental releases of oil from various sources on and around rigs. These will occur with increasing frequency as production begins. It has been written of a fully developed offshore field [Alpine, 1971]:

Spills varying in size from a few gallons to many barrels are endemic to the Gulf of Mexico. . . U. S. Coastguard's reconnaissance flights report 3 to 7 pollution incidents every week.

In 1971 there were 1436 pipeline breaks in U.S. coastal waters, spilling 897,685 gallons of oil [Boesch et al., 1974], an average of 625 gallons (19 bbl) per spill. Most of these breaks arose from nearshore corrosion of old pipelines; corrosion in arctic waters is very slow, but there is the additional danger of pipeline disturbance by ice keels. Another source of minor spills will be the vastly increased density of shipping in the Beaufort Sea, especially supply vessels and icebreaking tenders. These may lose oil from engine cooling loops, propeller shaft glands, leaks while fuelling from tank barges, etc.

Thus, in the absence of major accidents we can still expect a release of oil into the environment, perhaps of order 100 bbl/year. Our remarks on the spread of oil from a major blowout can also be taken to apply, on a much reduced scale, to the fate of oil from these minor releases.

6.2 Inner Fast Ice Zone

The simplest type of ice morphology is just a smooth, level ice sheet, as is found in the inner part of the fast ice zone. We assume that the various phenomena occurring directly over the blowout site (burning of oil in situ, damming by the melted-out pool, mixing and emulsification into the water column) have not prevented a significant fraction of the oil from spreading beyond the immediate limits of the blowout zone. The blowout site is visualized on this scale as a point source emitting a uniform flow of non-emulsified oil which is presented directly to the surrounding ice surface (i.e., emulsified and dispersed oil is assumed to come out of the water column relatively quickly and join the oil which is spreading across the ice surface). Assuming that the oil forms a coherent slick—and the evidence is that although the emitted stream may begin to spread as droplets, these tend to join up into lenses and then slicks [Keevil and Ramseier, 1975]—our problem is to estimate the slick thickness and thus the oiled area per unit volume emission.
The author took part in the offshore oil spill experiment carried out by NORCOR some 30 km from Cape Parry on April 8, 1975 [Dickins et al., 1975]. The first of the two spills was under a smooth ice floe, and the oil came to rest in a coherent slick of thickness \((0.56 \pm 0.08)\) cm. The oil was Norman Wells crude, and studies by Rosenegger [1975] and Mackay et al. [1975] have shown that this is the equilibrium thickness of a drop or lens of Norman Wells crude lying at rest under an ice sheet. The thickness of a lens of large diameter is given by the balance of surface tension versus buoyancy forces around the edge of the lens:

\[
H^2\delta \rho g = 2T(1 - \cos \theta)
\]  

(21)

where \(\delta \rho =\) density difference between oil and sea water
\(T =\) surface tension at oil/ice interface
\(\theta =\) oil/ice contact angle, measured within the oil

Norman Wells crude has a specific gravity of 0.847, but its other properties appeared to be quite variable, presumably due to the presence of surfactants. It was found that the surface tension, especially, had a wide range, but that the most common values were between \(1.5\) and \(2 \times 10^{-2}\) N m\(^{-1}\), with \(\theta\) in the range \(140^\circ\) to \(170^\circ\). This gives a range \(0.55\) to \(0.68\) cm for \(H\). Beaufort Sea oil may have different properties from Norman Wells, and other oils tested seemed to have a higher \(H\), up to 1 cm. We therefore take 0.5 cm to 1.0 cm as a feasible range for \(H\), with 0.56 cm as our favoured value. It should be noted that Wolfe and Hoult [1974] offered a value of 0.7 cm, but this was based on heat flow reasoning which ignored surface tension. The area covered by various volumes of oil can now be estimated (Table 8).

| TABLE 8 |

| AREAS COVERED BY SPILLS OF VARYING MEAN THICKNESS \(H\). |

<table>
<thead>
<tr>
<th>Volume</th>
<th>(H = 5) mm</th>
<th>(H = 5.6) mm</th>
<th>(H = 10) mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 barrel (0.1591 m(^3))</td>
<td>31.8 m(^2)</td>
<td>28.4 m(^2)</td>
<td>15.9 m(^2)</td>
</tr>
<tr>
<td>1 cubic meter</td>
<td>200 m(^2)</td>
<td>179 m(^2)</td>
<td>100 m(^2)</td>
</tr>
<tr>
<td>(4 \times 10^5) bbl (blowout)</td>
<td>12.7 km(^2)</td>
<td>11.4 km(^2)</td>
<td>6.4 km(^2)</td>
</tr>
</tbody>
</table>

The actual dimensions of the slick depend entirely on the surface currents during the period of the oil release. The Cape Parry spill took place in a current of 0.10 m s\(^{-1}\), and although this current defined the direction of spread it did not cause the oil to continue spreading after it had reached its equilibrium thickness. The final shape of the slick could be idealized as an elongated ellipse with the spill point as one focus and the major axis pointing downstream. On this reasoning, in an area of steady currents a "standard blowout" would cause a slick roughly
6 km by 2 km, the longer axis pointing downstream. However, currents are never steady, certainly not for a period of a year, so the actual shape of the slick is indeterminate. Once the slick has reached its equilibrium thickness the spreading stops and, in winter, the oil becomes fixed by new ice growing below it and through it. If the source sends out oil in pulsations one can envisage a fresh ice surface which has sandwiched one slick acting as the collecting surface for a second slick, thus building up a multilayer sandwich and reducing the oiled area per unit volume. The extent to which this can happen is uncertain, because as each layer is added the very low thermal conductivity of the oil will cause the rate of ice growth to decrease considerably. We assume that gas will vent through the hole generated directly over the blowout.

For the inner shorefast ice zone, then, we have a reasonable idea of the area that will be affected by a blowout provided it occurs after freeze-up. The dimensions of the affected area are quite small, and provided the oil does not spread as far as the tidal cracks it will sit in place throughout the winter. Unfortunately such oil is not easily accessible for clean-up, since ice growing through the oil tends to divide the slick into cells, making drilling and pumping ineffective. Of critical importance is the date of break-up of the shorefast ice. NORCOR and other studies have shown that in early summer the oil migrates to the upper ice surface through expanded brine drainage channels, making it available for clean-up, but if this happens after break-up the oiled floes will become widely distributed and the oil may find its way onto the open sea.

6.3 Outer Fast Ice Zone

The outer fast ice zone is characterized by a morphology of ridges and hummocks, but the ice itself is stationary. In this case the morphology acts to reduce the area of the spill and to make clean-up easier. Kovacs and Mellor [1974] describe rugged fields of rubble or hummocks in this zone, generated in early fall by pressure of the polar pack on the young fast ice. Such a zone is the least unpleasant place for a blowout to occur, provided most of the oil release occurs after the rubble field has consolidated itself (otherwise the oil will become intimately bound up in the deformed ice). In such a field one can easily imagine a roughness scale of 5 cm on the ice bottom; the oil will collect in pools and pockets whose position can be estimated from the top surface morphology and which can more easily be tapped and pumped out. The actual area affected will be less, only 1.3 km² for a standard blowout in an area of vertical roughness scale 5 cm. Elsewhere in the outer fast ice zone the ridging will also act to reduce the area of a spill, but to a lesser extent.

The rough ice in this zone can be seen in Figure 23, which is SLAR imagery taken in Mackenzie Bay during the Argus flight of April 26, 1975. The smooth inner fast ice zone, spreading from the shore, is criss-crossed by ice roads serving artificial islands; there is then an outer zone of rougher ice before the wide lead noted in Section 3.3 marks the transition to moving ice.
6.4 Shear Zone

In the shear zone the real problems begin. First, the 20 m contour itself would be an extremely hazardous place to have a blowout, because a wide lead opens here early in the spring, allowing the oil to spread freely into open water and to distribute itself over great distances. Farther out into the winter shear zone we find heavy ridging, many leads, and vigorous and rapid ice movement. The measurements of Hibler et al. [1973, 1974a] have shown that under wind stress and internal stress transmitted from the polar pack there can be rapid shearing motions as well as alternate convergence and divergence of the pack in periods as short as a day, which cause opening and closing of lead systems. The long-term average motion of the ice is westward with the Beaufort Gyre, and from the drift track of ice island T-3 in this area [Campbell and Martin, 1973] we estimate a long-term drift of about 1000 km/year with a complete circuit of the Beaufort Gyre in 7 to 10 years.

In such an area, the first question concerns the scale of a blowout in relation to the frequency of leads and ridges. Figure 24 is a SLAR image which shows the very sharp tracery of intersecting ridges found in first-year ice; Figure 25, from farther north, shows a much more complex mixture of first- and multiyear ice with leads, rafting, and rubble fields. We can approximate the ice conditions by assuming that, except for pressure ridges and leads, the ice surface is smooth with the oil containment factor given in Table 8. Let us consider first a simply connected environment, the smallest space scale that consists only of smooth ice. If \( \mu \) is the mean number of ridges per km, such a scale has a mean diameter of \( 1/\mu \) and, supposing it is square, is surrounded by a length of \( 4/\mu \) of ridging. We need to estimate a specific containment factor \( V \) for a ridge, defined as the mean volume of oil that can be retained per unit ridge length. This is an extremely difficult quantity to estimate. The laboratory experiments of Moir and Lau [1975] suggest that oil will not build up behind a ridge as it does behind a boom in open water, but that instabilities will arise which allow the oil to pass under the keel. A full-scale experiment is needed to see whether this happens in practice. The nearest approach to such an experiment was the second offshore spill at Cape Parry, made near a small pressure ridge. Unfortunately the quantity of oil spilled was inadequate to test whether the pool that was growing behind the ridge would continue to grow without instability. The pool was formed because of the presence of a depression in front of the keel, a long wavelike feature that either represented part of the keel structure or else was generated by differential ice growth due to snow drifting against the surface ridge. If a keel is indeed ineffective as a boom it may still have a large containment factor due to the presence of such deformed ice in its "foothills."

Unfortunately, not enough keels have been profiled to determine whether these depressions are a universal phenomenon. As a basis for speculation we may consider the actual pool developed in the Cape Parry spill as being typical. According to the diver, its dimensions were 6 feet in width by 4 inches deep, giving \( V = 0.19 \text{ m}^2 \) per meter linkage. Our simple environment thus has an area of \( 1/\mu^2 \), which can contain a volume \( H/\mu^2 \) of oil under its level ice. Its bounding ridge can retain \( 4 V/\mu \) of oil. Taking
Fig. 23. SLAR images, 26 April 1975. Scale 1:250,000. Aircraft heading 052°, center of picture 69°49'N, 134°23'W. Smooth fast ice near shore, with vehicle tracks; scattered floes embedded in fast ice farther from shore; rough ice in outer fast ice zone; then wide lead marking edge of shear zone. North end of Richards Island at bottom; Hooper Island at far left.
Fig. 24. SLAR images, 26 April 1975. Scale 1:250,000. Aircraft heading 052°, center of picture 70°56'N, 134°30'W. First-year ice in shear zone with linear tracery of pressure ridges.
Fig. 25. SLAR images, 26 April 1975. Scale 1:250,000. Aircraft heading 180°, center of picture 71°54'N, 139°00'W. Polar pack, with smooth floes (light) and rough, possibly multiyear, floes (dark), rubble field (black matrix around floes) and polynyi (white).
9 ridges per km as typical of heavy ridging, and \( H \) as 0.56 cm, we have an area of 12,000 m\(^2\), which has a capacity of 69 m\(^3\) of oil (434 bbl) under level ice, and 84 m\(^3\) (528 bbl) retained against ridges. Thus more oil is contained against ridges than under the level ice, especially as we would normally expect \( V \) to be much greater than our estimate.

We have still to consider leads and the effect of ice motion. Taking the figure found in our aerial survey (0.72 leads per km) as typical, we have approximately one lead per 5 - 10 ridges. Thus if oil is spilled at a fixed point relative to the ice, it will have to cross about 3 to 5 ridges--i.e., 3 to 5 simple environments--to reach a lead, which will require between 2900 and 4800 bbl of oil. A small instantaneous spill, even in the shear zone, is therefore unlikely to get into a lead. However, since the ice is in motion, a continuous release, as from a blowout, will be in perpetual motion relative to the ice surface. It will thus "paint" a strip of oiled ice, a strip which will become more sinuous and Gordian in character the farther it progresses from the "paintbrush." The oil will enter leads and as lead systems are created and destroyed it will become incorporated into the subsequent new pressure ridges.

If \( V \) is the velocity of ice drift and \( \Omega \) is the rate of oil emission, the width \( W \) of the oiled swath traced out in smooth ice is given by

\[
W = \frac{\Omega}{VH}
\]  

(22)

\( W \) varies inversely with \( V \), so that the swath progressing from the blowout site grows wider when the ice drift slows. The long-term mean drift in the eastern and southern Beaufort Sea has been given as 2.2-2.6 km/day by Coachman and Barnes [1961]; 2.7 km/day from the drift track of T-3 taken over a year [Campbell and Martin, 1973]; and 2.4 km/day from a year's average of monthly means [Coachman, 1969]. The monthly means in the last reference shows an increase to about 4 km/day in September and October, the months of lowest ice concentration when rapid response to wind stress is possible. All of these long-term means are much lower than the instantaneous mean velocity which is required for equation 22. Dunbar and Wittmann [1963] defined the difference in a coefficient of meandering, the ratio of net distance covered to a long-term distance made good, which they calculated for various ice stations. The only one in the southern Beaufort Sea was T-3 (1959-60 drift), with a coefficient of 2.0, but other coefficients in the Beaufort Gyre ranged from 1.5 to 10.0 with a mean of 3.9. The shear zone to the northeast of Mackenzie Bay, where drilling is proposed, exhibits rapid ice movement in spring and summer, with frequent eastward excursions of the pack; using satellite photographs Marko [1975] reports velocities of 10 km/day or more as common. We conclude that the long-term westward drift of about 2.4 km/day should be multiplied by a meander coefficient of between 2 and 4 to obtain the actual distance travelled by the ice and hence \( V \) for equation 22. Taking the limits of \( \Omega \) as 2500 and 1000 bbl/day, and the limits of \( H \) as 0.56 cm (smooth ice) and 2 cm (to include a containment factor for ridging) we obtain, in Table 9, estimates for \( W \).
TABLE 9

SWATH WIDTHS $W$ LAID DOWN IN SLICKS OF THICKNESS $H$ UNDER ICE MOVING AT $V$ BY A BLOWOUT EMITTING AT RATE $\Omega$.

<table>
<thead>
<tr>
<th>$\Omega$ (bbl/day)</th>
<th>$V$ (km/day)</th>
<th>$H$ (cm)</th>
<th>Swath width $W$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>4.8</td>
<td>0.56</td>
<td>5.9</td>
</tr>
<tr>
<td>1000</td>
<td>4.8</td>
<td>2.0</td>
<td>1.7</td>
</tr>
<tr>
<td>1000</td>
<td>9.6</td>
<td>0.56</td>
<td>3.0</td>
</tr>
<tr>
<td>1000</td>
<td>9.6</td>
<td>2.0</td>
<td>0.8</td>
</tr>
<tr>
<td>2500</td>
<td>4.8</td>
<td>0.56</td>
<td>14.8</td>
</tr>
<tr>
<td>2500</td>
<td>4.8</td>
<td>2.0</td>
<td>4.1</td>
</tr>
<tr>
<td>2500</td>
<td>9.6</td>
<td>0.56</td>
<td>7.4</td>
</tr>
<tr>
<td>2500</td>
<td>9.6</td>
<td>2.0</td>
<td>2.1</td>
</tr>
</tbody>
</table>

It is clear that $W$ is small under all possible assumptions, and in fact in moving ice it is no longer valid to consider the blowout site as a point source. Topham [1975] found that bubble plumes in 23 m and 60 m of water generated circular areas of diameter 40 m and 72 m, respectively, at the water surface, which were swept by the bubble field and which constitute the point source. A blowout in which oil and gas come up together would emit oil to the environment from the edges of this highly turbulent area. The two depths are typical of the inner shear zone and are quite close to the depths of the proposed 1976 drilling sites. Since the diameter of the source exceeds the width of a fully oiled swath we draw the important conclusion that the paintbrush does not have enough paint to draw a full black line; i.e., the swath will take the diameter of the source rather than $W$ and will not be oiled to its full carrying capacity. There will be a patchy oil coverage over this wider swath, but the patchiness will be distributed over a greater area. Consider a blowout in 60 m of water. The swath is now of minimum diameter 72 m, and with a 9.6 km/day drift the area of ice affected in a year is 252 km$^2$, twenty times the area estimated in Table 8 for a stationary ice cover. Within this area the oil contamination is equivalent to a continuous cover of thickness only 0.6 cm (at 2500 bbl/day). Not only is the swath wider and more patchy, but the meander results in a very sinuous swath track which may spread 1000 km to the westward in a year yet be 2000 to 4000 km long. In this way a low concentration of oil is distributed over a very wide geographical area, both in latitude and longitude, making clean-up extremely difficult.

In this new view a keel has no special status; since the oil coverage is insufficient to build a coherent slick of equilibrium thickness under smooth ice, there is no incentive for the oil to pile up behind keels.
Depressions near keels will still act as gathering points for oil deposited in their "watershed area," but less effectively than they would if the oil were positively spreading. Thus drilling and pumping in the neighbourhood of ridges is not likely to be very effective. In addition we must consider the effect of gas, which in stationary ice vents form a hole directly over the blowout. With moving ice it is not known whether the gas will spread under the ice until it finds a crack, or whether its pressure will continually break the ice over the blowout, resulting in a trail of shattered ice blocks and brash ice which coincides with the oiled swath. Icebreaking experience with the Alexbow has shown that it is far easier to break ice from below than from above, and pressures of 0.1 atm. have been quoted as sufficient. If continual breakage occurs much of the oil will end up mixed in with the loose blocks and brash ice; it will be accessible for clean-up for a few hours before becoming frozen into a matrix of fresh ice [Lewis, 1970]. If breakage does not occur, the spreading gas may fill pockets and depressions in the ice bottom, destroying the containment factor of a keel completely.

With an essentially random mechanism for oil deposition the percentage that will enter existing leads will be only slightly greater than the percentage coverage of open water, since drainage into leads from the periphery of floes is only a local phenomenon. Thus in winter less than 5% of the oil will be deposited in existing leads. In summer, as seen in Figure 3, almost all the oil will be in open water if the blowout site is south of the summer ice margin, while if the site is well inside the ice margin it may only be 10%. In winter the existing leads will close within a few days, and the young ice and oil in them will be built into pressure ridges. We thus expect to see oiled pressure ridges, with a weaker structure than normal ridges which will be an early source of oiled meltwater pools in summer. Leads which open in an oiled ice cover after deposition has occurred will accumulate little oil, since once the oil has frozen in, its capacity for horizontal movement or drainage is slight. The "lead-matrix pumping" mechanism of Campbell and Martin [1973], whereby closure of one set of leads pumps oil longitudinally into another set of leads, is not likely to occur since lead closure takes place initially at a chain of contact points which retain the oil so that it is either built into the ridge or forced over or under the adjacent level ice.

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I wish to express my grateful thanks to the officers and crew of Maritime Command Argus 10728 for their skill and good humour; to Dr. R. E. Francois of the Applied Physics Laboratory, University of Washington, for furnishing UARS sonar profiles; and to Mr. H. Hengeveld of Atmospheric Environment Service, Downsview, Ontario, for furnishing AES laser profiles and for assisting in their analysis.
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ANTARCTIC SEA ICE DYNAMICS AND ITS POSSIBLE CLIMATIC EFFECTS

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ABSTRACT

Ice extent charts prepared from satellite images by the U.S. Naval Fleet Weather Facility and passive microwave emission data from the Nimbus V satellite were examined for the winters of 1973 and 1974 to determine the variation between the two years of the heat loss by the atmosphere because of variations in sea ice extent and concentration. The microwave data indicate that most of the area within the ice edge is less than 80% ice covered even during the coldest part of the year, probably because of ocean currents, waves, and swell, and convergence and divergence in the atmospheric forcing fields. Since the winter heat and moisture transports from open water are about two orders of magnitude larger than from an equal area of sea ice, even small areas of open water within the ice edge can greatly affect the energy exchange. These new data are compared with the assumption of 100% ice cover within the ice edge and with previously assumed mean values for the total area covered by ice in calculating the heat lost by the atmosphere during the winter period in high southern latitudes.

A rapid decrease in sea ice extent observed during the winter of 1973 is correlated with a nearly real-time adjustment by the atmosphere to the change in the heat loss caused by the removal of the ice. This example indicates that sea ice dynamics is influential not only in long-term climate, but in synoptic-scale weather patterns as well.
INTRODUCTION

A case for the importance of Antarctic sea ice to global climate has been made by Fletcher [1969]. In his study a thermal model of the southern heat sink was used to compute the annual variation of the poleward gradients which force the zonal circulation of the atmosphere. The annual wind patterns calculated using the available data on the extent of the sea ice, estimated average values from atlases, and other sources [Treshnikov, 1967] were found to correspond well with observed winds at the given latitudes.

One of the main features of the ice cover is its ability to effectively shut off the transfer of heat and moisture from the ocean to the atmosphere. According to experimental measurements made in both the Arctic and the Antarctic [Badgley, 1961; Allison, 1973], the flux of heat and moisture over sea ice is two orders of magnitude less than over open ocean water at its freezing point. Possibly the sea ice cover exhibits a "positive feedback" influence on the atmosphere. Laevastu [1965] has found a preponderance of cyclones over areas where there is a large latent and sensible heat flux from the ocean to the atmosphere; areas where there are many anticyclones are associated with a reduced flux. If, for the sake of argument, we associate an anticyclone with the outbreak of a mass of cold polar air, then possibly the sea ice cover would exhibit a positive-feedback influence on the air mass; i.e., a cold air mass that could create ice would be maintained longer than expected by the growth of the ice cover because of the reduced flux of heat from the ocean. Also, the white-grey sea ice reflects more solar radiation than the dark ocean, thereby reducing the surface heating and sustaining the anomaly.

From this type of argument, Fletcher suggests that the variability in ice extent may be an important factor in amplifying the climatic effects of small changes in global heating. His model calculations indicate this effect should have a greater influence on thermal forcing during fall and winter than during summer. Streten [1973], basing his conclusion on the limited data on sea ice extent then available to him, gives at least general qualitative support for stronger circulation with greater ice extent. Budd [1975] finds that Antarctic regions with the coldest temperatures are also those regions of greatest sea ice extent. All these studies, however, have relied on data
available for the latter part of the winter season or on seasonal averages from several years of observation. Therefore, it would seem important to obtain more detailed observations of the fluctuations of the ice cover in order to establish the connection between these changes and changes in the atmosphere. This understanding may then be useful in understanding climatic changes and the role of the ice cover in modifying the properties of the atmosphere.

In this paper our use of new data for 1973 and 1974 allows a previously unobtainable estimate of sea ice extent and concentration within the ice edge. The winter seasons were of special interest, and we calculated the heat lost by the atmosphere south of 50°S using the new data; a comparison with results based on previously assumed data is carried out. Along lines similar to Vowinckel's [1966], we divided the Antarctic region into four longitudinal sectors to emphasize the somewhat different sea ice and heat budget regime in each sector.

VARIATIONS IN ICE EXTENT

Previous winter data have been sporadic, based as they are on observations from coastal stations, aircraft, and ships that all sample extremely small areas for short time intervals [Treshnikov, 1967]. Other observations have been based on meteorological satellite data (visible and infrared) from which time averaging removes the effect of clouds to produce a 5-day CMB (composite minimum brightness). However, the CMB data are available only for the latter part of the winter (late September on) because of the lack of sufficient light to illuminate the areas of interest during the middle of winter.

The new data for 1973 and 1974 from ESMR, the Electronically Scanning Microwave Radiometer, on board the Nimbus V satellite removes these difficulties since it measures the electromagnetic emission at a wavelength of 1.55 cm (~19 GHz). The emission is not dependent on the visible light received by the ice surface, so the time of year is of no consequence. The radiation at these wavelengths penetrates the clouds of low liquid-water content that are prevalent at high latitudes, and thus the emission from the surface can be received
at the satellite without being stopped by the cloud cover. The microwave emission is dependent on the surface temperature of the emitting surface and its emissivity; and because of the strong contrast between the emissivity of sea ice of ocean water ($\varepsilon_{\text{ice}} \approx 0.95$ compared with $\varepsilon_{\text{water}} \approx 0.4$), the ice boundary and the relative amounts of ice and water within the ice boundary (ice concentration) can be identified. The satellite makes enough passes to obtain the state of the ice cover and the ice boundaries around the entire continent in a single day [Gloersen et al., 1974; NASA, 1972].

The ESMR data are available in several formats, each used in a slightly different manner from the others. The first of these formats is the information on ice extent for the winter period. Using mostly ESMR data, the U.S. Navy Fleet Weather Facility (FLEWEAFAC) prepares weekly maps of both the Northern and the Southern Hemisphere showing the position of the ice edge and, during periods of interest to shipping, the variation of the concentration for certain areas within the ice edge. This information is prepared from the so-called first-look ESMR picture, a black and white photograph of the microwave brightness temperature variations of the individual orbit swaths of the satellite. Because of the large difference in brightness temperature between sea ice and water, the ice edge and ice properties can be readily identified. These swaths are geometrically distorted, but a grid overlay is available so that reference can be made to the global locations. The FLEWEAFAC transfers the information from 3-5 days of satellite orbits to a polar stereographic map centered at each pole and indicates the location of the ice edge averaged over this period as the location for that week.

For the present study, maps of the Southern Hemisphere from December 1972 through December 1974 were measured by planimetry to give ice extent, the total area bounded by the ice edges, as a function of time. These data are shown in Figure 1 for the winters of 1973 and 1974, for some of the earlier satellite data, and for the mean monthly extent given by Treshnikov [1967]. To obtain the area involved in sea ice coverage, the area of the Antarctic continent ($14.1 \times 10^6$ km$^2$) should be subtracted from the area given on this figure.

The large peak for the 1973 curve in the last half of April (Fig. 1) is probably not real but due to measurement errors in the ice extent for the
period. The ice extent data for March, April, and the first part of May 1973 were determined by the authors from limited first-look ESMR photographs, FLEWEAFAC maps not being available for that period.

In comparing the extents for 1973 and 1974 with the mean extent of Treshnikov, some cautious conclusions can be drawn. Treshnikov's areas apparently are not the ice extent as defined above; there may have been some compensation for concentration below 100% ice cover within the ice edge. According to Pease [1975], the extent given in the Soviet Atlas of the Antarctic corresponds to a maximum value of $19.9 \times 10^6$ km$^2$; compare this with the maximum value of $18.8 \times 10^6$ km$^2$ shown for Treshnikov's estimates. If we assume that Treshnikov's estimate is based on the Atlas extent values, the area of sea ice he gives appears to be the total area covered by sea ice and corresponds to a slightly greater extent (at less than 100% concentration) than Figure 1 shows. The true comparison, by taking the ratio of the Atlas values and the area given by Treshnikov, corresponds to an approximate 6% increase.
(1 × 10^6 km^2 at maximum extent) over the mean monthly extent given in Figure 1. Thus it could be argued that the extents given by Treshnikov that we plot in Figure 1 should be increased by approximately 6%. However, the mean extents are still lower than the observed values by approximately 20%.

Treshnikov indicates that maximum extent is reached in mid-September. The 1973 maximum was later, at the end of September, and 1974 had an earlier maximum, near the end of August. Although Treshnikov's estimated mean values may be difficult to compare with individual years, the variation between the two years of known data for the timing of maximum extent is considerable, from 4 to 5 weeks. With Fletcher's model, the difference between 1973 and 1974 would correspond to a change in atmospheric heat loss of about 8%-10%, with the greater heat loss occurring in 1974, the year with the earlier maximum. This comparison assumes that there was no great difference in the concentrations within the ice edge between the two years, an assumption that will be checked when concentration for 1974 is calculated.

It is also of interest that the growths for 1973 and 1974 were nearly coincident until the middle of August, when the extent in 1974 changed by approximately 2 × 10^6 km^2 within two weeks. It is difficult to argue that a change of this magnitude over such an extensive area could be caused by thermodynamic factors alone, and one might conclude that advection related to wind stress on the ice field [e.g., Gordon and Taylor, 1975] must have contributed to the increase in extent over this short time. The wind stress could be responsible for opening leads in the ice pack and increasing its extent while decreasing the concentration within the boundary. Exposed to a cold atmosphere deep in the pack, the leads would rapidly freeze over, producing in time an ice pack of about the same concentration but of greater extent. Analogous studies of Arctic situations [Hibler, 1974; Coon et al., 1974; Ackley and Hibler, 1975] have indicated that sea ice will diverge under certain synoptic conditions. The fluctuations in ice extent recorded in Figure 1 are erratic, but seem to occur on time scales comparable to the fluctuations in synoptic scale weather, within the limitations imposed by a once-a-week sample. This point is examined more closely in a later section.

While some of the differences between the Treshnikov curve and the measured values for 1973 and 1974 can be questioned because of the confusion
regarding ice concentration, the measured curves give the same value for ice extent as the mean value for mid-April. If these values are corrected for concentration, it is likely that a considerable discrepancy would exist between the Treshnikov value assumed (already corrected for concentration) and the actual value of ice area for 1973 and 1974 during the early part of the growth season (April). The 6% adjustment in the mean value curves still would not bring it up to the value obtained for the ice extent in winter 1973 and 1974, leading to the conclusion the annual variation of ice cover is greater than previously indicated. However, whether this result corresponds to more ice (actual area covered by sea ice) and thus greater atmospheric heat loss depends on the concentrations observed in 1973 and 1974, which are discussed in a later section.

Using the total extent curves in Figure 1 as a basis, one could argue that ice extent for 1974 did not differ much from the 1973 extent until late August. However, if the Antarctic continent is divided into four sectors (Figure 2) and the data for those regions considered discretely (Figure 3), the regions show significant variations between the two years that are not reflected in the total extent. For example, according to Figure 3 the Weddell Sea ice had consistently greater extent in 1973 than in 1974, while the Ross Sea showed an opposite trend. (The breadth of these distribution differences is particularly evident in Figure 4, which shows the ice extent at the first of August.) The Amundsen-Bellingshausen sector had more ice in early winter and less ice in late winter of 1973 than it had in the same periods of 1974.

Examination of the change in ice extent from week to week shows little correlation between sectors for periods of heavy growth or loss. This lack of correlation and the variation in each sector from one year to another indicate that an examination of the ice growth and decay in the individual regions may be a more useful approach to understanding the relationship between the ice cover and the atmosphere than just a consideration of the total ice cover. The differences in the ice distribution as shown in Figure 3 would certainly contribute to shifting the influence of the pack on the atmosphere from one side of the continent to the other. According to Pease [1975], some ice atlases show consistently larger extents in either western longitudes or eastern longitudes than do other atlases. The data in Figure 3 indicate that
Fig. 2. Map of the Antarctic region showing the definition of the four sectors used in this paper.

Fig. 3. Sea ice extents in the four sectors shown in Fig. 2 for the winter months of 1973 and 1974.
Fig. 4. Comparison of sea ice extents for about the first of August in 1973 and 1974.

these fluctuations are real and that there can be a shift from one side of the continent to the other in ice extent between years and within a given year. To determine whether these patterns are a consistent long-term feature or are just fortuitously seen in these two years requires more data. What it does indicate is that the previous estimates from the various atlases can be either consistent with or quite different from a single year's observation, and that what were assumed to be discrepancies could actually be a real annual variation of the ice cover.

Evidence exists that links some atmospheric phenomena to the propagation of anomalies around the ice cover. In Budd's [1975] investigation, a large air temperature anomaly prevailing over a large geographical area was accompanied by an anomalously large ice extent. There was also some indication that the temperature anomalies propagated as much as 100 degrees longitude per year and come in pairs, a positive anomaly being associated with a negative one farther around the coast. If future data substantiate this behavior,
then global climate models may have to incorporate a time-varying ice distribu-

tion regionally around the continent in order to give a more realistic 

picture of the ice-atmosphere interaction in the Southern Hemisphere. This 
effect will be further examined in flux calculations of the heat loss by the 
atmosphere later in this paper.

Another contrast between the two years is shown in Figure 5. Here the 

change in ice extent for each year from the beginning to the end of winter is 

shown for two similar dates for each year. The 1974 map (Fig. 5b) displays 
the trend previously predicted from atlas data; that is, the ice extents as 
the season advances to the time of maximum extent in September are "nested," 
with the later margin encompassing the earlier one at nearly all points around 
the continent. Some deviation from this behavior is seen in the Ross Sea 
area, where the extents are nearly coincident; basically, however, the ice 
extent for June lies enclosed within the extent for September.

The 1973 data (Fig. 5a) indicates a different distribution with several 
crossovers between the curves and considerable variation from the mean situ-

ation of nested extents. This behavior is especially evident from about 120°E 
estward to the Antarctic Peninsula (60°W). Over segments of 30-40 degrees 
longitude, the time of maximum extent from 120°E to 60°W varied, in order, 
from June to July (not shown in the figure) to September and back to June near 
the Peninsula. The boundary transformed from a roughly symmetric boundary at 
approximately 65°S during June, to a sharply angular one during September, 
the ice boundary showing large excursions to higher and lower latitudes than 
65°S as a function of the longitude between 120°E and 60°W. If the ice edge 
does form a boundary that provides the thermal contrast necessary to increase 
the baroclinicity of the atmosphere in its vicinity [Gibson, 1974], this 
change should also affect the atmospheric heat loss over large regions.

We can conclude from these studies that the regional changes in ice 
distribution show large variations and, as later data in this paper show, 
these changes can be of considerable importance in the manner that the atmos-

phere responds to changes in the net distribution.
Fig. 5. A comparison of the sea ice extents for days in the winter of (a) 1973 and (b) 1974.
VARIATIONS IN ICE CONCENTRATION WITHIN THE ICE EDGE

As noted previously, the variation in ice concentration is an important factor in considering the total area covered by sea ice. It may be difficult to make an estimate of the magnitude of the polar heat sink and its seasonal and annual variations unless some measure of the ice concentration is made, because a change in ice extent that is accompanied by divergence producing open water within the ice boundary could actually leave the heat exchange with the atmosphere unchanged. Observations of the concentration changes were previously unobtainable on synoptic scales during the winter period because the cloud cover obscured the ice cover for satellite observations, the ground station information was sparse, and ships were not present during that season. The ESMR data give information on the ice concentration because the received signal is proportional to the relative amount of open water and sea ice within the sampled area (typically a square 32 km on a side).

Gloersen et al. [1974] have shown the relation for the fraction of open water is

\[ F = \frac{T_b - eT_s}{T_w - eT_s} \]  

(1)

where \( F \) is the open water fraction, \( T_b \) is the observed brightness temperature, \( T_s \) is the surface temperature, \( T_w \) is the sea water brightness temperature (\( \sim 131^\circ K \)), and \( e \) is the emissivity of annual sea ice (0.95). Since snow and freshwater ice are low-loss dielectrics while sea ice has medium- and high-loss characteristics, eq. (1) applies only to mixtures of sea ice and sea water.

Unlike the ice extent information, which was compiled from first-look ESMR images, the variation in ice concentration was obtained from the processed pictures provided by NASA in either photographic or digital form. This information typically is the brightness temperature averaged from several hundred readings for a particular location over a 24-hour period. In the digital form we used, the brightness temperature is given on a polar stereographic projection at a scale of 1:20,000,000, and the data are averaged over a square with sides of approximately 120 nautical miles (\( \sim 190 \text{ km} \)) at 60°S. The number of readings depends somewhat on the track of the satellite.
To obtain an average ice concentration for each sector (Fig. 2), the brightness temperatures are averaged between the longitudinal boundaries of each sector and latitudinally between the Antarctic continent and the 150°K brightness temperature contour. Assuming for $T_s$ a climatic average surface temperature from Schwerdtfeger [1971], the choice of 150°K to define the ice edge corresponds to an ice concentration of about 14%. According to eq. (1) the ice concentration does not drop to zero until $T_b$ is near 131°K. However, storm action can raise the brightness temperature to higher than 131°K, and the 131°K contour showed large excursions into lower latitudes (north of 50°S) and enclosed many isolated patches of ice and ocean. The ice edge is characterized by strips and pieces of loosely consolidated pack ice separated from the main pack by winds, currents, waves, and swell. Because the true edge is so indistinct, FLEWEAFAC for its purposes defines the ice boundary as the point at which ice concentration falls below 1 okta (12%). Thus, the choice of the 150°K contour as the ice edge is consistent with the ice forecasting definition.

Figure 6 shows ice concentration during the 1973 winter in the four sectors identified in Figure 2. Values are plotted only for days that data were collected in all sectors. These concentrations were obtained, as described above, by averaging the $T_b$ between the 150°K contour and the Antarctic continent for one sector on one day, using the climatic average surface temperature for that sector at that time of year [Schwerdtfeger, 1971] in eq. (1) with the average $T_b$, and solving for the average ice concentration. (Note that the ice concentration is actually $1 - F$.)

One sees in Figure 6 the large variability in average concentration for the four sectors. One might expect, since long-term wind forces act over all regions for extended periods, that the average ice concentration would be roughly the same in all sectors and that, because of storms and other wind and ocean activity, a given region would manifest a fairly large day-to-day variation. In fact, however, the concentration variation is large between regions at any given time, while in each sector the concentrations tends not to vary a great deal (possibly a result of the spatial averaging); and the long-term average values for the different regions are quite distinct from each other, while the day-to-day variation in each region is usually less than the variation from region to region.
Fig. 6. Ice concentrations within the pack ice boundary for the available data in 1973 for each of the four sectors.

The other surprise in the data is that the concentrations everywhere are so low: a maximum of only 80% and some averages as low as 50% ice. The ice concentration is calculated with climatic average surface temperature (a long-term average) and therefore may be in error for the particular time studied. However, Gloersen et al. [1974] have estimated that an uncertainty in the surface temperature of ±5°K leads to an uncertainty of only ±6% in the estimate of the ice concentration. The temporal and spatial variations shown in Figure 6 are well beyond these error limits. Our values indicate considerably more water than previously estimated by, for example, Treshnikov [1969], who assumed about 6% average open water during the winter. Because freshly deposited snow, icebergs in the pack, and clouds with a high water content tend to lower brightness temperature [Gloersen et al., 1974], the method used here possibly overestimates the open water in the pack, so that the estimates given in Figure 5 should be regarded as the smallest possible concentration. However, the spatial and temporal variations shown in Figure 5 are beyond any reasonable error estimation, and those values indicate considerably more water than was previously thought to be present.
Because of these large open water values and the regional variation of the ice concentration, estimates of the heat loss and interaction with the atmosphere must consider these effects as well as ice extent. The variation in concentration between regions also indicates that the fluctuations in ice extent referred to in Figures 1 and 3 should be corrected by region for comparison of the total ice area change on a seasonal and annual basis, and ideally the correction should be made point by point.

**ATMOSPHERIC HEAT LOSS AS FUNCTION OF ICE EXTENT AND CONCENTRATION CHANGES**

In this section the information on ice extent changes is used to estimate the variation in heat loss by the atmosphere in the high latitudes of the Southern Hemisphere. We used the thermal model of Fletcher [1969], in which he calculates the energy budget over a unit area of sea ice, ocean, and continental ice. Fletcher defines an atmospheric heat gain as the absorbed solar radiation minus the long-wave loss to space plus the heat flux to the surface from below. The absorbed solar radiation is the sum of the solar radiation absorbed in the atmosphere and the solar radiation absorbed at the surface. In this context the surface could be the Antarctic continent, open ocean, or ice-covered ocean. The heat flux to the surface from below is the flux due to conduction from below and the latent heat of melting at the surface. From the surface flux balance equation, one may deduce that this flux from below is just the negative of the latent and sensible heat fluxes and the net surface radiation flux. The net surface radiation is the difference between the solar radiation absorbed at the surface and the outgoing long-wave radiation from the surface. At high polar latitudes the atmospheric heat gain is a negative quantity. We will define the atmospheric heat loss as the negative of the atmospheric heat gain, resulting in positive quantities for the heat loss.

Fletcher finds the atmospheric heat gained as a function of season over the unit areas of sea ice, ocean, and continent. Digital estimates from the curves of these parameters on a weekly basis and the weekly measured values for ice extent are used to compute the atmospheric and oceanic heat loss as a function of time for the two years of winter data. The boundary
for the calculation is the 50°S latitude circle. The results for the total atmospheric heat loss from 1973 ice data are shown as curves 2 and 3 in Figure 7, together with the heat loss computed from average values for ice area given by Treshnikov (curve 1). Both curves 1 and 2 assume 100% ice cover; both show greater heat lost by the atmosphere than is shown by curve 3, the 1973 data revised to include the effect of varying concentration. It is clear that including the ice concentration modifies the heat transfer significantly.

If we assume that these estimates are not too bad and compare Treshnikov's average with 1973 values, we see that the heat loss per week from the atmosphere is of the order of from $1 \times 10^{16}$ to $5 \times 10^{16}$ kilocalories greater on the average than in 1973. Since the average total energy of a cyclone is of the order $0.5 \times 10^{16}$ kilocalories [Sellers, 1965], these differences, if real, could contribute enough energy to the atmosphere to intensify or create storms or significantly strengthen or weaken the mean flow.

Figure 7 also shows the relative minimum in the heat loss curves occurring in June, at the start of deep winter. This feature results from choosing 50°S as the outer boundary for the computation: it completely encloses the

![Diagram](image-url)

**Fig. 7.** Heat lost by the atmosphere south of 50°S for winter months of 1973, and average values from Treshnikov [1967]. Curves 1 and 2 assume 100% ice cover within the ice edge; curve 3 takes into account a reduced ice concentration.
ice-covered portion of the ocean during all periods of the year, but in so doing it also includes a large portion of icefree ocean. A boundary at higher latitudes, closer to the ice edge, would minimize the area of open ocean and reduce this effect.

According to Fletcher, this early minimum heat loss arises because the total heat lost to space during the winter is supplied directly by emitted long-wave radiation from the ocean; therefore, the heat lost by the atmosphere over open ocean is relatively small during winter. This effect dominates the heat loss by the atmosphere during the early part of the winter since the ice extent is well below its possible maximum while, at the same time, the heat loss per unit area over open ocean has already reached a minimum value. The product of the minimum heat loss coefficient with an extensive open ocean area produces the relative minimum in the total atmospheric heat loss curve of Figure 7. As the winter progresses, the general trend is for atmospheric heat loss to increase as the ice extent increases to the maximum in each region. The heat loss by the atmosphere decreases rapidly in September once the equinox is past because the radiative heating leading to surface warming rapidly increases, even though the ice is at near-maximum extent through the end of September.

In general, under the assumption of 100% ice cover within the ice edge, the 1974 heat loss by the atmosphere follows quite closely that of 1973, as would be expected from the small variation in the total ice extents shown in Figure 1. For this reason, the calculated value for the 1974 total heat loss is not plotted in Figure 7. However, there are significant variations between regions, as would be expected from the regional differences in ice extent between 1973 and 1974 shown on Figure 3. These regional data are compared in Figure 8 for the two years, with the assumption that the ice boundary defines the region of 100% ice cover. The corrected heat loss data using the average concentration information for 1973 are also given. There are significant variations of total heat loss from region to region, and they persist throughout the winter periods: the Weddell and Ross Sea areas differed by $1-2 \times 10^{16}$ kcal/week between the two winter seasons, the Weddell showing less heat loss in 1974 and the Ross more. Evidence of the earlier maximum ice extent in 1974 is seen during the latter part of August, which showed sharp increases in the
heat loss for the Weddell and Bellingshausen areas (and continuation of no noticeable change from the usual upward trends in the other regions).

In the Bellingshausen region, 1973 shows an unusual heat loss curve: a second minimum following an anomalous reversal in the cooling trend in the mid-July period. The "normal" effect is for cooling to continue from the June minimum until the equinox in mid-September or later as shown on the heat loss curve for 1974 in this region and the curves for both 1973 and 1974 for all the other regions. The corrected heat loss curve for 1973, when the concentration within the ice boundary is computed, also reflects the downturn in the heat loss that the uncorrected value does. Referring to Figures 3 and 6, this second minimum follows the large decrease in both extent and concentration during a ten-day period in mid-July 1973 for this region. The relationship of this ice loss to the synoptic weather pattern will be examined in the next section.

Fig. 8. Heat lost by the atmosphere in each sector for the winters of 1973 and 1974, including the corrections due to measured ice concentrations for 1973. A second minimum in the heat loss curves caused by an ice loss in 1973 is noted on the curves for sector I.
EFFECT OF ICE VARIATIONS ON ATMOSPHERIC ACTIVITY

Weather charts and hemispheric satellite mosaics for the ice loss in July 1973 in the Bellinghausen Sea show a blocking high at the surface over South America for an unusually long period (10 days). The cloud pattern suggests that large amounts of warm air were pumped from lower latitudes into the Bellinghausen Sea area on the western side of the Antarctic Peninsula, a likelihood strengthened by reports of sharp increases in air temperature from the coastal stations on the Peninsula during this period. The ice loss and reduction in concentration are connected with this transfer of air from lower latitudes, although the relative contributions of in situ melting and wind-induced divergence and advection to warmer water areas have not been determined yet. The effect of this single synoptic event then apparently was reinforced by subsequent adjustment in the forcing of the ice cover once it was placed into this less extensive, less concentrated state.

The rest of the period until mid-September, usually characterized by an increase in ice extent, was instead a period of continuing ice recession and low concentration. These effects are summarized in Figure 9, where the values for ice extent, concentration, corrected area of pack, and atmospheric heat loss are plotted. We conjecture that the atmosphere responded to this decrease in heat loss (the second minimum referred to earlier and also shown in Figure 9) by an increase in storm activity once the ice cover was removed. This is indicated in Figure 10, where the ice extents in the Bellinghausen area are plotted during the 1973 and 1974 winter periods and the number of low days per week (the total for one week of the product of the number of lows and the days they were present) over the same periods are plotted below. This figure shows that for the early part of the winter season the 1973 ice extent exceeded the 1974 extent and the number of low days during this period was lower for 1973 than for 1974. Once the large ice loss referred to in Figure 9 occurred, the ice extent for 1973 fell below the similar period in 1974 and the number of low days per week also reversed and rose above the 1974 level at about the same time as the ice loss period.

A possible explanation for this activity is that the increase in heat flux over the newly icefree areas either sustained storms that otherwise would
have died out over the ice cover or strengthened the weak frontal activity. A further feedback can be postulated to then sustain the ice cover at the lower level of ice concentration by consideration of Hibler's [1974] theoretical model of the ice response to atmospheric forcing. His model is supported by experimental evidence from observations taken on the Arctic pack [Hibler et al., 1974].

In this model the ice acts as a viscous fluid with bulk and shear viscosity coefficients dependent on the ice resistance between floes which is probably a function of the ice concentration. The model predicts that in the high viscosity limit (high value of ice concentration) the ice will generally diverge under high atmospheric pressure and converge under low atmospheric pressure. However, when the viscosity is low (low concentration), the water...

Fig. 9. Ice pack characteristics, atmospheric heat loss, and number of low days per week for sector I (Amundsen-Bellingshausen) for 1973.
Fig. 10. Atmospheric heat losses and number of low days per week for the Amundsen-Bellingshausen sector (sector I) in winters of 1973 and 1974. The increase in the number of low days in 1973 coincides with an increase in available heat from oceanic sources (decrease in atmospheric heat loss shown in the bottom curve).

stress exerts a greater influence, and the opposite trend is observed; i.e., the ice will diverge in low pressure. This model at present is constructed for an infinite boundary solution, so that the behavior at the ice boundary is not predicted. In other areas of the Antarctic, we have found some qualitative support for the behavior in the high viscosity case predicted by the model (unpublished).

With these cautions, the model may be used to explain the behavior seen in the Bellingshausen area during 1973. Under "normal" conditions, the ice in winter is sufficiently concentrated to act like a highly viscous material, so that lows entering the pack tend to cause the ice to converge, reducing the oceanic contribution of heat and moisture to the atmosphere and thereby
increasing the overall atmospheric heat loss. However, the ice cover may respond to some synoptic event (such as, in 1973, the blocking high transferring heat from lower latitudes) by lessening in extent and decreasing substantially in concentration. This looser ice cover then responds to lows by diverging, which, by exposing the ocean, allows greater transfer of heat and moisture to the atmosphere and causes a relative reduction in the atmospheric heat loss. The increase in available energy then strengthens the depression or provides enough input to sustain it longer. Once the ice cover is sufficiently loose, the process is self-sustaining; i.e., the increase in lows causes more ice divergence, which causes more lows to form, and so on, until subsequent thermodynamic effects or an adjustment by the atmosphere allows the ice cover to increase its compactness and return to the "normal" winter response.

Although more details are needed to verify the interaction postulated here, the data given in Figure 10 indicate that, at least for this part of the Antarctic sea ice region, there is a close correspondence between the properties of the ice cover and the behavior of the atmosphere.

CONCLUSIONS

In this report we have investigated the variability of the sea ice cover in the Antarctic by means of ice extent data from ESMR pictures for two winters. These ESMR images provide a resolution previously unavailable without sacrificing the total view and fine time scale necessary for studying the interaction of the ice cover with the atmosphere. The influence on the ocean of long-term variations in the ice cover has not been treated here, but it is an important area of investigation for the future.

The data for 1973 and 1974 reveal widespread differences in sea ice extent and concentration within the ice edge. Concentration is considerably lower than previously assumed, indicating that the Antarctic pack is strongly divergent. The heat balance from Fletcher's model, when corrected to account for ice concentration, gives atmospheric heat losses for 1973 of between $1 \times 10^{16}$ and $5 \times 10^{16}$ kilocalories per week less than those predicted using atlas values for ice extent during the winter.
A single synoptic event, such as a blocking high, may trigger ice events that change the character of atmospheric activity over a region for the remainder of the winter (Fig. 9). These changes may affect the atmosphere over other regions, so that the atmospheric variations that appear to be random events may in fact be a response to some short-term change in the ice cover.

It can be seen that the state of the ice cover is a major factor in atmosphere-ice interaction. According to the evidence given here and elsewhere, the same atmospheric driving force will cause opposite responses in ice of low concentration and ice of high concentration. The ice cover, especially in winter, can therefore provide either a positive or a negative feedback mechanism for the growth of atmospheric disturbances, with the sign of the feedback determined by the state of the ice cover when forced by the atmosphere.

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Is the stress tensor symmetric for sea ice at all scales? It is usually treated that way in a continuum approach. When one does a moment of momentum analysis on a control volume and lets the control volume shrink to zero, the result is $\sigma_{ij} = \sigma_{ji}$ in the absence of magnetism, electricity, etc. But can we use this result for studying the dynamics of floating ice?

Let us start with a control volume (hereafter referred to as c.v.) and write down the momentum equation as well as the equation for the angular momentum. The c.v., which comprises several floes, consists of the four control surfaces shown in Figure 1 plus the top and bottom control surfaces that coincide with the top and bottom of the floes.

The stress, $\sigma$, has the unit of force per unit length. The unit is not the usual force per unit area because we are mainly concerned with a two-dimensional stress field, $\sigma_{i,j}$, where $i,j = 1, 2$. Thus $\sigma$ is the three-dimensional stress integrated over the ice thickness [Rothrock, 1970]. The two-dimensional $\sigma$ is more meaningful since ice thickness is highly variable (R. Pritchard, personal communication, 1975).

$S_1$, $S_2$ represent components of surface stress per unit surface area acting on the top and bottom ice surfaces, and $B_1$, $B_2$ represent body force per unit surface area. Thus

\[
S_1 = \tau_{a_1} + \tau_{w_1} - \rho_1 gh \frac{\rho_i}{\rho_w} \frac{\Delta h}{\Delta x_1}
\]

\[
S_2 = \tau_{a_2} + \tau_{w_2} - \rho_1 gh \frac{\rho_i}{\rho_w} \frac{\Delta h}{\Delta x_2}
\]

\[
B_1 = -\rho_1 gh \frac{\Delta h}{\Delta x_1} + \rho h f u_2
\]
Here $\tau_a$ and $\tau_w$ are the air stress and water stress; $\rho_i$ and $\rho_w$ are the densities of ice and water, respectively; $h$ is the thickness of ice; $\rho$ is the mesoscale density of the floes, which is less than or equal to $\rho_i$; $f$ is the Coriolis parameter; and $u_1$, $u_2$ are the average velocities of the material in the c.v. in the $x_1$ and $x_2$ directions. The terms $-\rho_1gh \frac{\Delta h}{\Delta x_1}$ and $-\rho_1gh \frac{\Delta h}{\Delta x_2}$ are stresses from the water pressure acting on the inclined undersurface of the ice [Nye, 1973b]. The terms $-\rho_1gh \frac{\Delta h}{\Delta x_1}$ and $-\rho_1gh \frac{\Delta h}{\Delta x_2}$ are due to gravity, while $\rho_1 f u_2$ and $-\rho_1 f u_1$ are due to the rotation of the earth with respect to an inertial reference frame, or Coriolis force [Kane, 1968].

![Diagram](image-url)

Fig. 1. Control volume analysis
The momentum equations are then [Rothrock, 1970]

\[ \rho h \frac{Du_1}{Dt} = \frac{\Delta C_{11}}{\Delta x_1} + \frac{\Delta C_{12}}{\Delta x_2} + S_1 + E_1 \]

(1)

\[ \rho h \frac{Du_2}{Dt} = \frac{\Delta C_{21}}{\Delta x_1} + \frac{\Delta C_{22}}{\Delta x_2} + S_2 + E_2 \]

(2)

where \( \frac{Du_1}{Dt} \) and \( \frac{Du_2}{Dt} \) are the substantial derivatives or accelerations.

The above equations can be obtained either directly from Newton's law of motion, \( \sum F = ma \), in a Lagrangian sense or from a complete momentum balance on the c.v. in a Eulerian sense, wherein the momentum flux into and out of the c.v. shown in Figure 1 includes terms such as \( \rho u_1^2 h \), \( \rho u_1^2 h + \Delta(\rho u_1^2 h) \), and \( m u_1 \) (the \( x_1 \) momentum that comes into or out of the c.v. from precipitation, freezing, and melting). If we combine these with a mass balance, we will get the same results by the use of \( \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla = \frac{D}{Dt} \).

Referring to Figure 1, let us assume that the line of action of each stress (or body force per unit surface area) goes through the center of the surface (or body) on which it acts. Using the Eulerian approach, which is more convenient here, we can write down the principle of angular momentum as follows:

Increase in the storage + net rate of outflow of angular momentum = sum of torques

Since the increase in the storage of angular momentum in the c.v. about point 0 is zero (corresponding to the \( \frac{\partial}{\partial t} \) part of the \( \frac{D}{Dt} \)), the equation of angular momentum about point 0 is then

\[ -\frac{1}{2} \Delta x_2 \cdot \Delta(\rho u_1 u_2 h) \Delta x_1 = \frac{1}{2} (\Delta x_1)(\Delta x_2) [2(\sigma_{21} - \sigma_{12}) + \Delta(\sigma_{21}) - \Delta(\sigma_{12})] \]

or

\[ 2(\sigma_{21} - \sigma_{12}) + \Delta(\sigma_{21}) - \Delta(\sigma_{12}) + \Delta(\rho u_1 u_2 h) = 0 \]

(3)

where \( \Delta(\rho u_1 u_2 h) \) is the \( \mathbf{u} \cdot \nabla \) part of the substantial rate of change of angular momentum.
If we take the limit and let the $c,v$ shrink to zero, then (3) gives $\sigma_{21} = \sigma_{12}$. But when we do not let $\Delta x_1, \Delta x_2 \to 0$, then $\sigma_{21} \neq \sigma_{12}$ in general. And if $\Delta(\rho u_1 u_2)$ is negligible, $\sigma_{21} \neq \sigma_{12}$ is still a possible result. Now let us do assume that the change of angular momentum is negligible; then for $\Delta x_1 = \Delta x_2$, we will get an equation of angular momentum about point $A$:

$$
2(\sigma_{21} - \sigma_{12}) + 2\Delta(\sigma_{21}) - 2\Delta\sigma_{12} + \Delta\sigma_{22} - \Delta\sigma_{11} + (S_2 + B_2 - S_1 - B_1)\Delta x_1 = 0
$$

(4)

For large values of $\Delta x_1$, $\sigma_{21}$ cannot be equal to $\sigma_{12}$ unless $\Delta\sigma_{22} - \Delta\sigma_{11} + (S_2 + B_2 - S_1 - B_1)\Delta x_1 = 0$, which is not an identity. The difference between $\sigma_{21}$ and $\sigma_{12}$ gets small as $\Delta x_1$ gets small. We believe that for the Arctic Basin ice cover it may be incorrect to let $\Delta x_1 \to 0$ because it would imply a continuum without any scale limitations. It seems that when a limiting procedure is taken, one should use $\Delta x_1 + \ell$ instead of $\Delta x_1 \to 0$, where $\ell$ is the scale at which the material can be considered as a continuum [Nye, 1973a]. In the case of modeling air or water (laminar flows only) or solid metals, the last term in equation (4) is usually $(B_2 - B_1)\Delta x_1$ and the scale $\ell$ at which the material is treated as a continuum is very small. Furthermore, $B_2,B_1$ are usually small, too (in most cases they are not the driving force); therefore, $(B_2 - B_1)\Delta x_1$ is negligible compared with $\sigma_{1j}$, and (3) and (4) give $\sigma_{21} = \sigma_{12}$ as a solution.

When we come to ice dynamics, it can be seen from (2) that $S_2 + B_2$ is the same order of magnitude as $\frac{\Delta\sigma_{21}}{\Delta x_1}$, or $(S_2 + B_2)\Delta x_1 = 0(\Delta\sigma_{21})$. The scale at which the motion of the sea ice can be considered as continuous may not even exist [Nye, 1974]; but if we do assume that it exists, it is probable that it is large—on the order of 100 km—and $\Delta\sigma_{21}, \Delta\sigma_{12}$ can be very significant compared with $\sigma_{21}$ and $\sigma_{12}$. Thus $\Delta\sigma_{21} = 0(\sigma_{21})$, so $(S_2 + B_2)\Delta x_1 = 0(\sigma_{21})$. The last term in (4) therefore cannot be neglected. Figure 2 shows a comparison of $\Delta\sigma$ with $\sigma$ for a large-scale continuum such as sea ice. It is represented in a way to correspond roughly to a kind of typical air stress field over the whole Arctic.

What seems to be improper in using the differential equation obtained from $\Delta x_1 \to 0$ is the fact that when it is integrated numerically the grids...
have to be so large that it is no better than taking a large c.v. and making momentum and angular momentum balance about it to start with. It seems suspect to throw away the higher order terms by first shrinking the c.v. to zero and then during calculation blow the c.v. up to a large size again without considering the effects of (4).

Therefore, with a large-scale continuum we should probably look into something other than $\sigma_{i j} = \sigma_{ji}$. We can write $\sigma_{ij} = \sigma_{ij}^S + \sigma_{ij}^A$, where $\sigma_{ij}^S$ is the symmetric part of the stress tensor and $\sigma_{ij}^A$ is the antisymmetric part of the stress tensor.

Other authors have concerned themselves with the effects of antisymmetric stress. Mindlin and Tiersten [1962] investigated the effects of couple stresses in linear elasticity by extending Cosserat's classic work on the subject; Condiff and Dahler [1964] studied the fluid mechanical aspects of antisymmetric stress to account for some structural aspects of fluid flow. Erdogan [1972] studied the dynamics of polar fluids in which fluids contain structures such as ions, atoms, molecules, bubbles, and particulate matters. In the works just mentioned, the basic equations of
motions are derived in such a way as to include a couple-force vector as well as a couple-stress tensor. Evans [1975] considered arctic pack ice as composed of a series of discrete interacting floes, each of which behaves essentially as a rigid body except for local deformations at floe boundaries. In his work, a spin vector is included which, if not equal to the field vorticity, will give rise to an antisymmetric stress. In the present report, however, we are not including a couple force acting on ice and therefore there would be no couple stress.

Letting $\Delta x_i \to \lambda$ we get from (3)

$$\sigma_{ij} \neq \sigma_{ji}$$

and from (1) and (2)

$$\rho \frac{Du_i}{Dt} = \frac{\partial' \sigma_{ij}}{\partial' x_j} + S_i + B_i$$

Here the symbol $\partial'$ is defined as follows:

$$\lim_{\Delta x_i \to 0} \frac{\Delta \sigma_{ij}}{\Delta x_i} \frac{1}{\Delta x_j} = \frac{\partial' \sigma_{ij}}{\partial' x_j}$$

It is a partial derivative in a mesoscale sense. Equations (5) and (6) can be applied to a large-scale continuum whether it is treated as elastic, plastic, elastic-plastic, or fluid.

For the sea ice the acceleration term is small [Rothrock, 1970] and we have

$$\frac{\partial' \sigma_{ij}}{\partial' x_j} + S_i + B_i = 0$$

$$\sigma_{ij} \neq \sigma_{ji}$$

If one is treating the sea ice as a mesoscale fluid, then one can write

$$\sigma_{ij} = (-\rho + \lambda \nabla' \cdot u) \delta_{ij} + u \left( \frac{\partial' u_i}{\partial' x_j} + \frac{\partial' u_j}{\partial' x_i} \right) + \sigma_{ij} A$$
Here we may assume

$$\sigma_{ij}^A = \eta \left( \frac{\partial'^u_i}{\partial x_j} - \frac{\partial'^u_j}{\partial x_i} \right)$$

as a first try, in which $\lambda$ and $\mu$ are constants, $u$'s are velocities, and $\eta$ is a coefficient which depends on the length scale $\lambda$ (when $\lambda \to 0$, $\eta \to 0$). What we have done now is to enlarge the definition of a stress tensor for mesoscale materials by putting in a nonzero $\sigma_{ij}^A$.

It should be pointed out that $\frac{\partial'^u_i}{\partial x_j} - \frac{\partial'^u_j}{\partial x_i}$ is not the spinning of individual floes. It consists only of differentials (or, rather, differences) of velocities that correspond to the vorticity of the mesoscale velocity field. Since we are concerned only with the differences of $u$'s, or the second "derivative" of relative distances, it does not violate the principle of material indifference [Truesdell and Toupin, 1960].

The important point to consider is that for the Arctic Ocean and parts of the Antarctic oceans $\lambda$ can be large. Recent aircraft and satellite data [Campbell et al., 1974, 1975] show that the Beaufort Sea has a significant variation of floe size, with many large floes, up to 60 km in diameter, in the eastern part and much smaller ones in the western part. During recent aircraft flights (Campbell, to be published) between Greenland and the North Pole, many large aggregates composed of numerous small and large floes were observed which had dimensions on the order of 100 km. Marko and Thomson [1975] have noted the presence of large-scale, spatially rectilinear leads separated by distances of approximately 100 km through satellite imagery of the ice-covered Canada Basin in the Arctic Ocean. This is a further evidence that the sea ice as a continuum has a very large scale.

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MISGIVINGS ON ISOSTATIC IMBALANCE AS A MECHANISM FOR SEA ICE CRACKING

by

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INTRODUCTION

The AIDJEX model assumes that the pack ice can be modeled as an elastic-plastic material and that, on the scale considered by the model, the ice is densely fractured [Coon et al., 1974]. It further assumes that enough cracks are formed by other processes that the model can ignore how cracks are formed. The model then seeks to describe the ice motion by deformation and movement at the pre-existing cracks.

The mechanism of how cracks are formed is not an easy problem for materials as complex as sea ice. Our purpose here is to examine one of the invoked mechanisms, isostatic imbalance as presented by Schwaegler [1974], since we think this particular mechanism needs closer examination based on a more realistic picture of the physical properties of the ice. This mechanism has already been referred to in articles as a cracking mechanism [Colbeck et al., 1975; Rothrock, 1975; Coon et al., 1974] even though several assumptions in the analysis require rethinking.

To summarize his work briefly, Schwaegler [1974] finds that by using a model containing thickness variations similar to those observed on multi-year ice [Hibler et al., 1972; Ackley et al., 1974] the isostatic imbalance from the thickness variations leads to bending stresses large enough to cause cracking. In his analysis, the ice floe is assumed to be a homogeneous, isotropic, linear elastic plate of variable thickness. By various formulations of beams on elastic foundations, he finds that the stress induced within the floe exceeds the cracking stress over a range of roughness element sizes that are typically found in multiyear ice.
PHYSICAL PROPERTIES OF MULTIYEAR ICE

Along with the documented thickness variations of multiyear ice, calculations of this type should also consider the evidence that multiyear ice is an inhomogeneous medium with measurable density variations. Examples of the variation are shown in Figure 1, the vertical profiles of a multiyear floe obtained during the 1971 AIDJEX pilot study [Ackley et al., 1973]. Further evidence for these variations is given by Cox and Weeks [1974] from observations obtained during the 1972 pilot study.

In a previous paper [Ackley, et al., 1974], we compared a model for predicting multiyear ice thickness assuming constant density for the ice (equivalent to the homogeneous, isotropic assumption used by Schwaegler) with one using a variable density. The variable density used in the predictive model was based on the data shown in Figure 2. In this figure, the effective freeboard at each measured point (ice freeboard plus the snow depth in ice equivalent) is plotted against the density calculated from assuming isostatic equilibrium at each point, i.e., solving the following equation for the ice density:

\[ \rho_1 T + \rho_s s_d = \rho_w (T - f) \]  (1)

Here \( \rho_1, \rho_s, \rho_w \) are the densities of ice, snow, and water, and \( T, f, \) and \( s_d \) are the ice thickness, freeboard, and snow depth.

The density–freeboard relation was then obtained by fitting the least-squares line to the data over the range shown in the figure. The equation for this line is:

\[ \rho_1 = -194 f' + 974 \]  (2)

where \( \rho_1 \) is in kg m\(^{-3}\) and \( f' \) is the effective freeboard defined by

\[ f' = f + \frac{<\rho_s>}{<\rho_1>} s_d \]  (3)

In this equation \( <\rho_s> \) and \( <\rho_1> \) are the mean ice and snow densities for the floe. A relation of this type compares favorably with the measurements reported by Cox and Weeks [1974], where a drop in the mean salinity (and hence
Fig. 1. Brine volume and density profiles of multiyear sea ice.
less brine volume and a lower density) of the top meter of multiyear ice was observed as the freeboard increased. Using eq. (2), a predicted thickness curve was constructed and compared with the measured values of thickness obtained by drilling. Figure 3 shows the predicted values compared with the measured values for (a) the constant density assumption and (b) the variable density assumption. This figure clearly indicates that a better estimate of the measured thickness is obtained by considering a systematic variation in the density. The standard errors between the measured and predicted values were 0.92 m for the constant density assumption and 0.45 m for the variable density.
Fig. 3. Comparison of predicted thickness (using freeboard) of multiyear ice assuming constant density (a) and a variable density linearly dependent upon freeboard (b).

EFFECTS OF VARIABLE DENSITY ON THE CALCULATED BENDING STRESSES

We may now compare the bending stresses induced by assuming a homogeneous or constant density model with those present assuming a variable density model. The bending stresses are due to forces of the type $\rho_i \Delta T g$ (per unit area), where $\Delta T$ is the deviation in the thickness between what should be there, based on isostasy, and what is there (the measured thickness). Since the standard deviation $\sigma_T$ between the measured and predicted thickness values is halved (from 0.92 m to 0.45 m) by using a statistical variable density model consistent with the physical properties of the ice, we would expect that the predicted bending stresses, in the mean, will also be approximately halved since the $\Delta T$ values would be reduced.

For a more quantitative assessment of the effect of density variations on bending stresses we need to examine the individual stress values at each point since the ice may be "crackable" at only a few locations where the bending stresses are at the extremes. To make such a comparison we utilized the profile shown in Figure 3 and calculated the deflections using an infinite beam approximation with the loading at each point, $z$, given by $g <\rho_i > \Delta T (x)$ for the constant density case and $g \rho_i (x) \Delta T (x)$ for the variable density case. A Green's function approach [Greenberg, 1971, p. 30] was used for the calculation with a finite symmetric digital filter used to approximate the infinite
Green's function. The convolution operation necessitated the loss of a number of endpoints, at both ends of the profile, equal to one-half the length of the filter. The comparison between the two models is shown in Figure 4. This figure compares the second derivative, $d^2w/dx^2$ of the deflection, $w$, at each point. In the analysis given by Hetenyi [1946], this value is proportional to the bending moment from which the maximum stress (on the outside surfaces of the beam) is calculated according to the following equations:

$$ M = -\frac{EI d^2w}{dx^2}, \quad \sigma = \frac{MT/2}{I} \tag{4} $$

Here $M$ is the induced bending moment, $E$ the Young's modulus, $I$ the moment of inertia, $\sigma$ the stress at the extremities, and $T$ the thickness of the beam.

To account, to some extent, for the thickness variations, the stresses were recalculated with a variable moment of inertia based on the thickness variations after using a constant $I$ to calculate the bending moment. For this case the stress is proportional to $\frac{1}{T^2} \frac{d^2w}{dx^2}$, which is plotted in Figure 5.

In these figures the appropriate values of $(d^2w/dx^2)$ and $\frac{1}{T^2} \frac{d^2w}{dx^2}$ corresponding to the failure stress used by Schwaegler (3 bars) are indicated by the light lines labeled "critical cracking values" (note the break in scale for Figure 5). The Young's modulus used in these calculations is also the same as that given by Schwaegler. The figure indicates that the extreme values of the calculated stresses are of the order of 50%-60% of the cracking value for the constant density model but, by using the variable density, are reduced to 25% or less of the cracking value. We conclude that the mean and extreme values of stress in multiyear sea ice due to thickness variations cannot account for the cracking of the ice. Stresses from other sources must account for 75% or more of the cracking stress when the density variations are taken into account.

**DISCUSSION**

The main purpose of this note was to compare the effects of density variations in the elastic bending model for sea ice beams. Two other points should also be mentioned. First, the Young's modulus used here and by
Fig. 4. Second derivative of calculated deflection using profile in Fig. 3 assuming (a) constant density and (b) variable density linearly dependent on freeboard.
Fig. 5. Second derivative of calculated deflection divided by thickness squared using profile in Fig. 3 assuming (a) constant density and (b) variable density linearly dependent on freeboard.
Schwaegler is not very representative of sea ice. Weeks and Assur [1967] indicate that this value is 10%-50% higher than the data from dynamic measurements (acoustic and seismic) would indicate for sea ice and is an order of magnitude higher than the extreme values seen for beam tests on sea ice [Tabata, 1967]. From eqs. (4), we see that the stresses generated are directly proportional to the modulus used and that a proportional decrease in the modulus will lead to similar stress reductions, i.e., 10%-50% less than those shown in Figure 4.

The second argument is related to the reduction in modulus for static tests, such as the beam experiments, as compared with the dynamic measurements. This effect has been attributed to creep processes occurring during the loading period which are not present to the same degree during the time period of an acoustic or seismic test. The experiments by Tabata indicate the relaxation time for this process (qualitatively, the time to onset of steady-state creep) is of the order of ten minutes. Given this observation, we must question the validity of a purely elastic analysis especially when the physical process that causes the thickness variation loading to occur is examined. This process is probably a combination of small changes in ablation and accretion, rafting, brine drainage, or melt pond formation—that is, incremental loading changes occurring over periods of days, weeks, or months. If large stresses do occur within the ice, the properties are such that creep should occur within the ice and thereby continuously reduce the stress level, especially during the time periods, probably weeks or months, that the stresses are present and the incremental fashion in which they are probably applied. If the only factor we assume is that the modulus is consistent with the beam experiments on sea ice (an order of magnitude lower than that used in Figures 4 and 5) and even if the loads are applied instantaneously, the resulting stresses are an order of magnitude lower than those shown in Figures 4 and 5 and only of the order of 2%-5% of the value necessary to crack the ice.

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A DESCRIPTION OF A MODEL OF THE ATMOSPHERIC BOUNDARY LAYER

by
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There now exist several models of the atmosphere in which the boundary layer is stably stratified. In this paper we discuss three of these models—by Babileva et al. [1967], Brown [1974], and Businger and Arya [1974]—and compare their results with data obtained during the 1972 AIDJEX pilot study.

The steady-state, horizontally homogeneous, barotropic boundary layer is described by the following equations of motion:

\[
\begin{align*}
\frac{\partial u'w'}{\partial z} - f\bar{v} + \frac{1}{\delta} \frac{\partial p}{\partial x} &= 0 \\
\frac{\partial v'w'}{\partial z} - f\bar{u} + \frac{1}{\delta} \frac{\partial p}{\partial y} &= 0
\end{align*}
\]

(1)

where \( \bar{u} \) and \( \bar{v} \) are the mean velocity components in the \( x \) and \( y \) directions, respectively, and \( u'w' \) and \( v'w' \) are the components of the momentum flux.

The momentum flux can be expressed in terms of an eddy diffusion coefficient as

\[
\begin{align*}
-u'w' &= K \frac{\partial \bar{u}}{\partial z} \\
-v'w' &= K \frac{\partial \bar{v}}{\partial z}
\end{align*}
\]

---

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Differentiating (1) once with respect to $z$, we get

$$\frac{d^2\eta}{dz^2} + \frac{\sigma}{K} = 0, \quad \frac{d^2\sigma}{dz^2} - \frac{\eta}{K} = 0$$

Here the dimensionless variables are defined as

$$\sigma = \frac{-u' \omega'}{u_*^2}$$

$$\eta = \frac{-v' \omega'}{u^2}$$

$$K_* = \frac{K_k}{u_* L_1}$$

$$\xi = \frac{z}{L_1}$$

$$L_1 = \frac{k u_*}{f}$$

where $f$ is the Coriolis parameter, $u_*$ is the dynamic (friction) velocity, and $k$ is von Kármán's constant.

Equations (2) were used to calculate turbulent characteristics in the boundary layer by Babileva et al. [1967] and Businger and Arya [1974]. To solve (2), Businger and Arya used the following expression for the eddy diffusion coefficient:

$$K_* = \frac{k \xi \exp \left( - \frac{|V_g| \xi}{u_*} \right)}{(1 + \beta \xi u_*)}$$

and the boundary conditions

at $\xi = \xi_0$ \hspace{1cm} $\eta = \exp \left( - \frac{|V_g|}{u_*} \xi_0 \right)$

$\sigma = 0$;

at $\xi = H_*$ \hspace{1cm} $\eta = 0$,

$\sigma = 0$. 

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Here \( \mu_* = \frac{u_*}{fL} = \frac{\alpha_k}{T_0} \frac{\omega^2}{f u_*^2} \) is a stability parameter, \( \beta \) is a constant equal to 4.7 \( \pm \) 0.5, and \( H_* \) is the boundary layer thickness. This expression for \( K_* \) is derived from a semiempirical fit to the observed vertical temperature profile in order to account for the vertical change in \( K_* \). Equations (2) can then be solved iteratively starting with an assumed \( V_g/u_* \) profile.

Babileva et al. [1967] solve (2) by relating the eddy coefficient to the turbulent energy production. To calculate \( K_* \) they use the equation for the balance of turbulent kinetic energy (TKE) in dimensionless form:

\[
\frac{n^2 + \sigma^2}{K_*} - \mu + \beta \frac{d}{dz} k_* \frac{db}{dz} = E
\]

where term I is the production of TKE by shear force, term II is the production of TKE by the buoyancy force, term III is the transport term, and term IV is the dissipation of TKE. Actually the divergence of the eddy transport of TKE is approximated by assuming an eddy diffusivity for transport proportional to the eddy diffusivity for momentum, \( \beta K_* \). The following ad hoc assumptions are used to close the set of equations:

\[
K = \beta b \lambda
\]
\[
E = \sigma b^{3/2} \lambda
\]
\[
\lambda = K \frac{\partial \psi}{\partial z}
\]
\[
\psi = \left( \frac{\partial u}{\partial z} \right)^2 + \left( \frac{\partial v}{\partial z} \right)^2 - \alpha_T \frac{\partial}{\partial z} \frac{\partial \theta}{\partial z} + \alpha_b \frac{1}{K} \frac{d}{dz} \left( \frac{Kdb}{dz} \right)
\]

Here \( b \) is the TKE, \( \lambda \) is a mixing length, \( K \) is the eddy diffusivity for momentum, and \( \alpha_T \) and \( \alpha_b \) are proportionality constants. The heat flux is calculated from

\[
\frac{d^2 \mu}{dz^2} = 0
\]

which is obtained by differentiating the equation of temperature diffusion once with respect to \( z \).
The following boundary conditions are used in the numerical solution of equations (2), (6), (7), and (8):

\[ \text{at } \xi = \xi_0 \quad \eta = 1, \sigma = 0, \mu = \mu_0, b = 1; \]
\[ \text{at } \xi = \infty \quad \eta = 0, \sigma = 0, \mu = 0, b = 0. \]

Equations (2) can then be solved numerically by iterating until a distribution at \( K_*(z) \) compatible with the TKE balance is found.

Calculated profiles of stress are shown in Figure 1. The geostrophic drag coefficient, \( u^*/G \) (\( G = |V_g| \)) and the geostrophic departure angle \( \alpha \) are shown plotted against \( \text{Ro} \) for various stabilities in Figures 2 and 3.

The results of the models can also be graphed as geostrophic drag coefficient versus geostrophic departure angle, with stability and roughness
Fig. 2. Geostrophic drag coefficient, $\gamma = \frac{u^*_x}{G}$, versus $\ln R_0$ for various values of stability parameter, $\mu$. Dashed line is according to Deacon [1974].

Fig. 3. The angle $\alpha$ versus $\ln R_0$ for various values of stability parameter, $\mu$. Dashed line is according to Deacon [1974].
as variable parameters (Figure 4). There is close agreement with the similarity-derived curve fit to AIDJEX data by Brown [1974]. These curves follow a similarity relation obtained by patching a stratified log layer to an Ekman-Taylor outer layer modified by secondary flow:

\[
\frac{\nu^*}{G} = \frac{1}{C_2} \left[ \sin \alpha + C_1 (\cos \alpha - \sin \alpha) \right] \tag{9}
\]

Here \( C_1 \) and \( C_2 \) are functions of \( \delta/h \) and \( z_1/h \), where \( \delta \) is equal to \((2K_E/f)^{1/2}\), the Ekman layer characteristic scale, \( z_1 \) is the surface layer characteristic scale, and \( h \) is the patching height.

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Fig. 4. Geostrophic drag coefficient versus geostrophic departure angle \( \alpha \) for various values of stratification, \( \nu \), and \( \ln \text{Ro} \).
When \( \ln \phi = 9 \), there is correspondence in roughness between the two models (\( z_1 \) then is equal to \( z_0 \) in the similarity theory), and both can approximate the data fairly well. This model and the resistance law, (9), with \( C_1 = 2, C_2 = 60 \), fit the data trend well throughout the stable-neutral regime. However, the results of this paper's model suggest a rapid increase in \( u_*/G \) for \( x \rightarrow 0 \) in the unstable portion. This would correspond to a decreasing \( C_2 \) in (9), an acceptable possibility since there is no a priori reason for the ratios \( \delta/k \) and \( z_1/h \) to remain constant. To date, the available data are inadequate to denote such a trend in the unstable regime.

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In March and April 1974 the Arctic Lead Experiment was conducted at Barrow, Alaska, to measure the sensible heat flux from leads. During the winter months the air temperature is from -20°C to -40°C, while the water temperature is from -1°C to -2°C. This creates intensive convective heat transfer from the water to the air when leads form in the ice cover (10^2 to 10^3 W m^-2). Measurements were taken both over artificial leads made by pumping water into a pond on top of the ice and over natural leads near Point Barrow. Wind and temperature profiles were measured with a moving instrument array in the unmodified air upwind and in the thermally modified air downwind of the lead. The instruments used were hot film anemometers, cup anemometers, and fast response thermocouples.

Most of this study was concerned with the unmodified air profiles that were obtained by the group from the University of Washington. The downwind profiles were taken by a group from Oregon State University and are being evaluated by them, but some preliminary results are given. The unmodified air profiles were fit to diabatic profile theories to obtain the values of $u_*$, $\theta_*$, $z_0$, and $\theta_0$ from which the air stress and the heat flux were calculated. There is an error analysis of these parameters that takes into account instrumental errors, the goodness of the fit of the measured profile to the calculated profile, and the uncertainties in the von Kármán constant, the value $K_h/K_m$, constants in the diabatic profiles, and the stability parameter, $z/L$. This analysis indicated that there was a mean percentage uncertainty in $u_*$ of 38% and in $\theta_*$ of 43%.

(*) The thesis is available in the AIDJEX library and in the Department of Atmospheric Sciences.
The average roughness over the ice was 0.3 mm and the mean value for the drag coefficient, $C_{10}$, was $(0.8 \pm 0.4) \times 10^{-3}$.

The downwind profiles were used to calculate the heat flux in two ways. In one, the additional heat content of the air was used to find the heat flux, $F_h$, from the relation

$$F_h = \frac{c_p}{\rho} \int_0^\delta \rho U \Delta T \, ds$$

where $c_p$ and $\rho$ are the specific heat and the density of the air, $U$ is the downwind wind speed, $\Delta T$ is the difference between upwind and downwind air temperatures, $x$ is the fetch of the wind over the open water, and $\delta$ is the height of the thermally modified boundary layer. Since the error in $\Delta T$ must be well below 0.1°C we were unable to use the $\Delta T$ found directly and had to use the top of the modified profile for the upwind temperature.

A second method was to use the profile fitting procedure with only the lowest measurement level, 10-15 cm above the surface, and assign a value of $z_0$ by creating a second level at the desired height with zero wind speed and an air temperature equal to the water temperature. This allows us to find $u_*$ and $\theta_*$ from which the heat flux was calculated. A value of $z_0 = 0.2$ mm was found to cause good agreement between the two methods for those profiles in which the top of the modified layer was apparent. With this second method we found the heat flux for all of the other profiles, the values ranging from 110 to 406 W m$^{-2}$.

The bulk heat transfer coefficient, $C_s$, is defined by the relation

$$F_h = \frac{c_p}{\rho} C_s U_2 \Delta T$$

where $U_2$ is the wind speed at 2 meters and $\Delta T$ is the water-air temperature difference. A mean value of $C_s$ was found to be $(3.3 \pm 0.6) \times 10^{-3}$. The results indicate $C_s$ is not well correlated with the fetch or with the stability of the unmodified air, but it is well correlated with the downwind stability modelled as $\Delta T/U_2^2$. This indicates a stability dependent
heat transfer coefficient, $C_s'$, could be used and would be given by

$$C_s' = 2.6 + 0.11 \frac{U_2^2}{\Delta T}$$

($U_2$ in m sec$^{-1}$, $\Delta T$ in °C). Such a formulation of $C_s'$ reduces the mean error in (1) from 15% to 7%.

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DISCLINATIONS AND CATASTROPHES IN THE VECTOR
AND TENSOR FIELDS OF SEA ICE

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ABSTRACT

The line singularities called disclinations, which appear in
the structure of liquid crystals, also occur as moving points
in the two-dimensional vector and tensor fields, like stress
and strain rate, associated with drifting sea ice—or, indeed,
associated with any two-dimensional deforming material such
as a layer of the atmosphere or ocean. A disclination, being
a topological structure, cannot be created or destroyed except
by encounter with another of opposite strength. In the three-
dimensional space-time (x, y, t) the tracks of these stable
moving entities constitute a linear framework giving form to
the changing vector or tensor field that fills the regions
between them. Since they indicate the structure of the field
rather than its quantitative details the tracks of the disclin-
ations may be useful in characterizing the fields of the AIDJEX
model.

There is a connection between disclinations and three of
the elementary catastrophes classified by Thom's theorem. Disclin-
ations of strengths $\frac{1}{2}$ and $-\frac{1}{2}$ in two-dimensional stress fields,
which are identical with positive and negative isotropic points,
are associated respectively with the hyperbolic and elliptic
umbilic catastrophes. At an instant when a positive and a nega-
tive isotropic point merge together we have a parabolic umbilic.
The other elementary catastrophes (fold, cusp, swallowtail, and
butterfly) have simple meanings, via geometrical optics, in
stress (or other) fields that can be represented by an Airy
stress function, but when body forces are present such interpr-
etations, although still possible, are less direct.
1. INTRODUCTION

The results from the AIDJEX model consist mainly of two-dimensional distributions, varying in time, of various vector and tensor quantities. Such fields typically contain examples of disclinations, a particular kind of point singularity. In this paper we show how the properties of disclinations follow from their topology, and we suggest that their positions and motions may be useful as a way of characterising the fields of the AIDJEX model.

The word disclination was first used by Frank [1961] to describe the line singularities observable under the microscope in nematic and cholesteric liquid crystals (see, for example, de Gennes [1974]). Frank [1958] had earlier called them "disinclinations" by analogy with the dislocations in solid crystals. Dislocations and disclinations are in fact closely related and, indeed, in the French language the word dislocation is now used to embrace both categories of line singularities.

We shall see that disclinations in two-dimensional tensor fields are related to three of the seven "elementary catastrophes" classified by Thom's theorem. The remaining four elementary catastrophes are associated with other singular properties of the fields.

2. VECTOR FIELDS

Figure 1 shows a region of a vector field which is continuous in the sense that the two Cartesian components of the vector are everywhere continuous. If a circuit (broken line) is drawn around the origin and followed once around clockwise, the direction of the vector will be seen to rotate by $2\pi$ anticlockwise. The circuit may be altered, but the property described remains fixed so long as the circuit continues to encircle the origin. At the origin itself the azimuth of the vector field is indeterminate and hence the magnitude of the vector is zero. Figure 1 shows a disclination of strength $-1$. In general, the rotation of the vector may be $2\pi n$ clockwise, where $n$ is an integer, and we say that we have encircled a disclination of strength $n$. The eight disclinations in Figure 2 are all of strength...
Fig. 1. A disclination of strength $n = -1$ in a vector field. When the broken-line circuit is followed once around clockwise, the vector direction turns one revolution anticlockwise.

$n = -1$

Fig. 2. Disclinations of strength $n = +1$ in a vector field. Diagrams (a) to (h) and back to (a) show a continuous transition from pure convergence, through a clockwise vortex, pure divergence, to an anticlockwise vortex, and back to pure convergence.

$n = +1$
+1. It is important to notice that right-handed and left-handed vortex-like structures, with or without convergence or divergence are all of strength +1. Figure 3 shows disclinations of strengths ±2. If a circuit encircles more than one disclination, the rotation of the vector is given by the sum of their strengths. For example, in Figures 4a and 4b the circuits shown (broken lines) enclose total disclination strengths of +1 and +2 respectively.

If the two-dimensional vector field changes continuously with time its disclination points will move, but they cannot disappear except by meeting others of opposite strength (or by reaching the boundary of the field). This, their most important property, follows from the combination of continuity and the circuit definition. Consider, for example, an isolated disclination of strength +1. For a circuit around it the total rotation of the vector must remain $2\pi$, even though, as time changes, the individual infinitesimal rotations along the path alter continuously. Only when the circuit actually passes through the point disclination does the total rotation of the vector become indeterminate. Thus, at the instant that the disclination crosses from the inside to the outside of the circuit the total rotation changes discontinuously from $2\pi$ to 0.

Several disclinations of strengths $n_1, n_2, \ldots, n_m$ may combine and will then produce a disclination of strength $n_1 + n_2 + \ldots + n_m$. In particular, two disclinations of strengths +1 and -1 which combine will annihilate one another.

A consequence of this analysis is that while one may pass continuously through the eight configurations of Figure 2, the pattern of Figure 1 (a stagnation point) is essentially different. It cannot disappear spontaneously by itself, but only by combination with one of the patterns of Figure 2. The disclinations are stable moving entities: the fundamental particles of changing two-dimensional vector fields.

As a practical matter a disclination, being a topological feature of the field, is detectable even in the presence of noise. For instance, in Figure 1 the magnitude of the vector is zero at the origin and so its
Fig. 3. Disclinations of strength $n = \pm 2$ in a vector field.

Fig. 4. Combinations of disclinations.
direction will be ill defined in the near neighbourhood. But by drawing the circuit large enough to avoid the ill-defined region one can be sure that there is a disclination somewhere within it, even though one does not know exactly where. Of course, if two disclinations are too close together, noise will make it impossible to resolve them; only their combined strength (which may be zero) will be known.

How often should we be able to see disclinations in typical vector fields such as those of the AIDJEX model? Taking any one vector field, say the ice velocity, $y$, at any instant, one could draw the contours of one component $u$ and note the zero-level contour. Wherever this crosses the zero-level contour of the other component $v$ (as it will normally do, provided the field is large enough), there will be a disclination. If the crossing is a simple one, the dislocation strength will be $\pm 1$.

Because both components will vary linearly through zero in the neighbourhood of the crossing, it follows that the length of the vector $\sqrt{u^2 + v^2}$ also varies linearly with distance from the disclination. So, on the landscape whose height above a datum level shows the length of the vector, although the summits and bottoms, or immits,* will normally be locally parabolic, there will be certain points where an immit reaches the zero level, and these places will be locally conical.

As time progresses the zero-level contours of $u$ and $v$ will normally migrate, and so will their crossings—thereby showing in another way why the disclinations are persistent entities. There will be occasional times, however, when two intersections of the same two zero-level contours come together and mutually annihilate; at the moment of annihilation the two contours are tangential to one another (Figure 5). Likewise there will be occasional times when two new intersections, and therefore two new disclinations of strengths $\pm 1$, suddenly appear. Consequently, on a contour plot of the length of the vector, an isolated, locally conical zero cannot

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Fig. 5. Coalescence of intersections of two zero-level contours.

lift off and become a parabolic non-zero minimum; it can only do this by encounter with another. For the same reason new locally conical zeroes can only appear in pairs.

The zero-level contour of one component, say u, will occasionally contract to a point (summit or immit) or will contain a saddle-point (so that the contour crosses itself), but at this critical instant the zero-level contour of v will not usually pass through that point. Such coincidences are infinitely unlikely to occur in a given field evolving over a given period of time; they correspond to special types of encounter between disclinations of strength ±1. Infinitely more unlikely again is the coincidence in space and time of a zero-level saddle point for u with a zero-level saddle-point for v. By drawing lines $u = 0$ and $v = 0$ on Figures 6a and 6b one can see that this doubly infinitely unlikely event is a disclination of strength ±2.

We conclude that, in the vector fields of the AIDJEX model, if they are large enough, in the changing two-dimensional vector fields associated with floating sea-ice generally, or in those associated with any continuous two-dimensional deforming material such as a layer of the atmosphere or ocean, disclinations of strength ±1 will normally occur. At certain instants they will be born, or will coalesce, in pairs; but disclinations
Fig. 6. Vector disclinations of strengths $\eta = -2$ and $+2$ correspond to the coincidence in space and time of zero-level saddle points in both components of the vector.
of higher order will not occur. However, they may sometimes appear to occur because of lack of resolution. In the language of catastrophe theory [Thom, 1975; Berry, 1976] disclinations of strength \( \pm 1 \) are "structurally stable" (their topology is unaltered by a small general perturbation) in the two-dimensional space \((x,y)\). Their birth or coalescence in pairs is structurally stable in \((x,y,t)\). But disclinations of strength greater than 1 are structurally unstable in \((x,y,t)\). To observe them in \((x,y,t)\) would require two additional dimensions or degrees of freedom, such as would be given by adjusting two independent parameters within the AIDJEX model.

These remarks apply to situations, such as we have in the AIDJEX model, which are "generic"—there is nothing special about them. They do not apply to nongeneric situations, such as those we commonly set up for analytical solution, where the problem is purposely simplified to have various sorts of symmetry. In such specially contrived cases there is no limit to the strength of the disclinations that could be produced. This is an important distinction between the real, generic, world and our invented nongeneric models which are amenable to analysis. Computer modeling of problems with complex boundaries and driving forces captures an aspect of the real world that is commonly lost in the artificial world of analysis with its symmetries and resulting degeneracies.

3. TENSOR FIELDS

The same topological reasoning may be applied to two-dimensional tensor fields, such as those of stress or strain rate, where the tensor is of the second rank and symmetric. At each point at a given instant there are then two principal directions, and we may draw two orthogonal families of trajectories which are everywhere parallel to them. Thus, for example, in Figure 7 the full lines give the direction of the algebraically greater of the two principal values of the tensor, while the broken lines refer to the direction of the lesser principal value. The diagrams show disclinations of strengths \( n = \pm \frac{1}{2}, \pm 1, \pm \frac{3}{2} \). The half-integer values of \( n \) occur because a rotation of \( \pi \) rather than \( 2\pi \) is sufficient
Fig. 7. Disclinations of strengths $n = \pm \frac{1}{2}, \pm 1, \pm \frac{3}{2}$ in a tensor field.
to bring the principal axes into self-coincidence. At the singularity itself the two principal values are equal and the azimuth of the trajectories is indeterminate.

If the tensor under consideration is stress, the disclinations of strengths \( n = +\frac{1}{2} \) and \(-\frac{1}{2}\) are known in the theory of photoelasticity as positive and negative isotropic points respectively [Frocht, 1941; Jessop and Harris, 1949]. At them the shear stress vanishes and the normal stress, which may be tensile or compressive irrespective of the value of \( n \), is the same in all directions. (The distinction between the two kinds of points is thus nothing to do with the difference between tension and compression.) Because the azimuth of the principal stresses is indeterminate at an isotropic point, isoclinics (loci of constant azimuth of the principal stresses) of all azimuths radiate from it. If the azimuth is conventionally taken as increasing clockwise, and if the azimuth label on the isoclinics also increases clockwise, we have a positive isotropic point; if the label increases anticlockwise the point is negative. (In a photoelastic experiment between crossed polariser and analyser the isoclinics appear as dark lines; when the polariser and analyser are rotated the isoclinics near an isotropic point, which look like a two-bladed propeller, rotate in the same direction as the polariser-analyser if the point is positive and in the opposite direction if it is negative.)

The case \( n = 1 \) (Figure 8) is special, just as it is for the vector disclination, in that it contains a continuous series of patterns. This happens because at any point the azimuth of the tensor direction \( \alpha \) differs from the polar angle \( \theta \) of the point merely by a constant, the different patterns corresponding to different values of this constant. In all other cases, changing the additive constant in the relation between \( \alpha \) and \( \theta \) merely rotates the diagram.

To answer the question of how often one might expect to find tensor disclinations in a changing two-dimensional field we may proceed by analogy with the vector field, where the disclinations were the intersections of the zero-level contours of the two Cartesian components. We shall assume that the tensor field, like the vector field, is continuous in the sense that its Cartesian components are everywhere continuous. In terms of the
Fig. 8. A disclination of strength \( n = +1 \) in a tensor field. This is one of the continuous series of patterns possible in this special case.

Cartesian components \( S_{xx}, S_{yy}, S_{xy} \) representing any symmetric second-rank tensor, the condition for an isotropic point is

\[
\begin{align*}
S_{xx} - S_{yy} &= 0, \\
S_{xy} &= 0.
\end{align*}
\]  

(1)

So, if we draw the zero-level contours of \((S_{xx} - S_{yy})\) and \(S_{xy}\), the intersections will denote the isotropic points.

Since both \((S_{xx} - S_{yy})\) and \(S_{xy}\) will vary linearly near such an intersection, the difference of the two principal values \(S_1\) and \(S_2\) of the tensor, which is given by
\[ \left| S_1 - S_2 \right| = \sqrt[4]{(S_{xxx} - S_{yy})^2 + 4S_{xy}^2}, \]

will vary linearly with the radial distance from an isotropic point. The surface whose height above a datum level gives \( |S_1 - S_2| \) will have the same property as the one for the modulus of a vector, that the zeroes are conical rather than parabolic minima.

As time progresses the intersections of the zero-level contours of \( (S_{xx} - S_{yy}) \) and \( S_{xy} \) will migrate, but it is at once clear that they will not disappear except in special circumstances. When they do disappear, or are created, it will be in pairs, corresponding to two zero-level contours becoming tangential (Figure 5). On the surface representing \( |S_1 - S_2| \) the locally conical zeroes will move about, but will not disappear except by encounter in pairs.

The situation is the same as for vectors except that for tensors the intersections of the zero-level contours mark disclinations of strength \( \pm \frac{1}{2} \). The same arguments about structural stability then follow. Tensor disclinations of strength \( n = \pm \frac{1}{2} \) will normally be present in \((x,y)\) at any given instant \( t \). They will move with \( t \), and at particular \((x,y,t)\) values there will be an event of creation or annihilation. Special sorts of encounters (corresponding to a summit, immit, or saddle in the zero-level contour of one of the quantities) are not structurally stable in \((x,y,t)\) and therefore will not occur in a generic situation such as we are dealing with. They would occur if an extra degree of freedom (such as changing a parameter in the model) were introduced. Disclinations of strength \( n = \pm 1 \) would need yet another degree of freedom.

4. ISOTROPIC POINTS AND THE UMBILIC CATASTROPHES

We have already borrowed from catastrophe theory the idea of structural stability. Now we wish to establish a relation between the singularities of two-dimensional time-varying tensor fields and Thom's seven "elementary catastrophes" [Thom, 1975; Berry, 1976]. We do this by first associating with the tensor field \( S_{ij} \) (\( i, j = x, y \)) \( (S_{ij} = S_{ji}) \) at any instant a surface
whose height above the plane \((x,y)\) is specified by the function \(f(x,y)\), the slope of the surface being everywhere small. Thus the tensor \(S_{ij}\) with its three independent components is replaced by the single scalar \(f\) in a way we shall explain later. The normals to the surface \(f(x,y)\) generate envelopes and it is in this space of the normals \((X,Y,Z)\) -- the "control" space -- that we recognise the catastrophes. The connection between the tensor field and the catastrophes is thus made in two stages: first constructing the scalar surface \(f(x,y)\) and then constructing its normals.

There is a particularly close relation between the isotropic points (disclinations with \(\pi = \pm \frac{1}{2}\)) we have been discussing and the three "umbilic catastrophes" (elliptic, hyperbolic, and parabolic). We shall take this first, leaving until later the identification of the remaining four elementary catastrophes with other singularities of the tensor field.

The directions of principal curvature of \(f(x,y)\) at each point \((x,y)\) are made identical with the principal directions of the tensor \(S_{ij}\). This does not specify \(f(x,y)\) completely, but it is enough for the present purpose. If the principal directions of \(S_{ij}\) at \((x,y)\) make angles \(\alpha\) and \(\alpha + \frac{1}{2} \pi\) with the \(x\) axis, we then have

\[
\tan 2\alpha = \frac{2f_{xy}}{f_{xx} - f_{yy}}, \tag{3}
\]

where the subscripts here denote partial derivatives. Given the tensor field, and therefore given \(\alpha(x,y)\), \(f(x,y)\) is then any one of the solutions of the partial differential equation

\[
\tan 2\alpha(x,y)(f_{xx} - f_{yy}) - 2f_{xy} = 0. \tag{4}
\]

A disclination in the tensor field, where \(\alpha\) becomes indeterminate, corresponds in \(f(x,y)\) to a point where the two principal curvatures are
equal and the surface is locally spherical. In other words, isotropic points in the tensor field denote locally spherical points in the surface.

The next step is to draw the normals to the surface. If the surface were a wavefront propagating in a uniform medium according to the laws of geometrical optics, the normals would be the rays. Clearly, the rays from the neighbourhood of an isotropic point (either positive or negative) will tend to focus at the centre of the osculating sphere, and we ask what is the nature of the ray field near the focus.

A general feature of any ray field is the presence of caustic surfaces, which are envelopes of the rays. Rays from neighbouring points on the original wavefront meet on the caustic surfaces to produce the weakest and most elementary kind of focussing. In a cup of coffee on a sunny day one sees two caustic lines meeting at a cusp; the surface of the coffee is a generic section through two caustic surfaces that meet in a cusp line. The cusp line is the next higher order of singularity of focus. Similar patterns can be produced in a variety of ways— for example, whenever light from a point source is refracted or reflected from a wavy surface. We see the results as the bright patterns on a white tablecloth after sunlight has passed through a glass of wine, as the dancing networks of light on the bottom of a sunlit swimming pool, or as the similar patterns formed by reflection on the underside of a bridge over water. Berry [1976] has made a detailed study, in the context of catastrophe theory, of such patterns as they would be formed at infinity, or focussed in the focal plane of a lens, such as the eye. This case can be vividly realised by viewing a distant street light through a drop of rain on a window pane, or on one's spectacle lenses (for best results the glass should be slightly dirty so as to ensure an irregularly shaped drop). Dr. Berry and I have taken many photographs of these patterns with the aid of a laser.

One of the applications of Thom's theorem in mathematical topology is to classify the possible types of focussing singularities, or catastrophes, that can occur. The number is limited. Besides the caustic surface, which corresponds to the fold catastrophe, and the cusp, there are
the swallowtail, the elliptic umbilic (Figure 9a), and the hyperbolic umbilic (Figure 9b). These are structurally stable, in the sense that they will occur naturally without any special preparation of the surface. The parabolic umbilic and the butterfly, which complete the list of the seven elementary catastrophes, require another variable parameter ("control variable"); and we can readily introduce one, namely time, by allowing our surface $f(x,y)$, along with the associated tensor field, to change continuously. The parabolic umbilic and the butterfly will then occur naturally in the field of normals at particular instants of time.

We return to the question of the nature of the ray field near the focus associated with an isotropic, locally spherical, point. I had speculated in a note circulated to the AIDJEX modeling group that the tensor disclinations of strengths $n = -\frac{1}{2}$ and $\frac{1}{2}$, the negative and positive isotropic points, were associated respectively with the elliptic and the hyperbolic umbilic catastrophes. Dr. Berry (unpublished) has now proved that this is correct. The proof, which is not straightforward, consists of first constructing surfaces having the lines of curvature of Figure 7 ($n = -\frac{1}{2}$ and $\frac{1}{2}$), and then showing that the forms of the caustic surfaces formed by the rays in the immediate neighbourhood of the focus do indeed have the correct analytical forms of the elliptic and hyperbolic umbilic in control space $(X,Y,Z)$.

Because the parabolic umbilic requires a further dimension, say the time dimension, and represents a transition between an elliptic and a hyperbolic umbilic, it is natural to suppose that it corresponds to the coalescence or creation of two disclinations with $n = -\frac{1}{2}$ and $\frac{1}{2}$. In problems in photoelasticity where one has both a positive and a negative isotropic point it may be possible to make them coalesce by altering the relative dimensions of the specimen; this again would give a parabolic umbilic.
Fig. 9. (a) An elliptic umbilic catastrophe, and (b) a hyperbolic umbilic catastrophe, in the space \((X,Y,Z)\) of the normals. (after Berry)

5. A RELATION BETWEEN THE TWO-DIMENSIONAL FIELDS OF A SYMMETRIC TENSOR AND THE OTHER FOUR ELEMENTARY CATASTROPHES

First we sketch the presumed relations between the surface \(f(x,y)\), the wavefront in our optical model, and the remaining four elementary catastrophes: the fold, cusp, swallowtail, and butterfly. We shall find that this involves properties of \(f(x,y)\) which are not simply the directions of its curvatures, and so we shall then have to consider more fully how to construct \(f(x,y)\) from the original tensor field \(S_{ij}(x,y)\).

We want to know precisely what feature of the surface \(f(x,y)\) produces each of the catastrophes in the space of its normals (the control space). A fold catastrophe or simple caustic is associated with every point of the surface. A cusp is presumed to arise from points in the surface (Figure 10a) where the trajectory defining the direction of a principal curvature
Fig. 10a. The full lines show the direction of one of the two principal curvatures in the surface, while the broken lines are contours of the magnitude of this principal curvature. A cusp line (shown in section on the right) in the envelope of the normals to the surface is produced from the dot-dash line in the surface.

Fig. 10b. The notation is the same as for Fig. 10a, but at P the contour is tangential to the trajectory and also crosses it. Correspondingly, in the space of the normals (right-hand diagrams), two cusps coalesce to form a swallowtail catastrophe.

Fig. 10c. As for Figs. 10a and 10b, but at Q there is a higher-order contact between the contour and the trajectory. In the space of the normals there is a butterfly catastrophe. The figure shows only one of the possible unfoldings (patterns of development) of the singularity.
is tangential to one of the contours of its magnitude. Thus, as one follows the trajectory the absolute magnitude has a maximum (cusp pointing towards the surface) or a minimum (cusp pointing away from the surface).

Suppose now the contours have the form of Figure 10b. Trajectory A meets first a maximum and then a minimum; similarly for trajectory B; so that two cusp lines are generated by the normals to the surface. But when we reach trajectory C the maximum and the minimum have merged together and there is no turning point. The point P, where a trajectory is tangential to the contour and also crosses it, is a singularity associated with the merging of two cusp lines (one pointing up and the other down). This must therefore be the swallowtail. The configuration will evidently occur naturally at certain points in any generic two-dimensional field \((x,y)\). A swallowtail is structurally stable in the three-dimensional \((X,Y,Z)\) space of the normals. Note that only one of the two principal curvatures, and therefore only one (local) sheet of the caustic, is involved. This makes it different from the umbilics, where both curvatures are involved and merge their identities.

Figure 10c shows the next higher order contact between the trajectories and the contours, where the separation varies as the fourth power of the distance. Evidently three cusp lines are involved and we are dealing with the butterfly. This configuration will not occur naturally at any given instant of time in the space \((x,y)\), but if we consider time as an additional variable it will occur at a particular instant. In agreement with this the butterfly is structurally stable in the four-dimensional space-time of the normals \((X,Y,Z,t)\). Again only one of the two curvatures is involved.

We have seen that disclinations with strength greater than \(\frac{1}{2}\) are not structurally stable in \((x,y,t)\) and will therefore correspond to structurally unstable configurations in the \((X,Y,Z,t)\) space of the normals. We can deduce that the associated catastrophes will be of co-dimension (dimension of control space) greater than 4.

The connection has been made between the changing surface \(f(x,y)\) and the seven elementary catastrophes. Now we must consider how to construct
\( f(x, y) \) from a given two-dimensional tensor field, such as a field of stress or strain-rate. Because we are constructing a field of a single scalar \( f(x, y) \) from a quantity \( S_{ij}(x, y) \) that has three independent components, we can expect there to be some arbitrariness in the procedure. However, there is a way of doing it which is particularly natural in the case of stress and so we shall use stress as an example.

If there are no body forces and the material is in equilibrium (as in most photoelastic experiments, for example), the stress components \( \sigma_{xx}, \sigma_{yy}, \sigma_{xy} \) satisfy

\[
\begin{align*}
\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} &= 0, \\
\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} &= 0.
\end{align*}
\]

If the scalar function \( \phi(x, y) \), the Airy stress function, is defined by

\[
\sigma_{xx} = \frac{\partial^2 \phi}{\partial y^2}, \quad \sigma_{yy} = \frac{\partial^2 \phi}{\partial x^2}, \quad \sigma_{xy} = -\frac{\partial^2 \phi}{\partial x \partial y},
\]

equations (5) are automatically satisfied. Now choose \( f(x, y) \), the height of the surface, to be equal to \( \phi(x, y) \). The directions of the principal axes of stress now make an angle of \( \frac{1}{2} \pi \) with the directions of principal curvature of the surface \( \phi(x, y) \). This means that the trajectories coincide, but that the magnitude of the curvature corresponding to one set of trajectories gives the principal stress in the direction of the other set.

Isotropic points for the stress correspond to locally spherical places in the Airy stress function surface, and hence to elliptic, hyperbolic, and parabolic umbilics. Noting that contours of equal magnitude of principal curvature of the \( \phi \) surface are identical with contours of equal magnitude of principal stress, we can now reinterpret Figures 10a, 10b, and 10c in terms of stress rather than curvature of the surface. If the full lines show the direction of one principal stress, the broken lines are the contours
of the other. For example, the cusp condition is that the direction of
the principal stress $\sigma_1$ is parallel to a contour of the principal stress
$\sigma_2$. With this interpretation the singularities of the stress field shown
in Figures 10a, 10b, and 10c correspond respectively to the cusp, the
swallowtail, and the butterfly.

We now turn to the general case of two-dimensional stress where
there is a distribution of body forces and the material is not in
equilibrium—as in the AIDJEX model. The momentum equations may be
written as

\[
\begin{align*}
\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + F_x &= 0, \\
\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + F_y &= 0,
\end{align*}
\]

(7)

where the surface traction $F = (F_x, F_y)$ represents the resultant of the
applied surface tractions (air stress, water stress, Coriolis force, and
ocean tilt) together with the acceleration term.

More generally, we may take $\sigma_{ij}$ to represent any two-dimensional
field of a symmetric tensor (strain-rate, for example), perform the
differentiations in equations (7) and interpret $F(x, y)$ as the (supposedly
known) vector field necessary to satisfy the equations.

To associate a scalar field with the tensor $\sigma_{ij}$ and to make this as
nearly as possible the Airy stress function, a possible procedure is the
following. Define two scalar potentials $\phi_1$ and $\phi_2$ by

\[
F_x = \frac{\partial \phi_1}{\partial x}, \quad F_y = \frac{\partial \phi_2}{\partial y},
\]

(8)
and define $\phi$ by

$$\sigma_{xx} = \frac{\partial^2 \phi}{\partial y^2} - \phi_1, \quad \sigma_{yy} = \frac{\partial^2 \phi}{\partial x^2} - \phi_2, \quad \sigma_{xy} = -\frac{\partial^2 \phi}{\partial x \partial y}. \tag{9}$$

Then equations (7) are automatically satisfied.

One can express $\phi_1$ and $\phi_2$ in terms of the divergence and curl of $\vec{F}$ by first noting that

$$\text{curl } \vec{F} = \frac{\partial^2}{\partial x \partial y} (\phi_2 - \phi_1), \quad \text{div } \vec{F} = \frac{\partial^2 \phi_1}{\partial x^2} + \frac{\partial^2 \phi_2}{\partial y^2},$$

and then making the transformation

$$\begin{align*}
\psi_1 &= \frac{1}{2}(\phi_1 + \phi_2), \\
\psi_2 &= \frac{1}{2}(-\phi_1 + \phi_2),
\end{align*} \quad \begin{align*}
\phi_1 &= \psi_1 - \psi_2, \\
\phi_2 &= \psi_1 + \psi_2,
\end{align*}$$

so that

$$\text{curl } \vec{F} = \frac{\partial^2 \psi_2}{\partial x \partial y}, \tag{10}$$

and

$$\text{div } \vec{F} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \psi_1 - \left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right) \psi_2. \tag{11}$$

Hence, if the field $\vec{F}$ is such that $\text{curl } \vec{F} = 0$, and so is derivable from a scalar potential, (10) shows that we may take $\psi_2 = 0$; and then, from (11), $\psi_1$ must be chosen to satisfy Poisson's equation

$$\nabla^2 \psi_1 = \text{div } \vec{F},$$
div $\boldsymbol{F}$ being regarded as a known function of position. If, on the other hand, curl $\boldsymbol{F}$ is non-zero—as in the AIDJEX model—$\psi_2$ is a solution of (10) which can then be inserted in (11) to find $\psi_1$. However, the more direct interpretation of $\phi_1$ and $\phi_2$ is in terms of the original definition (8).

If equations (9) are written in terms of $\psi_1$ and $\psi_2$,

$$\begin{align*}
\sigma_{xx} &= \frac{\partial^2 \phi}{\partial y^2} - \psi_1 + \psi_2, \\
\sigma_{yy} &= \frac{\partial^2 \phi}{\partial x^2} - \psi_1 - \psi_2, \\
\sigma_{xy} &= -\frac{\partial^2 \phi}{\partial x \partial y},
\end{align*}$$

(12)

we can see that $\psi_1$ and $\psi_2$ represent an isotropic stress and a pure shear stress respectively. If the body-force field $\boldsymbol{F}$ has a zero curl, we can eliminate it by subtracting from the real stress distribution $\sigma_{ij}$ a distribution $\psi_1$ of isotropic stress. If $\boldsymbol{F}$ does not have a zero curl, we need to subtract distributions both of isotropic stress $\psi_1$ and of pure shear stress $\psi_2$. When this is done the remaining stress field is given by an Airy stress function.

The subtraction of an isotropic stress makes no difference to the positions or signs of isotropic points, and so the umbilic singularities remain unmoved. But because the isotropic stress $\psi_1$ is not spatially uniform it does alter the form of the contours of equal stress magnitude; it therefore alters the positions of the cusp, swallowtail, and butterfly singularities. The subtraction of the pure shear stress $\psi_2$ alters the position of both the umbilic and the other singularities.

These are complications which tend to erode the directness of the connections this paper is concerned to make. So it is important to recall in conclusion that the presence of isotropic points and their
intimate association with the three umbilic elementary catastrophes was established quite independently of the discussion in this section of the Airy stress function.

In summary, a generic two-dimensional field of a symmetric tensor possesses isotropic points, or disclinations of strengths $\pm \frac{1}{2}$, which associate quite naturally with the elliptic, hyperbolic, and parabolic umbilic catastrophes. To associate the other elementary catastrophes with the field is simple when the field in question is stress and when there are no body forces, for then we can use the Airy stress function. If the field is stress and there are body forces, or if the field is a general one, the association with the non-umbilic catastrophes is less simple, but it can still be done by transforming the field to another one which is representable by an Airy stress function.

ACKNOWLEDGMENTS

I am particularly grateful to Dr. Michael Berry for the many instructive conversations about catastrophe theory we have had together, and I should also like to thank Mr. Francis Wright for the helpful work he has done on the wave-front surfaces associated with the elliptic and hyperbolic umbilics. This paper was written while I was a visitor to the AIDJEX modeling group.
REFERENCES


Jessop, H. T., and F. C. Harris. 1949. Photoelasticity: Principles and Methods, pp. 74-77, Cleaver-Hume Press, London. For good examples of positive and negative isotropic points, see Fig. 154, p. 161, which illustrates a railway coupling hook--one of the few generic shapes to be found in any book on photoelasticity.

A COORDINATE SYSTEM FOR TWO-DIMENSIONAL STRESS AND STRAIN-RATE AND ITS APPLICATION TO THE DEFORMATION OF SEA ICE

by

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ABSTRACT

A state of two-dimensional stress or strain-rate may be represented in a three-dimensional coordinate system. This has the advantage of retaining directional information that is lost if one specifies only the first and second invariants. Pure shear differs from pure convergence or divergence by possessing not only a magnitude but also a direction; this simple fact explains in essence why states near pure shear occur in sea ice so much more frequently than states near pure convergence or divergence. A framework is suggested in which to discuss the (small) strain history of an element of sea ice.

1. PURPOSE OF THE PAPER

In representing a state of two-dimensional strain-rate as it occurs in the AIDJEX model it has become customary to specify either the principal values \( \dot{\varepsilon}_1, \dot{\varepsilon}_2 \) or the two invariants

\[
\begin{align*}
\dot{\varepsilon}_I &= \dot{\varepsilon}_1 + \dot{\varepsilon}_2, \\
\dot{\varepsilon}_{II} &= \dot{\varepsilon}_1 - \dot{\varepsilon}_2, \quad (\dot{\varepsilon}_1 > \dot{\varepsilon}_2).
\end{align*}
\]

In this way one can conveniently categorise the deformation as uniaxial extension, pure shear, pure convergence, or whatever intermediate type of behavior it may be.
Of course, the advantage over using Cartesian components is that the arbitrary orientation of the coordinate system is conveniently suppressed. But, at the same time, by using only two of the three independent quantities that specify the strain-rate, one loses some information, namely, the direction in which the deformation takes place.

We show in this paper that there are some advantages in using a coordinate system to represent strain-rate, or stress, that retains the directional information. First we define the system and then we suggest some applications.

2. A COORDINATE SYSTEM IN STRAIN-RATE SPACE

Choose a Cartesian coordinate system \((x,y)\) in real space and start with the strain-rate components \(\dot{\varepsilon}_{xx}, \dot{\varepsilon}_{yy}, \dot{\varepsilon}_{xy}\), as given. So far as possible, our definitions will follow those of the AIDJEX modeling group. Thus define the first strain-rate invariant by

\[
\dot{\varepsilon}_I = \dot{\varepsilon}_{xx} + \dot{\varepsilon}_{yy},
\]

and the second by

\[
\dot{\varepsilon}_{II} = + \sqrt{[\dot{\varepsilon}_{xx} - \dot{\varepsilon}_{yy}]^2 + 4\dot{\varepsilon}_{xy}^2}.
\]

Define \(\dot{\varepsilon}_{xx}'\) and \(\dot{\varepsilon}_{yy}'\), components of the strain-rate deviator, by

\[
\dot{\varepsilon}_{xx}' = \dot{\varepsilon}_{xx} - \frac{1}{2}(\dot{\varepsilon}_{xx} + \dot{\varepsilon}_{yy}) = \frac{1}{2}(\dot{\varepsilon}_{xx} - \dot{\varepsilon}_{yy})
\]

and

\[
\dot{\varepsilon}_{yy}' = \dot{\varepsilon}_{yy} - \frac{1}{2}(\dot{\varepsilon}_{xx} + \dot{\varepsilon}_{yy}) = -\frac{1}{2}(\dot{\varepsilon}_{xx} - \dot{\varepsilon}_{yy}).
\]

Now set up a Cartesian coordinate system \((\dot{\varepsilon}_{xx}', \dot{\varepsilon}_{xy}, \frac{1}{2}\dot{\varepsilon}_I)\) (Figure 1a) so that any point \(P\) in the space so defined (to be called \(\dot{\varepsilon}\) space) represents a state of two-dimensional strain-rate. Clearly, the \(\dot{\varepsilon}_{yy}'\) axis is oppositely directed to \(\dot{\varepsilon}_{xx}'\). If the axes are rotated about the \(\dot{\varepsilon}_{xy}\) axis by 45°
(Figure 1b), the transformations are

\[
\begin{align*}
\frac{\dot{\varepsilon}_{xx}}{\sqrt{2}} &= \frac{1}{\sqrt{2}} (\dot{\varepsilon}_1' + \frac{1}{2}\dot{\varepsilon}_1), \\
\frac{\dot{\varepsilon}_{yy}}{\sqrt{2}} &= \frac{1}{\sqrt{2}} (-\dot{\varepsilon}_1' + \frac{1}{2}\dot{\varepsilon}_1),
\end{align*}
\]

and so we obtain an alternative Cartesian system in which the axes are \((\dot{\varepsilon}_{xx}/\sqrt{2}, \dot{\varepsilon}_{yy}/\sqrt{2}, \dot{\varepsilon}_{xy})\).

\[\text{Fig. 1. Cartesian, cylindrical polar, and spherical polar coordinates in strain-rate space.}\]

If, on the other hand, we treat the \(\frac{1}{2}\dot{\varepsilon}_1\) axis as a polar axis \((a)\) we find we have cylindrical polar coordinates \((r, \phi, z)\) which have the simple meanings \((\frac{1}{2}\dot{\varepsilon}_1', 2\gamma, \frac{1}{2}\dot{\varepsilon}_1)\), where \(\gamma\) is the angle between the direction of \(\dot{\varepsilon}_1\) and the \(x\)-axis in real space \((x,y)\). This follows from the relations, derivable from the Mohr circle construction,
Thus the \((\dot{\varepsilon}_{xx}, \dot{\varepsilon}_{xy})\) equatorial plane can be thought of as the plane of the Mohr circle.

Spherical polar coordinates \((\rho, \phi, \theta)\), where \(\phi\) is the azimuth and \(\theta\) the colatitude, are simply \((\frac{1}{2}\dot{\varepsilon}, 2\gamma, \theta)\); here \(\dot{\varepsilon}\), the total deformation rate, and \(\theta\), a measure of the relative rates of shearing and divergence, are defined by

\[
\dot{\varepsilon} = \sqrt{\dot{\varepsilon}_{I}^{2} + \dot{\varepsilon}_{II}^{2}}
\]

and

\[
\theta = \tan^{-1}(\dot{\varepsilon}_{II}/\dot{\varepsilon}_{I}),
\]

in accordance with the AIDJEX modeling group convention. The three coordinate systems are summarised in Table 1.

<table>
<thead>
<tr>
<th>TABLE 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>COORDINATE SYSTEMS FOR STRAIN-RATE</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Coordinate System</th>
<th>Variables</th>
<th>Degenerate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cartesian</td>
<td>(x, y, z)</td>
<td>(\dot{\varepsilon}<em>{xx}, \dot{\varepsilon}</em>{xy}, \frac{1}{2}\dot{\varepsilon}_{I})</td>
</tr>
<tr>
<td>Cylindrical polar</td>
<td>(r, \phi, z)</td>
<td>(\frac{1}{2}\dot{\varepsilon}<em>{II}, 2\gamma, \frac{1}{2}\dot{\varepsilon}</em>{I})</td>
</tr>
<tr>
<td>Spherical polar</td>
<td>(\rho, \phi, \theta)</td>
<td>(\frac{1}{2}\dot{\varepsilon}, 2\gamma, \theta)</td>
</tr>
</tbody>
</table>
Of course, one could multiply all the Cartesian coordinates by a common factor, but the reason for the factor $\frac{1}{2}$ in the defining the $\frac{1}{2} \hat{e}_{1}$ axis is the wish to make the axes representing $\hat{e}_{xx}$ and $\hat{e}_{yy}$ come out at right angles. The scale relation between the $\hat{e}_{xx}$ and $\hat{e}_{xy}$ axes is fixed by the wish to make the polar angle come out as $2\gamma$.

It will be noticed that the two principal strain-rates $\hat{e}_{1}$ and $\hat{e}_{2}$ have not so far played any part in these coordinate systems. Axes representing $\hat{e}_{1}$ and $\hat{e}_{2}$ lie in the azimuthal plane containing $\frac{1}{2} \hat{e}_{1}$ and $\frac{1}{2} \hat{e}_{II}$ (Figure 2). To assign values $\hat{e}_{1}$ and $\hat{e}_{2}$ to a given point in this strain-rate space one must first construct the azimuthal plane containing the point and then draw axes at $45^\circ$ to $\frac{1}{2} \hat{e}_{1}$ in this plane. This awkward procedure explains why $\hat{e}_{1}$, $\hat{e}_{2}$, and some third coordinate, such as $\gamma$, do not form a very convenient coordinate system.

3. A COORDINATE SYSTEM IN STRESS SPACE

We can now set up an analogous coordinate system for stress. The only difference arises because, by convention, $\sigma_{I}$ and $\sigma_{II}$ are defined differently from $\hat{e}_{I}$ and $\hat{e}_{II}$ by factors of 2. Thus, in terms of principal stresses $\sigma_{1}$ and $\sigma_{2}$,
\[ \sigma_I = \frac{1}{2} (\sigma_1 + \sigma_2), \]

and

\[ \sigma_{II} = \frac{1}{2} (\sigma_1 - \sigma_2), \quad (\sigma_1 > \sigma_2) \]

Therefore, we retain the labeling of all other axes and simply label the \( z \) axis \( \sigma_I \) and the radial distance \( \sigma_{II} \), without factors of \( \frac{1}{2} \) (Table 2, and Figure 3).

**TABLE 2**

COORDINATE SYSTEMS FOR STRESS

<table>
<thead>
<tr>
<th>System</th>
<th>( x, y, z )</th>
<th>( \sigma'<em>{xx}, \sigma</em>{xy}, \sigma_I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cartesian</td>
<td>( x, y, z )</td>
<td>( \sigma'<em>{xx}, \sigma</em>{xy}, \sigma_I )</td>
</tr>
<tr>
<td>Cylindrical polar</td>
<td>( r, \phi, z )</td>
<td>( \sigma_{II}, 2\gamma', \sigma_I )</td>
</tr>
<tr>
<td>Spherical polar</td>
<td>( \rho, \phi, \theta )</td>
<td>( \sigma, 2\gamma', \psi )</td>
</tr>
</tbody>
</table>

New definitions are

\[ \sigma'_{xx} = \sigma_{xx} - \frac{1}{2} (\sigma_{xx} + \sigma_{yy}) = \frac{1}{2} (\sigma_{xx} - \sigma_{yy}), \]

\[ \sigma'_{yy} = \sigma_{yy} - \frac{1}{2} (\sigma_{xx} + \sigma_{yy}) = -\frac{1}{2} (\sigma_{xx} - \sigma_{yy}), \]

\[ \sigma = +\sqrt{(\sigma_I^2 + \sigma_{II}^2)}, \]

\[ \psi = \tan^{-1}(\sigma_{II}/\sigma_I), \]

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and $\gamma'$ is the angle analogous to $\gamma$. The relation of the $(\sigma_1, \sigma_2)$ axes to those of $(\sigma_I, \sigma_{II})$ is shown in Figure 4.

4. THE DISTRIBUTION OF POINTS IN STRAIN-RATE SPACE

**Theory**

Each point in $\dot{e}$ space (Figure 1a) represents a strain-rate, and so it is natural to ask how the points corresponding to deforming sea-ice are distributed in this space. For example, a state of pure divergence ($\theta = 0$) or pure convergence ($\theta = \pi$) would be represented by a point on the positive or negative $\dot{e}_I$ axis, while a state of pure shear ($\theta = \frac{1}{2} \pi$) would lie in the equatorial plane. If sea-ice behaved as an incompressible material, the only possible deformation would be pure shear, and so the states would all lie in the equatorial plane.

To investigate this a little further let us see what would be implied if the points representing the strain-rates were distributed spherically symmetrically about the origin. It is not supposed that they really are distributed in just this way (for example, for the ice in the Beaufort Sea during the AIDJEX experiment), but the hypothesis is useful as a point...
of reference. The hypothesis means that, if we are given a range of $\dot{\varepsilon}_{xy}$ from $\dot{\varepsilon}_{xy}$ to $\dot{\varepsilon}_{xy} + \delta \dot{\varepsilon}_{xy}$, so giving a two-dimensional slice of the distribution (Figure 5a), the lines of equal probability form circles in the plane of the slice. Then, given the range $\dot{\varepsilon}_{xx}$ to $\dot{\varepsilon}_{xx} + \delta \dot{\varepsilon}_{xx}$, the distribution of $\dot{\varepsilon}_{yy}$ is symmetrical about zero. So a positive $\dot{\varepsilon}_{xx}$, for instance, does not bias the expected sign of $\dot{\varepsilon}_{yy}$ in either direction. In contrast, if the distribution had the form of an oblate spheroid, rotationally symmetrical about the $\frac{1}{2} \dot{\varepsilon}_I$ axis, rather than being spherical, the lines of equal probability on the slice would be elliptical (Figure 5b); and clearly a positive $\dot{\varepsilon}_{xx}$ would tend to associate with a negative $\dot{\varepsilon}_{yy}$, and vice versa. The strain-rates $\dot{\varepsilon}_{xx}$ and $\dot{\varepsilon}_{yy}$ would tend to have opposite signs.

In spite of these rather pedestrian conclusions it does not follow that with a spherically symmetrical distribution the principal strain-rates $\dot{\varepsilon}_1$ and $\dot{\varepsilon}_2$ are uncorrelated in sign. In fact the spherical distribution means that $\dot{\varepsilon}_1$ and $\dot{\varepsilon}_2$ tend to have opposite signs. The reason is as follows. Points where $\dot{\varepsilon}_1$ and $\dot{\varepsilon}_2$ have the same sign lie within a cone with axis $\frac{1}{2} \dot{\varepsilon}_I$ and semi-angle $45^\circ$, while points where $\dot{\varepsilon}_1$ and $\dot{\varepsilon}_2$ have opposite signs lie outside this cone. Therefore, with a spherically symmetrical distribution of points, the relative number having $\dot{\varepsilon}_1$ and $\dot{\varepsilon}_2$ with the same sign is the
Fig. 5. Hypothetical distributions of points in \( \dot{\varepsilon} \) space:
(a) spherically symmetrical, (b) oblate spheroidal.

proportion of the total solid angle that is within the cone, namely, 
\[ 1 - \frac{1}{\sqrt{2}} = 0.293. \]  
(To use an analogy, 29.3% of the Earth's surface lies north of latitude 45°N or south of latitude 45°S.)

A spherically symmetrical distribution in strain-rate space would mean that for a given small range of \( \dot{\varepsilon} \) the number of points with \( \theta \) between \( \theta \) and \( \theta + d\theta \) would be proportional to \( \sin \theta \ d\theta \). Thus, regardless of \( \dot{\varepsilon} \), the distribution of \( \theta \) would be proportional to \( \sin \theta \). The probability of pure divergence (\( \theta = 0 \), North Pole) or pure convergence (\( \theta = \pi \), South Pole) would be zero, while the maximum probability would be for pure shear (\( \theta = \frac{1}{2} \pi \), the Equator). We can see that the reason for this is simply that, while pure divergence or convergence is specified by only one parameter (the magnitude of the rate), pure shear has an additional degree of freedom, namely, its direction in space.

A manifestation of the same effect, because it springs from the different dimensionality of pure divergence or convergence and pure shear, is that, if we make a two-dimensional \((x,y)\) contour plot of \( \theta \), say for the AIDJEX area, the contour for pure shear \( \theta = \frac{1}{2} \pi \) will generally be a line—but the places where \( \theta = 0 \) or \( \pi \), if they occur at all, will be points (isotropic points). I have discussed the likelihood of such points and their behavior in the \((x,y)\) plane in another paper [Nye, this Bulletin].
If we wished, we could represent small strains in the same way as we have represented strain-rates. The space then has the useful property that successive small strains add together by simple vector addition. So it is possible to ask the question: how does the representative point for an element of the material move in strain space? Does it explore it by a quasi-random walk, or does it show more systematic behavior? (Because large strains do not commute I speak here only of small strains.)

A similar question could be asked about the movement in stress space (σ space) of the point representing the state of stress of a single element. The teardrop-shaped flow curve used at present by the AIDJEX modeling group, which is a curve in the σ\script{I}, σ\script{II} plane, becomes in σ space a surface of revolution about the σ\script{I} axis—a teardrop shape in truth. While states associated with pure compression or divergence are mere single points on the flow surface, states associated with pure shear form a complete girdle of points around it.

The normal flow rule for two-dimensional plastic stress states [Coon et al., 1974, p. 38] states that the vector in (\dot{ε}\script{I}, \dot{ε}\script{II}) space representing the plastic strain-rate is normal to the flow curve drawn in the parallel (σ\script{I}, σ\script{II}) space. We may restate the rule (still for two-dimensional stress states) as saying that the plastic strain-rate vector in our three-dimensional \dot{ε} space is normal to the flow surface in our parallel three-dimensional σ space.

Observation

Observations made during the AIDJEX main experiment can be made to give a distribution of points in \dot{ε} space. Although it is true that an incompressible material would give only points in the equatorial plane, the distribution should not be thought of as determined solely by the properties of the ice. On the contrary, the distribution is to do both with the ice and with the way it is driven.

To decide what to regard as a single strain-rate point one must choose a time interval. Using observations at the AIDJEX manned camps, Alan Thorndike (personal communication) has constructed curves of their position and velocity smoothed to suppress periods of less than about 8 hours, and has computed values of instantaneous strain-rate and of θ. A series of 197 values of θ
at 12-hour intervals during the period Day 121-220 (1 May to 8 August 1975) gave the histogram in Figure 6a. Comparing it with the normalised sin θ distribution (broken curve) that would correspond to a spherically symmetrical distribution of points in strain-rate space, we see that, although this is a passable first approximation, there are rather more pure shear "events" (θ = 90°) than would be given by a spherical distribution, and rather fewer events near pure divergence (θ = 0°) or pure convergence (θ = 180°).

Observations by Richard Hall (personal communication) made on Landsat pictures give a remarkably similar result. Using the measurement technique described by Nye [1975, pp. 129-132] and measuring strain over 98 one-day and 25 two-day intervals in the AIDJEX area during March, April, and May 1973 (24 cases) and during April and May 1975 (99 cases), he has calculated the 123 θ values whose histogram is shown in Figure 6b. Again, it is higher than the sin θ curve in the middle and lower at the sides.

One might be inclined to deduce from these results that the hypothetical spherical distribution is compressed somewhat towards the equatorial plane. But we must be careful not to assume too glibly that the distribution is rotationally symmetrical about the $\frac{1}{2}\xi$ axis. Indeed, the most recent results from Alan Thorndike's observations (personal communication) suggest that this may be far from the truth. Although over short enough periods the shearing deformation may take place in many different azimuths, summing the deformations over a longer period suggests there is a bias in the angle $2\gamma$. The present indications are that over successive 20-day periods the motion of a point in strain space is not at all isotropic, and as a result the cumulative shear is strongly biased in a particular geographical direction. When one remembers that the element in question is embedded in the general ice circulation and deformation pattern of the whole Arctic Ocean this is not altogether surprising. The distribution of strain-rate events in $\dot{\epsilon}$ space, say $P(\frac{1}{2}\dot{\epsilon}, 2\gamma, \theta)$ is evidently not simple. A possible hypothesis to test is that it may be expressed as a product of three distributions,
$P_1\left(\frac{3}{2} \dot{\varepsilon}\right) P_2(2\gamma) P_3(\theta)$ so that, for example, the distribution in $\theta$ is independent of the value of $\dot{\varepsilon}$ or $\gamma$.

![Histograms](image)

Figure 6. Histograms of $\theta$ from (a) manned camps and (b) LandSat.

- $\theta = 0^\circ$ pure divergence,
- $\theta = 45^\circ$ uniaxial extension,
- $\theta = 90^\circ$ pure shear,
- $\theta = 135^\circ$ uniaxial compression,
- $\theta = 180^\circ$ pure convergence.

The ordinate is the number of cases lying within a $20^\circ$ range of $\theta$. 
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REFERENCES


THE FINITE ELEMENT METHOD IN AIDJEX

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ABSTRACT
This note, intended for those interested in and familiar with the AIDJEX ice model finite difference scheme, describes in general terms the finite element method and notes an equivalence between the two methods.

INTRODUCTION
During the last ten years a class of computational techniques, known generically as the finite element method, has come to be used to calculate approximate solutions to almost all types of boundary value problems in continuum mechanics. The popularity of the finite element method is due to the ease with which various complicated shapes, boundary and interface conditions, and material response functions can be treated within the framework of a single scheme. There are widely used general purpose finite element computer programs which treat broad classes of boundary value problems in solid and fluid mechanics.

Finite element techniques are used to model the spatial dependence of the fields of interest (e.g., stress, velocity, temperature) in a problem. The resulting models are functions defined piecewise over an assemblage of subregions or elements. Typically the elements are of simple form (three or four sided, with either straight or curved sides) and the interpolating functions within each element are polynomials of fairly low order. These functions, called shape functions, are constructed so that the values of the fields everywhere are completely determined by their
values at certain points, called nodes, which are usually the vertices or midsides of the elements. The shape functions are such that the minimal required degree of element to element continuity exists in the model. For second order differential equations, such as govern the deformation of a continuum, the requirement is that the velocities be continuous with piecewise continuous first derivatives. The construction and techniques for manipulation of shape functions of this class, i.e., \( C^0 \) shape functions, have been developed for a range of element shapes as well as for various orders of approximating polynomials.

Boundary and initial-boundary value problems are treated by introducing the finite element interpolation of the dependent variables into a variational statement of the problem. The appropriate variational statement may be known (as is the case for conservative problems) directly from the physics of the problem or it may be the result of a Galerkin or weighted residual argument. The result of the substitution of the finite element model into the variational equation is a set of coupled equations in the unknown nodal point values of the dependent variables. If the problem is time independent the resulting equations are algebraic and are usually written in the form of (1), in which the coefficient matrix \( K \) is or is not a function of the nodal point unknowns \( y \) as the original problem is nonlinear or linear.

\[
K y = f \quad (1)
\]

In (1), \( f \) is called the load vector and depends on the forcing function in the original problem.

If the problem is transient, say second order in time, then the finite element equations are a coupled set of ordinary differential equations in time. These are usually written in the form of (2)

\[
M \ddot{u} + C \dot{u} + K u = f \quad (2)
\]

In (2) \( \ddot{u} \) is the time derivatives of the nodal point values. As in (1), the coefficient matrices \( M, C, \) and \( K \) may not depend on \( u \) (or \( \dot{u} \)). The finite element equations of motion (2) are integrated numerically with
respect to time. Central difference techniques are often used although implicit methods, requiring the solution of a coupled, linear algebraic system at each time step, are also employed.

The finite difference scheme by which the equations of the AIDJEX ice model are integrated is based on a certain arrangement of differencing and integrating around cells. This scheme, though no doubt well reasoned, is a somewhat arbitrary choice from among many possibilities. Because of their generality and widespread use, finite element techniques seem a worthwhile alternative to the difference scheme.

To help evaluate the attractiveness of this proposition, a comparison was made between certain calculations as they are made in the finite difference scheme and as they would be made in a finite element modeling of the ice model equations. The results are given below, and some details of the finite element development are given in the appendix.

FINITE ELEMENT CALCULATIONS

The calculations described are based on the formulation used in the finite element computer program HONDO. This code, written by Sam Key of Sandia Laboratories, calculates the large-deformation dynamic response of two-dimensional solids (either axisymmetric or plane). The kinds of material response provided for in the program include elastic, elastic-plastic, and viscoelastic in both small and large strain regimes.

All calculations in a finite element scheme are made element by element. The results are then assembled, producing the equations associated with nodal point unknown velocities. In the finite difference scheme the nodal point equations are written directly, the contribution from all cells (elements) connected to the node being calculated as the node is considered. This distinction accounts for the major differences in form between computer implementation of the two methods. Calculations for a typical element of the type used in the HONDO code are developed in (A1) through (A18) of the appendix. The assembly into nodal point forces corresponding to \( \text{div} \sigma \) is given by (A19).
The development is given for the calculation, within an element, of the deformation measures and of the forces corresponding to the distribution of \( \text{div } \sigma \). The \( \text{div } \sigma \) term is integrated numerically in the HONDO code; here it has been done analytically so that a comparison can be made of the explicit results.

The formulae for the components of deformation gradient (A12) and velocity gradient (A14) prove to be identical to the corresponding formulae in the two-dimensional finite difference scheme. The assembled nodal point force calculations (A19) are also identical to the corresponding quantities from the finite difference code.

**REMARKS**

Although only two of several relevant calculations have been compared, it is clear that the distinction between finite element and finite difference calculations is often more one of method than of substance. In light of this (less than astounding) revelation two observations can be made.

No improvement in the solution would result from a reformulation of the ice model computations on a finite element basis, and any effort along these lines would be a clear waste.

Less obvious, perhaps, is that the experience and confidence gained by using the finite difference code could be expected to remain relevant to any use of finite element techniques or programs which might appear attractive for extensions or modifications of the present model. The ease with which finite element techniques can be used to model complicated shapes with arbitrary variation in the mesh spacing is likely to be the motivating factor leading to any such use.
APPENDIX
FIRST ORDER ISOPARAMETRIC ELEMENTS

The mapping from the square parent element \((-1 \leq \xi, \eta \leq 1)\) to the element in \(x - y\) space (Figure A1) is written conveniently in terms of the \(C^0\) shape functions \(N_i(\xi,\eta)\):

\[
x(\xi,\eta) = N_i(\xi,\eta) \xi_i
\]

\[
y(\xi,\eta) = N_i(\xi,\eta) \eta_i
\]

(A1)

where \(\xi_i\) and \(\eta_i\) are the nodal point values of \(x\) and \(y\) and

\[
N_1 = \frac{1}{4} (1 - \xi) (1 - \eta) \quad N_3 = \frac{1}{4} (1 + \xi) (1 + \eta)
\]

\[
N_2 = \frac{1}{4} (1 + \xi) (1 - \eta) \quad N_4 = \frac{1}{4} (1 - \xi) (1 + \eta)
\]

(A2)

Because the functions \(N_i\) are continuous from element to element (although they are defined locally within each element), an assemblage of isoparametric elements will cover a region of \(x - y\) space with no gaps or overlaps.
Furthermore, continuous functions of $\xi$ and $\eta$ will also be continuous in $x$ and $y$. Note that either the material coordinates $X, Y$ or spatial coordinates $x, y$ can be treated as long as the nodal point values of each are known.

Consider the finite element interpolation of functions of $x$ and $y$, say $u(x,y)$ and $v(x,y)$. In terms of the nodal point values $u_1$ and $v_1$ we write

$$ u(x,y) = N_1[\xi(x,y), \eta(x,y)]u_1 $$

$$ v(x,y) = N_1[\xi(x,y), \eta(x,y)]v_1 $$

In (A3), the dependence of $\xi$ and $\eta$ on $x, y$ is implicitly defined by (A1). If we want to calculate the value of, say, $u$ at the centroid of the element we note that $\xi = \eta = 0$ and proceed directly, evaluating (A2) at $\xi = \eta = 0$ and then using (A3). Evaluation of $u$ at arbitrary values of $x, y$ is difficult but not often required. Differentiation of $u$ with respect to $x$ must be done using the chain rule and implicit function theorem. Integration of functions of $x, y$ over an element is conveniently done from

$$ \xi = -1 \text{ to } 1 $$

$$ \eta = -1 \text{ to } 1. $$

**DIFFERENTIATION**

To calculate, say, $\frac{\partial u}{\partial x}$ from (A3) we write $\frac{\partial u}{\partial x} = \frac{\partial N_1}{\partial x} u_1$, so that we actually must be able to differentiate only the shape functions. We have, for example,

$$ \frac{\partial N_1}{\partial x} = \frac{\partial N_1}{\partial \xi} \frac{\partial \xi}{\partial x} + \frac{\partial N_1}{\partial \eta} \frac{\partial \eta}{\partial x}. $$

(A4)
The quantities \( \frac{\partial N_1}{\partial \xi} \frac{\partial N_1}{\partial \eta} \) are easily found from (A2), but \( \frac{\partial \xi}{\partial x} \frac{\partial \eta}{\partial x} \), etc., must be calculated implicitly.

\[
\begin{bmatrix}
\frac{\partial \xi}{\partial x} & \frac{\partial \eta}{\partial x} \\
\frac{\partial \xi}{\partial y} & \frac{\partial \eta}{\partial y}
\end{bmatrix}
= \begin{bmatrix}
\frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\
\frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta}
\end{bmatrix}^{-1}
= \frac{1}{J} \begin{bmatrix}
\frac{\partial y}{\partial \eta} & -\frac{\partial y}{\partial \xi} \\
-\frac{\partial x}{\partial \eta} & \frac{\partial x}{\partial \xi}
\end{bmatrix}
\]

(A5)

\[J \equiv \frac{\partial x}{\partial \xi} \frac{\partial y}{\partial \eta} - \frac{\partial y}{\partial \xi} \frac{\partial x}{\partial \eta}\]

The derivatives \( \frac{\partial x}{\partial \xi} \), etc., are found from (A1) and (A2) to be

\[
\frac{\partial x}{\partial \xi} = \frac{\partial N_1}{\partial \xi} x_i, \text{ etc.}
\]

(A6)

Combining (A4) - (A6), we get

\[
\frac{\partial N_1}{\partial x} = \frac{\partial N_1}{\partial \xi} \left( \frac{1}{J} \frac{\partial y}{\partial \eta} \right) + \frac{\partial N_1}{\partial \eta} \left( -\frac{1}{J} \frac{\partial y}{\partial \xi} \right)
\]

\[
= \frac{1}{J} \left( \frac{\partial N_1}{\partial \xi} \frac{\partial N_j}{\partial \eta} - \frac{\partial N_1}{\partial \eta} \frac{\partial N_j}{\partial \xi} \right) y_j
\]

Finally

\[
\frac{\partial u}{\partial x} = \frac{1}{J} \left( \frac{\partial N_1}{\partial \xi} \frac{\partial N_j}{\partial \eta} - \frac{\partial N_1}{\partial \eta} \frac{\partial N_j}{\partial \xi} \right) y_j u_i
\]

(A7)

\[\equiv \frac{1}{J} D_{x1} u_i\]

Similarly

\[
\frac{\partial u}{\partial y} = \frac{1}{J} D_{y1} u_i
\]
where

\[
D_{y1} \equiv \left( \frac{-\partial N_1}{\partial \xi} \frac{\partial N_1}{\partial \eta} + \frac{\partial N_1}{\partial \eta} \frac{\partial N_1}{\partial \xi} \right) x_j
\]  

(A8)

The quantities \( J, D_{x1}, \) and \( D_{y1} \) are all that are required, then, to evaluate derivatives.

Expanding \( J \) leads to

\[
J = \left( \frac{\partial N_1}{\partial \xi} \frac{\partial N_1}{\partial \eta} - \frac{\partial N_1}{\partial \eta} \frac{\partial N_1}{\partial \xi} \right) x_1 \ y_j
\]

Expanding further and collecting terms, we have

\[
J = \frac{1}{8} \begin{pmatrix} 0 & 1-\eta & -\xi+\eta & -1+\xi \\ -1+\eta & 0 & 1+\xi & -\xi-\eta \\ \xi-\eta & -1-\xi & 0 & 1+\eta \\ 1-\xi & \xi+\eta & -1-\eta & 0 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix}
\]

or

\[
J = \frac{1}{8} \mathbf{x}^T \mathbf{P} \mathbf{y}
\]

Expanding further and collecting terms, we have

\[
J = \frac{1}{8} \left( J_0 + \xi J_1 + \eta J_2 \right)
\]

\[
J_0 = X_{13} Y_{24} - X_{24} Y_{13}
\]

\[
J_1 = X_{34} Y_{12} - X_{12} Y_{34}
\]

\[
J_2 = X_{23} Y_{14} - X_{14} Y_{23}
\]

From (A7) and (A8), using the notation introduced, the result is

\[
D_{y1} = \frac{1}{8} \frac{P \mathbf{x}}{z}
\]

\[
D_{x1} = -\frac{1}{8} \frac{P \mathbf{y}}{z}
\]
or in expanded form

\[
\begin{pmatrix}
\frac{1}{8} \left( \begin{array}{c}
X_{24} - \xi X_{34} - \eta X_{23} \\
-\xi X_{13} + \xi X_{34} + \eta X_{14} \\
-\xi X_{24} + \xi X_{12} - \eta X_{14} \\
X_{13} - \xi X_{12} + \eta X_{23}
\end{array} \right)
\end{pmatrix}
\begin{pmatrix}
Y_{24} - \xi Y_{34} - \eta Y_{23} \\
-\xi Y_{13} + \xi Y_{34} + \eta Y_{14} \\
-\xi Y_{24} + \xi Y_{12} - \eta Y_{14} \\
Y_{13} - \xi Y_{12} + \eta Y_{23}
\end{pmatrix} = \begin{pmatrix}
\frac{1}{8} \\
0 \\
0 \\
0
\end{pmatrix}
\] (A10)

Note that differentiation with respect to \( X,Y \) (instead of \( x,y \)) will lead to formulae identical to (A10) when \( (X,Y) \) are used in (A1)-(A9) in place of \( (x,y) \).

**STRAIN CALCULATION**

The calculation is done element by element. At each node the following quantities are either stored or calculated:

- \( X,Y \) the original nodal coordinates
- \( x,y \) the current nodal coordinates
- \( \dot{u}, \dot{v} \) the velocity components

The set of nodal point values are denoted thus: \( \mathbf{x} \) is the array of values \( x_1, \ldots \), etc.

**Interpolation**

Each of the six functions is represented in terms of the bilinear shape functions \( N_1 \) and the appropriate nodal values.

For example,

\[
\dot{u} = N_1 \dot{u}_1 \quad \text{and} \quad x = N_1 x_1, \text{ etc.}
\]

The \( N_1 \) are as defined by (A2).
Deformation Measures

The deformation gradient \( F_{ji} = \frac{\partial x^i}{\partial X^j} \) (tensor notation) and the velocity gradient \( D_{ji} = \frac{\partial x^i}{\partial X^j} \) are calculated. In general these derivatives will be rational functions of \( \xi \) and \( \eta \) [See (A7), (A9), and (A10).] Evaluating at the point \( \xi = 0 \), \( \eta = 0 \) yields the following specialization.

Deformation Gradient at \( \xi = \eta = 0 \)

\[
F^1_1 = \frac{\partial x}{\partial x} = \frac{8}{J_0} D_x x \quad F^2_1 = \frac{\partial y}{\partial x} = \frac{8}{J_0} D_x y \tag{A11}
\]

\[
F^2_2 = \frac{\partial x}{\partial y} = \frac{8}{J_0} D_y x \quad F^2_2 = \frac{\partial y}{\partial y} = \frac{8}{J_0} D_y y
\]

with \( x_k Y_1 \) used to evaluate \( X_{k1} \) and \( Y_{k1} \) in (A9), and \( D_x, D_y \) evaluated at \( \xi = \eta = 0 \). For example:

\[
P^1_1 = \frac{1}{(x_1 - x_2)(y_2 - y_4) - (x_2 - x_4)(y_1 - y_3)} [(y_2 - y_4)x_1 - (y_1 - y_3)x_2 - (y_2 - y_4)x_3 + (y_1 - y_3)x_4]
\tag{A12}

Velocity Gradient at \( \xi = \eta = 0 \)

\[
D^1_1 = \frac{\partial \dot{u}}{\partial x} = \frac{8}{J_0} D_x \dot{u} \quad D^2_1 = \frac{\partial \dot{v}}{\partial x} = \frac{8}{J_0} D_x \dot{v} \tag{A13}
\]

\[
D^2_2 = \frac{\partial \dot{u}}{\partial y} = \frac{8}{J_0} D_y \dot{u} \quad D^2_2 = \frac{\partial \dot{v}}{\partial y} = \frac{8}{J_0} D_y \dot{v}
\]

For example:

\[
D^2_2 = \frac{1}{(x_1 - x_2)(y_2 - y_4) - (x_2 - x_4)(y_1 - y_3)} [(x_2 - x_4)\dot{v}_1 - (x_1 - x_3)\dot{v}_2 - (x_2 - x_4)\dot{v}_3 + (x_1 - x_3)\dot{v}_4]
\tag{A14}
Note that the denominator \((x_1 - x_3)(y_2 - y_4) - (x_2 - x_4)(y_1 - y_3)\) has the value of twice the area of the element.

### INTEGRATION OF \(\text{div} \, \sigma\)

Generalized forces are calculated in each element. These forces are the coefficients of virtual nodal point motions (or test functions) in the virtual work expression. The virtual work expression may be interpreted as a consequence of applying Galerkin's method to the equations of motion.

\[
I = \iiint \text{div} \, \sigma \cdot \mathbf{\phi} \, da
\]

where \(\sigma\) is the stress tensor and \(\mathbf{\phi} = (\phi^1, \phi^2)\) are test functions.

Integration by parts gives

\[
I = \iiint \sigma : \nabla \mathbf{\phi} \, da = \iiint \sigma^{ij} \phi^1_i \phi^2_j \, da.
\]

Expanding the integrand, this becomes

\[
I = \iiint \left[ \sigma_{11} \frac{\partial \phi^1}{\partial x} + \sigma_{22} \frac{\partial \phi^2}{\partial y} + \sigma_{12} \left( \frac{\partial \phi^1}{\partial y} + \frac{\partial \phi^2}{\partial x} \right) \right] \, da. \quad (A15)
\]

\(\phi^1, \phi^2\) are interpolated as

\[
\phi^1 = N_1 \phi_1^1 \quad \quad \phi^2 = N_1 \phi_1^2,
\]

so that the integrand becomes

\[
\phi^T \sigma \phi^T \mathbf{z},
\]
with
\[ \phi^T \equiv \left[ \begin{array}{cccccc} \phi_1^1 & \phi_2^1 & \phi_1^2 & \phi_2^2 & \phi_3^1 & \phi_4^1 \\ \phi_2^1 & \phi_2^2 & \phi_2^1 & \phi_2^2 & \phi_3^1 & \phi_4^1 \end{array} \right] \]

\[ t^T \equiv \left[ \begin{array}{cc} \sigma_{11} & \sigma_{12} \end{array} \right] \]

and
\[
B = \left[ \begin{array}{ccccccc} \frac{\partial N_1}{\partial x} & 0 & \frac{\partial N_2}{\partial x} & 0 & \frac{\partial N_3}{\partial x} & 0 & \frac{\partial N_4}{\partial x} \\ 0 & \frac{\partial N_1}{\partial y} & 0 & \frac{\partial N_2}{\partial y} & 0 & \frac{\partial N_3}{\partial y} & 0 \\ \frac{\partial N_1}{\partial x} & \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} & \frac{\partial N_2}{\partial x} & \frac{\partial N_3}{\partial x} & \frac{\partial N_3}{\partial x} & \frac{\partial N_4}{\partial x} \end{array} \right]
\]

From (A7) and (A10) we see that each of the terms \( \frac{\partial N_1}{\partial x} \) or \( \frac{\partial N_1}{\partial y} \) has the form
\[
\frac{\partial N_1}{\partial x} = \frac{1}{J} \cdot \frac{1}{8} \left( Y_{kl} \pm \xi Y_{mn} \pm \eta Y_{pq} \right) \] (A17)

The integration over the area of an element is conveniently done with respect to \( \xi \) and \( \eta \) since the element boundaries are lines of constant \( \xi \) and \( \eta \).

\[ \iiint_{\text{element}} \cdot d\alpha = \int_{-1}^{1} \int_{-1}^{1} \cdot J \ d\xi \ d\eta \]

Using (A16) and (A17) in (A15) and integrating as above we note two important simplifications:

1. The \( \frac{1}{J} \) in \( \frac{\partial N}{\partial x} \) cancels the \( J \) in \( d\alpha = J \ d\xi \ d\eta \).

2. The integrals of \( \xi Y_{mn} \) and \( \eta Y_{pq} \) vanish.
Taking advantage of these and integrating explicitly,

\[ I = \phi^T f \]

where

\[
\begin{bmatrix}
\sigma_{11} Y_{24} + \sigma_{12} X_{24} \\
\sigma_{22} X_{24} - \sigma_{12} Y_{24} \\
\sigma_{11} Y_{13} + \sigma_{12} X_{13} \\
\sigma_{22} X_{13} - \sigma_{12} Y_{13} \\
\sigma_{11} Y_{24} + \sigma_{12} X_{24} \\
\sigma_{22} X_{24} - \sigma_{12} Y_{24} \\
\sigma_{11} Y_{13} - \sigma_{12} X_{13} \\
\sigma_{22} X_{13} + \sigma_{12} Y_{13}
\end{bmatrix}
\]

(A18)

The total div \( \sigma \) force at a node is obtained by summing the contributions from all elements surrounding the node.

Consider the node to be 0 and the four adjacent elements \( A, B, C, \) and \( D \). The nodes in the sketch below are labeled as nodes 1 through 4 in each element and globally 0 through 8.
The assembled loads for node 0 are

\[ f_x = f_A^5 f_B^7 f_C^1 f_D^3 \]

\[ f_x = f_A^6 f_B^8 f_C^2 f_D^4 \]

where the subscripts refer to the component number in (A18). Using global node numbers these forces are

\[
f_x = \frac{1}{2} \left[ -\sigma_A^{11} (y_2 - y_4) + \sigma_A^{12} (x_2 - x_4) + \sigma_B^{11} (y_2 - y_5) - \sigma_B^{12} (x_2 - x_5) \\
+ \sigma_C^{11} (y_5 - y_7) - \sigma_C^{12} (x_5 - x_7) - \sigma_D^{11} (y_4 - y_7) + \sigma_D^{12} (y_4 - y_7) \right]
\]

(A19)

\[
f_y = \frac{1}{2} \left[ \sigma_A^{22} (x_2 - x_4) - \sigma_A^{12} (y_2 - y_4) - \sigma_B^{22} (x_2 - x_5) + \sigma_B^{12} (y_2 - y_5) \\
- \sigma_C^{22} (x_5 - x_7) + \sigma_C^{12} (y_5 - y_7) + \sigma_D^{22} (x_4 - x_7) - \sigma_D^{12} (y_4 - y_7) \right]
\]
SEA ICE – PROCESSES AND MODELS

ICSI/AIDJEX Symposium
Seattle, USA • 4-9 September 1977

First Announcement and Call for Papers

A symposium on sea ice will be held at the University of Washington, Seattle, USA, 4 to 9 September 1977 under the auspices of the International Commission on Snow and Ice (ICSI) and the Arctic Ice Dynamics Joint Experiment (AIDJEX). The meeting is intended to deal mainly with large scale processes, and with the modeling of processes. It is hoped that contributions will cover the various models and roles of sea ice as a component in the ocean/atmosphere system, from the global scale down to the micro-scale, but it is not intended that engineering aspects of sea ice dynamics should be dealt with directly. The symposium will provide the first opportunity for presenting results from the AIDJEX main experiment, and from the developing ship and satellite studies of Antarctic sea ice.

Topics

Possible topics include the following:

- Observations of ice forces and ice movement
- Thermal aspects of sea ice
- Sea ice data obtained by remote sensing
- Ice edge and shear zone phenomena
- Effects of waves and swell on sea ice
- Effects of synoptic weather systems on sea ice and vice versa
- Long term changes in the extent of sea ice
- Role of sea ice in formation of Antarctic bottom water
- Roles of sea ice in numerical models of atmosphere and ocean
- Prediction schemes for sea ice

Associated phenomena such as precipitation and persistent summer stratus cloud may be the subject of a special symposium during the IAMAP Assembly, to be held in Seattle immediately before the ICSI-AIDJEX Symposium.

Papers

The Papers Committee will invite a small number of review papers and will select other papers for presentation and publication. Only those papers actually presented by their authors at the symposium will be published in the symposium proceedings.

Attendance

Prospective authors and others planning to attend the meeting should complete and return the preliminary registration slip below. Subsequent circulars will then be mailed to them. In the first instance, all correspondence should be addressed to:

Dr. Malcolm Mellor
Secretary, International Commission on Snow and Ice
Cold Regions Research and Engineering Laboratory
Hanover, New Hampshire 03755
U.S.A.

Name: _____________________________
Address: ___________________________

Proposed topic or title of paper: