A PRELIMINARY AIDJEX MODEL SIMULATION WITH 1972 DATA
--AIDJEX Staff ........................................... 1

ESTIMATING THE DEFORMATION OF SEA ICE
--A. S. Thorndike and R. Colony .......................... 25

WINTER ICE DYNAMICS IN THE NEARSHORE BEAUFORT SEA
--R. S. Pritchard, M. D. Coon, M. G. McPhee, and E. Leavitt .. 37

AN ESTIMATION OF THE VISCOUS WIND-DRIVEN CIRCULATION
OF THE ARCTIC ICE COVER OVER A TWO-YEAR PERIOD
--W. D. Hibler III and W. B. Tucker III ................... 95

CHARACTERISTICS OF ARCTIC STRATUS CLOUDS OVER THE
BEAUFORT SEA DURING AIDJEX
--K. 0. L. F. Jayaweera .................................... 135

TILT-METER MEASUREMENTS DURING COLLISION OF TWO FLOES,
1972 AIDJEX PILOT STUDY
--J. R. Weber ................................................ 153

SYNTHETIC APERTURE RADAR IMAGERY OF THE AIDJEX TRIANGLE
--L. Bryan, T. Farr, F. Leberl, and C. Elachi ............... 161

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Front cover: Onan, having heard that life is like a bus, waits for another drifting station.

Back cover: Meteorological observers release a pibal at Caribou.
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Arctic Ice Dynamics Joint Experiment
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The AIDJEX Bulletin aims to provide both a forum for discussing AIDJEX problems and a source of information pertinent to all AIDJEX participants. Issues—numbered, dated, and sometimes subtitled—contain technical material closely related to AIDJEX, informal reports on theoretical and field work, translations of relevant scientific reports, and discussions of interim AIDJEX results.

Bulletin 37 is one of the last you will receive before we close our doors in January. If you have been sitting on a report that you owe the AIDJEX world, delay no longer. No one wants to break your heart with a refusal in the Christmas season.

Any correspondence concerning the Bulletin should be addressed to

Alma Johnson, Editor
AIDJEX Bulletin
4059 Roosevelt Way N.E.
Seattle, WA 98105

"Hartley, I have the unpleasant task of informing you that your services are no longer required. However, your efforts have not been unappreciated. The company has therefore empowered me to grant you three wishes."
A PRELIMINARY AIDJEX MODEL SIMULATION
WITH 1972 DATA

by

AIDJEX Staff

1. SUMMARY

The ice cover in a portion of the Beaufort Sea was simulated with the AIDJEX sea ice model for the period 11 to 16 April 1972 (Julian days 102-107). The motion was driven by air stress fields derived from surface pressure data. The boundary of the region was determined by a ring of imaginary buoys whose initial positions were arbitrarily chosen to encircle a manned station and whose subsequent positions during the 6-day simulation were assumed to be wind driven. The initial conditions on stress and thickness distribution were homogeneous. The initial velocities were the local wind-driven velocities.

As the integration proceeded, stress, velocity, and thickness distribution were calculated in the interior of the region from the momentum equation, the constitutive laws, and the thickness distribution equation, as described below. The calculated velocities were generally directed to the northwest and reached 20 cm sec\(^{-1}\) at times. Toward the end of the run spatial gradients became marked, the velocity varying through 180° across the solution region. At no time was the divergence of internal ice stress large enough to affect the momentum balance.

This run was designed primarily as an exercise to test our capability to set the inputs in an acceptable form and perform the numerical integration, and in this objective we have succeeded.
2. DESCRIPTION OF MODEL EQUATIONS AND INPUT DATA

The rationale for the AIDJEX model has been given by Coon et al. [1974], and the numerical procedure for solving it appears in Pritchard and Colony [1976]. Here we record the equations and parameters used in this simulation.

**Momentum Equation**

The momentum equation

\[ m \ddot{\mathbf{y}} = \text{div} \, \mathcal{Q} + \mathcal{T}_a + \mathcal{T}_w - m f_c \mathbf{k} \times \mathbf{y} \]

is used to determine the ice velocity \( \mathbf{y} \), given the air stress \( \mathcal{T}_a \). The water stress \( \mathcal{T}_w \) is a known function of velocity; the stress \( \mathcal{Q} \) is found from the constitutive law; and \( \mathbf{k} \) is the unit upward vector. The mass per unit area \( m \) is assumed in this run to be a constant 300 gm cm\(^{-2}\). The Coriolis parameter is also assumed constant:

\[ f_c = 2 \Omega \sin \phi \]

where \( \Omega = 7.29 \times 10^{-5} \text{ sec}^{-1} \) and \( \phi = 76^\circ \).

**Water Stress**

The water stress model is a quadratic relation between traction and ice velocity relative to geostrophic flow:

\[ \mathcal{T}_w = \rho_w C_w G \mathcal{E} \mathcal{G} \]

where \( \mathcal{G} = || \mathcal{G} || \)

\[ \mathcal{G} = \mathbf{y} - \mathbf{y}_g \]

\[ \mathcal{E} = \begin{pmatrix} -\cos \beta & \sin \beta \\ -\sin \beta & -\cos \beta \end{pmatrix} \]

\( C_w = 0.0034 \)
\[ \beta = 0.41888 \text{ radians} = 24^\circ \]
\[ \rho_w = 1.000 \text{ gm cm}^{-3} \]

In this simulation, the geostrophic current \( V_g \) is assumed zero.

**Constitutive Laws**

To write these laws, we need stress invariants

\[ I \equiv 2\sigma_I \equiv \text{tr } \sigma \]
\[ II' = \text{tr } \sigma' \sigma' \]
\[ \sigma_{II} = \sqrt{-\text{det } \sigma'} = (1/2 \ II')^{1/2} \]

where the prime denotes the deviator.

The constitutive laws consist of

(i) the strain rate decomposition

\[ \dot{\varepsilon} - W \sigma + eW = D - D_p \]

where the total strain rate \( D \) and the spin \( W \) are defined in the usual way

\[ D_{ij} = 1/2 \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \]

\[ W_{ij} = 1/2 \left( \frac{\partial v_i}{\partial x_j} - \frac{\partial v_j}{\partial x_i} \right) \]

(ii) the elastic response, which determines the elastic strain \( \varepsilon \),

\[ \varepsilon = \frac{I}{4M_1} \frac{1}{\rho} + \frac{1}{2M_2} \sigma' \]

\[ M_1 = 10^9 \text{ dyn cm}^{-1} \]
\( M_2 = 1/2 \times 10^9 \text{ dyn cm}^{-1} \)

(iii) the flow rule which determines the plastic strain rate \( \dot{\varepsilon}_p \),

\[
\dot{\varepsilon}_p = \begin{cases} 
\lambda [\phi_1 \frac{1}{2} + 2\phi_2 II G'] & \lambda \leq 0 \\
0 & \lambda < 0 
\end{cases}
\]

in terms of

(iv) the yield constraint \( \phi \) for the teardrop yield curve

\[
\phi = II' - 1/2 \ I^2 \ (1 + \frac{I}{2p^*}), \quad I \leq 0
\]

The multiplier \( \lambda \) is found as part of the solution. The yield strength \( p^* \) is known in terms of the thickness distribution (see below).

**Thickness Distribution**

The thickness distribution \( G(h) \) is the fraction of area \( G \) covered by ice thinner than \( h \). It is governed by the equation

\[
\dot{G} + f \ G_h = \Psi - G \ \text{div} \ \gamma
\]

where the redistributor \( \Psi \) is

\[
\Psi(h, D_p) = |D_p| [a_0(\theta) \ H(h) + a_1(\theta) \ W_r(h, G)]
\]

\[
|D_p|^2 = 4(D_{p1}^2 + D_{p2}^2)
\]

\[
\theta = \arctan \left( \frac{-D_{p1} + D_{p2}}{D_{p1} + D_{p2}} \right)
\]

\( D_{p1}, D_{p2} \) are principal strain rates of plastic stretching, \( D_p \). The ridging law is

\[
W_r(h, G) = \frac{-A(h) + 1/k \ A(h/k)}{1 - 1/k}
\]
where

\[ k = 5 \]

\[ A(h) = B(G(h)) \]

\[ B = \begin{cases} \frac{G}{G^*} (2 - \frac{G}{G^*}), & \text{for } 0 \leq \frac{G}{G^*} \leq 1 \\ 1, & \text{for } 1 < \frac{G}{G^*} \end{cases} \]

\[ G^* = 0.15 \]

The ridging coefficient

\[ \alpha_r(\theta) = \left[ \frac{\sigma_I}{p^*} \cos \theta + \frac{\sigma_{II}}{p^*} \sin \theta \right] \phi = 0 \]

is evaluated at the point \((\sigma_I, \sigma_{II})\) on the yield surface. The opening coefficient is given by

\[ \alpha_0(\theta) = \alpha_r(\theta) + \cos \theta \]

The thermodynamic growth rates \(f\) are the climatological means given in Table 1.

**TABLE 1**

**GROWTH RATES \(f\) IN CM DAY\(^{-1}\) AS FUNCTION OF ICE THICKNESS AND TIME OF YEAR**

<table>
<thead>
<tr>
<th>Thickness (cm)</th>
<th>Day 102</th>
<th>Day 112</th>
<th>Thickness (cm)</th>
<th>Day 102</th>
<th>Day 112</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10.82</td>
<td>9.69</td>
<td>450</td>
<td>0.14</td>
<td>0.14</td>
</tr>
<tr>
<td>50</td>
<td>1.63</td>
<td>1.36</td>
<td>500</td>
<td>0.09</td>
<td>0.10</td>
</tr>
<tr>
<td>100</td>
<td>0.36</td>
<td>0.34</td>
<td>550</td>
<td>0.05</td>
<td>0.06</td>
</tr>
<tr>
<td>150</td>
<td>0.32</td>
<td>0.28</td>
<td>600</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>200</td>
<td>0.29</td>
<td>0.26</td>
<td>650</td>
<td>-0.01</td>
<td>-0.01</td>
</tr>
<tr>
<td>250</td>
<td>0.27</td>
<td>0.25</td>
<td>700</td>
<td>-0.03</td>
<td>-0.03</td>
</tr>
<tr>
<td>300</td>
<td>0.25</td>
<td>0.23</td>
<td>750</td>
<td>-0.05</td>
<td>-0.05</td>
</tr>
<tr>
<td>350</td>
<td>0.23</td>
<td>0.22</td>
<td>(\geq 800)</td>
<td>-0.06</td>
<td>-0.06</td>
</tr>
</tbody>
</table>

Once \(G(h)\) is known, the yield strength \(p^*\) is determined from an energetics argument assuming gravitational potential energy as the sole energy sink for plastic work. The expression is

\[ p^* = c \int_{0}^{h^*} h \left[ 1 - B[G(h)] \right] dh \]
where
\[ c = \hat{\rho} \hat{g} k, \]
\[ \hat{\rho} = 0.09 \text{ gm cm}^{-3}, \text{ and} \]
\[ \hat{g} = 981 \text{ cm sec}^{-2}. \]

**Air Stress**

Barometric pressure maps are computed over a region somewhat larger than the solution region on a rectangular grid with a 400 km mesh spacing. Data from land stations, the AIDJEX camp, and NWS pressure maps are used to fit a polynomial surface to the pressure field. A sixth order polynomial is used in each direction. Coefficients are found by a least squares fit of the data. Pressure maps are computed at six-hour intervals, at 0000Z, 0600Z, 1200Z, and 1800Z each day.

Geostrophic flow is computed as
\[ G_a = \frac{1}{\rho f_c} k \times \nabla p \]

where
\[ k = \text{unit upward vector} \]
\[ \rho = 0.00125 \text{ gm cm}^{-3} \]
\[ f_c = 2\Omega \sin \phi \]
\[ \Omega = 7.29 \times 10^{-5} \text{ sec}^{-1} \]

and \( \phi \), the latitude, is changed to reflect actual geographical location. The pressure gradient \( \nabla p \) is computed by differentiating the polynomial function.

The air stress is computed as
\[ T_a = \rho C_g^2 G_a B_a G_a \]

where
\[ C_g = 0.022 \]
\[ B_a = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \]
\[ \alpha = 20^\circ \]

\[ G_a = \|G_a\| \]

**Initial Conditions**

The initial thickness distribution is homogeneous and is given in Table 2. This distribution was chosen as typical of early spring from a two-year calculation of \( G(h) \) driven by strains of a triangle of Arctic drifting stations.

**TABLE 2**

**THE INITIAL THICKNESS DISTRIBUTION**

<table>
<thead>
<tr>
<th>( h (\text{cm}) )</th>
<th>( G(h,t_0) )</th>
<th>( h (\text{cm}) )</th>
<th>( G(h,t_0) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.</td>
<td>100</td>
<td>0.08</td>
</tr>
<tr>
<td>10</td>
<td>0.005</td>
<td>150</td>
<td>0.10</td>
</tr>
<tr>
<td>20</td>
<td>0.01</td>
<td>200</td>
<td>0.30</td>
</tr>
<tr>
<td>30</td>
<td>0.02</td>
<td>300</td>
<td>0.80</td>
</tr>
<tr>
<td>50</td>
<td>0.03</td>
<td>1000</td>
<td>1.00</td>
</tr>
<tr>
<td>70</td>
<td>0.05</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The initial ice stress is chosen arbitrarily to be

\[ g = (+0.1646 \times 10^5 \text{ dyn cm}^{-1}) \parallel z \]

a value that *violates* the yield condition. (This was an error, of course. After one timestep the stress is back to, or within, the yield curve.)

The initial velocity field is calculated as the wind-driven drift of each grid point, using the equation

\[ \tau^a + \tau^w - mf_c \mathbf{g} \times \mathbf{v} = m \frac{d\mathbf{v}}{dt} = 0 \]

(a truncated form of the momentum equation described above), starting from rest one day prior to the first day of the simulation.
Boundary Conditions

The boundary of the region is a smooth closed curve through a set of imaginary buoys. The buoys initially lie on a circle of 800 km diameter and are assumed to move at the wind-driven velocity subsequently. Velocities are given at three-hour intervals for each boundary mesh point from the wind-driven drift calculations. Linear interpolation is used to obtain velocity at times needed in the calculation.

Numerical Solution

Refer to Pritchard and Colony [1976] for a complete description. Briefly, time differencing uses the leapfrog scheme. Both the water stress and the Coriolis centering parameters $\theta_w$ and $\theta_c$ are 1/2. The constitutive law is integrated by Newton's method except for stress states near the vertex of the yield curve and other states that are pathological for Newton's method; for these states, a binary search is used. Thickness distribution is integrated by a modified Euler method along $(h,t)$ characteristics using time steps equal to $\Delta t = 500$ sec, as with the other equations. The grid is Lagrangian.

Yield strength is computed by the trapezoidal rule. The integrand is not interpolated to stop at the proper maximum thickness of $h^*$, but is carried to the next thicker regular grid point in $h$.

3. SAMPLE OUTPUT

The modeled quantities listed in Table 3 are shown in the graphs on the following pages. Further printed and graphical output has been documented and is available through the AIDJEX Data Bank.

Field plots are provided at these times:

<table>
<thead>
<tr>
<th>Cycle</th>
<th>Day/hour-minute/second</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>102/1716/40</td>
</tr>
<tr>
<td>100</td>
<td>103/1530/00</td>
</tr>
<tr>
<td>180</td>
<td>104/1343/20</td>
</tr>
<tr>
<td>260</td>
<td>105/1156/40</td>
</tr>
<tr>
<td>340</td>
<td>106/1010/00</td>
</tr>
<tr>
<td>420</td>
<td>107/0823/20</td>
</tr>
</tbody>
</table>
The units are cgs unless stated otherwise. The symbol used to display a tensor at a point within a field is a cross whose arms lie in the principal directions. The cross is considered to have two arms, not four. A dotted arm represents a positive principal value; a solid arm shows a negative principal value. Each plot has the parallels 75°N and 80°N and the meridians 140°W, 150°W, and 160°W drawn in.

Time histories are given for node 7,7, which occupied the same position as the main manned camp, Jumpsuit, at the beginning of the run. Node 7,7 is shown by a dot in Figure 10. The vector balance of forces at node 7,7 is presented at the same times as field plots.

TABLE 3
QUANTITIES PRESENTED IN FIGURES 1-13

<table>
<thead>
<tr>
<th>Variable</th>
<th>Figure Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Field Plot</td>
</tr>
<tr>
<td>Kinematics:</td>
<td></td>
</tr>
<tr>
<td>Velocity v</td>
<td>1</td>
</tr>
<tr>
<td>Velocity components v_x, v_y</td>
<td>2</td>
</tr>
<tr>
<td>Total strain rate D</td>
<td>3</td>
</tr>
<tr>
<td>Thickness Distribution:</td>
<td></td>
</tr>
<tr>
<td>G(h=50) - G(h=10)</td>
<td>4</td>
</tr>
<tr>
<td>G for h=10,20,30,50,70,100,150,200,300</td>
<td>5</td>
</tr>
<tr>
<td>Terms in Momentum Balance:</td>
<td></td>
</tr>
<tr>
<td>Air stress τ_a</td>
<td>6</td>
</tr>
<tr>
<td>Water stress τ_w</td>
<td>7</td>
</tr>
<tr>
<td>Divergence of ice stress, div σ</td>
<td>8</td>
</tr>
<tr>
<td>Force balance</td>
<td>9</td>
</tr>
<tr>
<td>Ice Stresses:</td>
<td></td>
</tr>
<tr>
<td>State (elastic or hardening or weakening)</td>
<td>10</td>
</tr>
<tr>
<td>Strength p^*</td>
<td>11</td>
</tr>
<tr>
<td>Pressure p = -σ_I</td>
<td>13</td>
</tr>
</tbody>
</table>
4. REFERENCES


Fig. 1. Ice velocity fields.
Fig. 2. Ice velocity at node 7,7 versus time.
Fig. 3. Principal values of total strain rate, shown dashed if positive (extending), solid if negative (contracting).
Fig. 4. Fraction of area covered by ice between 10 and 50 cm.
Fig. 5. Fraction of area $G(h)$ thinner than $h$ cm at node 7,7 versus time.
Fig. 6. The applied air stress field.
Fig. 7. The water stress field.
Fig. 8. Divergence of ice stress (div $g$), in units of dyn cm$^{-2}$.
Fig. 9. The force balance at node 7,7, in dyn cm⁻².
Fig. 10. Computational grid and state of the ice. Plus sign indicates hardening (plastic); minus sign indicates weakening (plastic); a cell with no sign is elastic.
Fig. 11. Ice strength $p^*$ in units of $10^6$ dyn cm$^{-1}$. 
Fig. 12. The ice strength $p^*$ at node 7,7 versus time.
Fig. 13. Ice pressure $p = -\sigma_I$ in units of $10^6$ dyn cm$^{-1}$. 
ESTIMATING THE DEFORMATION OF SEA ICE

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INTRODUCTION

In current treatments of the large-scale interaction between sea ice and its environment, an important role is assigned to the deformation of the ice pack. Deformation occurs by the three mechanisms of pressure ridging, the formation of open leads, and shearing along existing leads.

Pressure ridges are important, from a dynamical point of view, primarily because of the work required to form them. For a given deformation, the work done in ridging can be used to estimate the state of stress in the ice pack. Ridges are also important because of the large proportion of mass tied up in them. They affect the roughness characteristics of the top and bottom surfaces of the ice, and are thought to have anomalous rates of ice growth and melting.

The presence of leads has two consequences. The strength of the ice pack decreases as the area of leads increases. If the fractional area of leads is as large as 10–20%, it is thought that the pack offers no resistance to applied forces. From a thermodynamic view, a vigorous exchange of heat and moisture occurs over leads, and the ice grows rapidly as a lead begins to freeze over.

Some shearing displacement along a wide lead can occur with no effect, but as the displacement increases, the irregular lead edges will come into contact and locally large stresses will cause the ice to break. Long, narrow regions in which floes appear pulverized are thought to be regions of concentrated shear deformation.

Having defined a probability density function to describe the relative abundance of ice of different thicknesses, Thorndike et al. [1975] developed a model which parameterizes the three deformation mechanisms in terms of a given strain rate tensor. This model has been coupled with a mechanical constitutive law [see Coon et al., 1974] to calculate the state of stress in a differential element of sea ice. The input data for these calculations are estimates of strain rate components. In what follows we discuss one measurement set from which strain rate estimates can be made, the procedure for making the estimates, and how the estimated quantities should be interpreted.

The deformation mechanisms are active at the boundaries separating nearly rigid floes which have length scales of up to tens of kilometers. The motion on such a scale is piecewise rigid with discontinuities between the floes. Along the lines argued by Maykut et al. [1972], an element of sea ice with a length scale of 100 km can be expected to contain many floe-to-fllo discontinuities and therefore to have a well-defined average deformation. Such an element is still small compared with the wave length of the Nye and Thomas curve [1974; also Nye, 1975], shown in Figure 1,
which is presumably related to the scale of atmospheric systems, or approximately 1000 km.

One of the objectives of AIDJEX was to measure the deformation of a single 100 km element. To this end, positions were measured about 30 times a day at each of four stations using satellite positioning systems accurate to about ±60 m. Each station was established on a single floe, so the raw measurements define the path followed by four distinct floes. The measurement set discussed here begins on 1 May 1975 and ends on 1 October 1975, when the residents of one station observed at close hand the mechanisms of deformation discussed above. (Measurements from the other three camps continued until 1 May 1976.) The configuration of the measurement array at several times is displayed in Figure 2.

ESTIMATING THE DEFORMATION

The motion of a point defines its position \( x \) at some time \( t_b \) in terms of its initial position \( x \), at time \( t_a \):

\[
x = x(X, t_b, t_a).
\]

(1)

The function \( x \) can be approximated locally (at nonsingular points \( x \)) by the linear expression

\[
x = \sigma(t_b, t_a) + F(t_b, t_a)x,
\]

(2)

where \( F \) is called the deformation gradient. The vector \( \sigma \) is constant in space; it represents the rigid body translation of the system. Note that

\[
\sigma(t_a, t_a) = 0
\]

(3)

\[
F(t_a, t_a) = I, \text{ the identity.}
\]

The deformation gradient contains information about the strain and the rotation. It can be decomposed as

\[
F = DSR,
\]

where \( D = \sqrt{e} I \) and \( R \) is a rotation through a counterclockwise angle \( \theta \). The symmetric matrix \( S \) has principal direction \( \psi \) and determinant 1. Thus, in principal coordinates, \( S \) has the form

\[
\begin{pmatrix}
\alpha & 0 \\
0 & 1/\alpha
\end{pmatrix}.
\]

(4)

Under this decomposition, \( e = \det F \) is the ratio of the area of an element at \( t_b \) to its area at \( t_a \). \( S \) is related to the shear. For small deformations, \( \alpha \) is approximately 1 and \( S - I \) will have the form

\[
\begin{pmatrix}
\beta & 0 \\
0 & -\beta
\end{pmatrix},
\]

which is the familiar pure shear representation.

This decomposition of \( F \) differs from that of Pritchard [1974]. He defines a strain tensor \( E = DS - I \). Invariants related to area change and to shear can then be defined in terms of principal values of \( E \). The present decomposition is preferred here to illustrate in a later section that \( F \) can be estimated in several steps related to the kinematic quantities of rotation, divergence, and shear.
From Equation 2, $F$ can be written as $\partial z/\partial x$. Suppose we require an estimate of $F$ for some region $A$. The Green-Gauss theorem relates the average of $F$ inside $A$ to an integral around the boundary of $A$:

$$\int_{\text{region } A} F = \int_{\text{boundary of } A} x \times n,$$

where $n$ is the outward-directed unit vector normal to the boundary and $x$ is the tensor product. If $x$ is known at enough points $X$ along the boundary of $A$ to allow a good interpolation, then the right-hand side of Equation 5 can be estimated. Values of $x$ inside $A$ do not affect the estimate.

Another technique to estimate $F$ uses least squares theory. If pairs of values $(x_i^t, y_i^t)$ for $i = 0, \ldots, N$ are known, then estimates $\hat{c}$ and $\hat{F}$ for $c$ and $F$ (6 parameters) can be found to minimize

$$\sum_{i=1}^{N} [x_i^t - (\alpha + \beta y_i^t)]^2.$$

In this procedure, points near the vector mean make only small contributions to $F$.

(In the measurements discussed here an array of four points was used, one of which was close to the vector average of the triangle defined by the other three.)

The least squares approach is followed here because of the well-developed theory for constructing confidence intervals for the estimated quantities. For our data set the boundary integral method and the least squares method give similar results for $\hat{F}$.

We make the hypothesis that the measurement region defined with length scale 100 km can be viewed as a differential element $A$ of the ice pack over which Equation 2 holds, i.e., $x$ is a linear function of $X$. Since any three points in $A$ determine $F$, there are several possible ways to group the data to calculate $F$, using data from the four AIDJEX camps shown in Figure 2. Here we have calculated $F$ for each of the three interior triangles.

Figure 3 shows the time series of area relative to area on 1 May 1975 for each of the three inner triangles and for the least squares approximation. A favorable comparison between the curves would suggest that Equation 2 was a good representation of the motion of the array. Of course, the three results are not independent—each pair of interior triangles share a common side—so no particular importance should be attached to a similarity between the results from any two triangles. The results are striking for their poor agreement; they show few sustained periods in which the changes of area of all interior triangles have the same sign. On a shorter time scale, the daily change of area was calculated and correlations between triangles were made. The correlation coefficients for daily area change are: between BB-CA-BF and BB-CA-SB, $\rho = 0.38$; between BB-CA-BF and BB-BF-SB, $\rho = -0.35$; and between BB-CA-SB and BB-BF-SB, $\rho = 0.03$. One must conclude that the comparison is unfavorable and that Equation 2 is not a very close approximation to the actual motion.

**ESTIMATING THE NONLINEAR PART OF THE MOTION**

We wish to quantify the notion that over finite regions the linear Equation 2 is not a close representation of the motion. Our approach is to resolve the actual motion into linear and nonlinear parts and then to estimate the magnitude of each part. Define a velocity

$$v = v(X,t,\Delta) = z(X,t,t+\Delta) - X \Delta.$$

Then, motivated by the Nye and Thomas [1974] curve in Figure 1, we decompose the true velocity field $v(X,t,\Delta)$ into a smooth field and a perturbation:
\[ v = v_L + \omega, \quad (7) \]

where \( v_L \) varies nearly linearly over distances \( L \) or less. We have in mind \( L = 100 \text{ km} \). The spatial derivatives of \( v_L \) are continuous and nearly constant over \( L \).

Nye [1973] constructs a smooth velocity (roughly analogous to \( v_L \)) by convolving \( v \) with a boxcar kernel of length \( L \) and shows, in one dimension, that the derivative of the smooth velocity at \( x \) is identical to the difference of the true velocities measured at \( x \pm L/2 \). Note, however, that the two-dimensional analogy requires that measurements be made at all points on the boundary of the element. A second comment with respect to Nye's definition is that his strain on a length \( L \) involves smoothing the velocity with a kernel of length \( L \). For \( v_L \) to vary linearly over a length \( L \) in our definition, the kernel must have a length which is long compared with \( L \).

We would like to estimate \( v_L \) and its first derivatives, given measurements of \( v \) at a few discrete points. To be precise, we have measurements (denoted by a tilde) of the positions of several points (subscript \( j \)) at evenly spaced times (subscript \( i \)). Then the measured velocities are

\[ \tilde{v}_{i,j} = \frac{\tilde{x}_{i+1,j} - \tilde{x}_{i,j}}{\Delta}, \quad (8) \]

where \( t_{i+1} - t_i = \Delta; j = 1, \ldots, 4 \) stations; and \( i = 1, \ldots, N \) times. The measurements satisfy

\[ \tilde{v}_{i,j} = v(x_j, t_i, \Delta) + \epsilon_{i,j}, \quad (9) \]

where \( \epsilon_{i,j} \) is a measurement error. The position measurements in Equation 8 have errors of only a few tens of meters; velocity errors are about 50 m/\( \Delta < 0.05 \text{ cm sec}^{-1} \).

Estimates of \( v_L \) and its derivatives will be functions of the measurements, and the uncertainty in these estimates will depend on the number of measurements and on the variances of \( v \) and \( \epsilon \). The goal now is to estimate the variance of \( \omega \). The observed velocity is partitioned into the linear velocity, the nonlinear velocity, and the measurement errors:

\[ \tilde{v}_{i,j} = v_{L_{i,j}} + (\omega_{i,j} + \epsilon_{i,j}), \]

and, since \( v_L \) has linear variation,

\[ v_{L_{i,j}} = B_i + A_i \ x_{i,j}. \quad (10) \]

We want to find estimates \( \hat{B}_i \) and \( \hat{A}_i \) which minimize

\[ 4 \sum_{j=1}^{4} r_{i,j}^2, \quad (11) \]

where

\[ r_{i,j} = \tilde{v}_{i,j} - (\hat{B}_i + \hat{A}_i \ x_{i,j}). \]

For each interval \( (t_i, t_{i+1}) \) we can use standard least squares techniques to estimate \( \hat{B}_i \) and \( \hat{A}_i \) and the variance \( \sigma_{\omega}^2 \) of \( \omega + \epsilon \). One estimator of \( \sigma_{\omega}^2 \) is

\[ \sigma_{\omega}^2 = \frac{4 \sum_{j=1}^{4} r_{i,j}^2}{4 - 3}, \quad (12) \]

which has the chi-squared distribution with 4 - 3 degrees of freedom: 4 measurements, 3 parameters (1 in \( B_i \) and 2 in \( A_i \)) [see Mood and Graybill, 1963, p. 351]. The estimates of \( \sigma_{\omega}^2 \) for each time are plotted in Figure 4. Since estimates made this way have but one degree of freedom, very little confidence can be placed in this...
them. A small value of $\sigma^2$, for example, is very weak evidence that $\sigma_{w}$ was actually small at time $t_i$. Even at a time when the four measurements fit some linear field very closely, it would be a mistake to think that the parameters of that field gave particularly good estimates of the true velocity derivatives. A far safer course is to believe that $\sigma^2$ is constant over some time period, and to estimate it with many degrees of freedom as follows.

Successive intervals $(t_i, t_{i+1}), (t_{i+1}, t_{i+2})$ appear to be independent (Fig. 4), so that we can lump together $N$ points in time and take

$$s^2 = \frac{1}{4N - 3N} \sum_{i=1}^{N} \sum_{j=1}^{4} t_i^2$$

(13)

to be an estimator of $\sigma_w^2$ with $N$ degrees of freedom. Ninety percent confidence intervals for $\sigma_w$ are found as

$$90\% \text{ confidence interval: } \sqrt{\frac{s^2 N}{X_{0.95}}} < \sigma_w < \sqrt{\frac{s^2 N}{X_{0.05}}}$$

(14)

These confidence intervals are given in Table 1. Several time intervals $\Delta$ have been used to define the velocity.

<table>
<thead>
<tr>
<th>Time</th>
<th>No. of Points</th>
<th>$\Delta$ Days</th>
<th>$\sigma_w$, in cm sec$^{-1}$ (u-component)</th>
<th>$\sigma_w$, in cm sec$^{-1}$ (v-component)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-30 May 75</td>
<td>30</td>
<td>1</td>
<td>0.44 ± 0.12</td>
<td>0.48 ± 0.09</td>
</tr>
<tr>
<td>31 May-29 Jun 75</td>
<td>30</td>
<td>1</td>
<td>0.50 ± 0.10</td>
<td>0.51 ± 0.11</td>
</tr>
<tr>
<td>30 Jul-18 Aug 75</td>
<td>20</td>
<td>1</td>
<td>0.96 ± 0.24</td>
<td>1.45 ± 0.35</td>
</tr>
<tr>
<td>19 Aug-17 Sep 75</td>
<td>30</td>
<td>1</td>
<td>1.07 ± 0.23</td>
<td>1.17 ± 0.23</td>
</tr>
<tr>
<td>25 Apr-14 Jul 75</td>
<td>8</td>
<td>10</td>
<td>0.28 ± 0.11</td>
<td>0.19 ± 0.09</td>
</tr>
<tr>
<td>3 Aug-2 Oct 75</td>
<td>6</td>
<td>10</td>
<td>0.45 ± 0.21</td>
<td>0.50 ± 0.24</td>
</tr>
<tr>
<td>25 Apr-14 Jul 75</td>
<td>4</td>
<td>20</td>
<td>0.30 ± 0.17</td>
<td>0.25 ± 0.13</td>
</tr>
<tr>
<td>3 Aug-2 Oct 75</td>
<td>3</td>
<td>20</td>
<td>0.56 ± 0.36</td>
<td>0.61 ± 0.39</td>
</tr>
<tr>
<td>25 Apr-14 Jun 75</td>
<td>1</td>
<td>50</td>
<td>0.28 ± 0.21</td>
<td>0.21 ± 0.17</td>
</tr>
</tbody>
</table>

The values in Table 1 are typically 0.5 cm sec$^{-1}$. This exceeds the measurement error by an order of magnitude. Therefore, we attribute the tabulated values largely to the variance of $\omega$. We conclude from the table that the nonlinear part of the velocity field $\omega = u - u_L$ has a standard deviation in the spring of $\sigma_w = 0.4 \pm 0.1$ cm sec$^{-1}$. In the summer the standard deviation is larger: $\sigma_w = 1.1 \pm 0.3$ cm sec$^{-1}$. For longer times $\Delta$ the departures from linearity are smaller. For $\Delta = 20$ days, $\sigma_w$ equals 0.3 ± 0.1 cm sec$^{-1}$ in spring and 0.6 ± 0.3 cm sec$^{-1}$ in summer.

For a point of comparison, consider the full variance $\sigma^2$ of the velocity. Let an estimate of $\sigma^2$ be

$$\sigma^2 = \frac{1}{4N - N} \sum_{i=1}^{N} \sum_{j=1}^{4} (\omega_{i,j} - E_i)^2$$

29
The full variance of the velocity, $\sigma^2$, includes the linear and the nonlinear variation in $v$ across the array. From Table 2, where $\sigma^2$ appears, we conclude that the full variations in $v$ over 100 km are typically 1.2 cm sec$^{-1}$ in spring and 2.1 cm sec$^{-1}$ in summer for one-day velocities, and that the variability decreases for larger $\Delta$.

### Table 2. Estimated variability of velocity over 100 km. Tabulated value is estimated RMS value of $(v - \overline{v})$, with 90% confidence intervals.

<table>
<thead>
<tr>
<th>Time</th>
<th>No. of Points</th>
<th>$\Delta$ Days</th>
<th>$\sigma_\Delta$, in cm sec$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$(u$-component)</td>
</tr>
<tr>
<td>1-30 May 75</td>
<td>30</td>
<td>1</td>
<td>1.15 ± 0.15</td>
</tr>
<tr>
<td>31 May-29 Jun 75</td>
<td>30</td>
<td>1</td>
<td>1.15 ± 0.15</td>
</tr>
<tr>
<td>30 Jul-18 Aug 75</td>
<td>20</td>
<td>1</td>
<td>2.35 ± 0.35</td>
</tr>
<tr>
<td>19 Aug-17 Sep 75</td>
<td>30</td>
<td>1</td>
<td>1.90 ± 0.20</td>
</tr>
<tr>
<td>25 Apr-14 Jul 75</td>
<td>8</td>
<td>10</td>
<td>0.70 ± 0.20</td>
</tr>
<tr>
<td>3 Aug-2 Oct 75</td>
<td>6</td>
<td>10</td>
<td>1.00 ± 0.30</td>
</tr>
<tr>
<td>25 Apr-14 Jul 75</td>
<td>4</td>
<td>20</td>
<td>0.65 ± 0.25</td>
</tr>
<tr>
<td>3 Aug-2 Oct 75</td>
<td>3</td>
<td>20</td>
<td>0.65 ± 0.25</td>
</tr>
<tr>
<td>25 Apr-14 Jun 75 and 3 Aug-22 Sep 75</td>
<td>1</td>
<td>50</td>
<td>0.40 ± 0.20</td>
</tr>
</tbody>
</table>

We can deduce the linear variance using

$$\sigma^2 = \sigma_u^2 + \sigma_{\text{linear}}^2 .$$

### Spring 1975:

- $\sigma_u = 0.4$ cm sec$^{-1}$
- $\sigma_{\text{linear}} = 1.1$ cm sec$^{-1}$
- $\sigma^* = 1.2$ cm sec$^{-1}$

### Summer 1975:

- $\sigma_u = 1.1$ cm sec$^{-1}$
- $\sigma_{\text{linear}} = 1.8$ cm sec$^{-1}$
- $\sigma^* = 2.1$ cm sec$^{-1}$

Clearly, it will be impossible to estimate with confidence the coefficients of the linear variation $(A$ and $B)$ when the measurements are contaminated by nonlinear variations of about 50%. The situation does not improve for longer time scales.

Measurements of the displacement of sea ice can also be made using remote sensing techniques. From a pair of Landsat images, for example, it is possible to estimate the displacements of many identifiable points within a 100 km region over a one-day period. In an unpublished note describing such measurements, Nye and Hall report that the residuals (the difference between each displacement and the best linearly varying displacement field) are typically 0.32 km (0.37 cm sec$^{-1}$), based on five pairs of images and about 20 displacement measurements per pair, from March and April 1973. Our result of $\sigma_u = 0.4$ cm sec$^{-1}$ from spring 1975, and using a different measurement technique, confirms their value.

Confidence intervals for the velocity derivatives can be stated in terms of $\sigma_u$. If the problem is set up as $Z = \mathbf{x} \beta + \epsilon$, where $Z$ contains the measurement, $\mathbf{x}$ the geometry of the array, and $\beta$ the unknown coefficients, then the least squares solution for $\hat{\beta}$ is $\hat{\beta} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T Z$ and a 90% confidence interval for each coefficient is

$$\hat{\beta}_i \pm 1.64 \sigma_u \sqrt{\mathbf{C}_{ii}},$$

where $\mathbf{C} = (\mathbf{X}^T \mathbf{X})^{-1}$. For the array used here, with $\overline{x} = \overline{y} = 0$, the matrices are
Thus, $\sqrt{C_{11}} = 1/2; \sqrt{C_{22}} = 1.02 \times 10^{-2} \text{ km}^{-1};$ and $\sqrt{C_{33}} = 1.07 \times 10^{-2} \text{ km}^{-1}$. These quantities are roughly proportional to $n^{-4}$, and the last two are also proportional to $L^{-1}$.

A histogram is shown in Figure 5 of velocity derivatives for spring data (60 one-day periods) computed as above. Also shown are several confidence intervals. The 90% confidence interval is $\pm 0.7 \times 10^{-7} \text{ sec}^{-1}$. If such an interval is applied as an "error bar" to each calculated velocity derivative, it will frequently--about 40% of the time--be impossible to resolve, with confidence, even the sign of the derivative. In practice, quantities of most interest are combinations of velocity derivatives (divergence or curl, for example). Confidence intervals for those quantities can be constructed from $C_f$ and $C$. They generally can be expected to have estimation errors larger than the errors in the individual derivatives.

When the spatial derivative of velocity, $A$, is large, the confidence interval is smaller in a relative sense. This conclusion depends on the assumption that the nonlinear part of the velocity field is independent of the size of the linear variation. A test of contingency between the calculated derivatives of $V_L$ and the residuals $\hat{r}_{ij}^2$ supported this assumption.

Conclusions from this section are the following:

1. The nonlinear part of the 100 km daily velocity field has a standard deviation of 0.4 cm sec$^{-1}$. The linear part has a standard deviation of 1.1 cm sec$^{-1}$. These numbers are greater in the summer and smaller for longer time scales, but their ratio is always about 0.5.

2. Since the available measurements include the nonlinear part of the field, estimates of the linear coefficients have associated uncertainties of about $0.7 \times 10^{-7} \text{ sec}^{-1}$.

3. The absolute variance $C_{11}$ appears to be independent of the linear variation. Therefore, in a relative sense, large gradients may be estimated with errors as small as 20%.

**TOTAL DEFORMATION OVER 150 DAYS**

We have discussed the spatial derivatives of the velocity field, but the displacement field can be treated analogously, the only difference being an appropriate division by $A$ to get from displacements to velocities. Consider as an example the change in configuration after 150 days. The least squares solutions for $\hat{G}$ and $\hat{F}$ are
\[ \sigma(270,120) = \begin{pmatrix} -123.3 \text{ km} \\ -355.6 \text{ km} \end{pmatrix}, \quad \hat{F}(270,120) = \begin{pmatrix} 0.6840 \\ 0.3715 \end{pmatrix}, \]

and the sum of the squared residuals \((x - \hat{F}X)^T (x - \hat{F}X)\) equals 355 km².

To construct a confidence interval for the coefficients in \(F\), we take \(\hat{\sigma} = 19 \text{ km} \) and get \(\pm 0.31\) (using Equation 17 and \(\hat{\sigma}_{ii} = [100 \text{ km}]^{-1}\)). It seems difficult to interpret these confidence limits in terms of, say, divergence or shear.

Alternatively, examine the decomposition discussion above: \(F = DSR\). We find

\[
F = \begin{pmatrix}
0.9973 & 0 \\
0 & 0.9973
\end{pmatrix}
\begin{pmatrix}
0.7229 & -0.2940 \\
-0.2940 & 1.5028
\end{pmatrix}
\begin{pmatrix}
0.6342 & -0.7732 \\
0.7732 & 0.6342
\end{pmatrix}.
\]

Table 3 shows how much the total variance can be reduced by including divergence, shear, and rotation in the deformation. For instance, when the motion is fit by the best irrotational tensor, \(x = DSX\), the unexplained variance is 14,076 km². By including rotation, \(x = DSRX\), the variance is reduced further to 711 km². The effect due to rotation then is the difference, 13,365 km²; the effect due to shear is 4329 km²; and the effect due to divergence is 1 km². Tests for statistical significance confirm that \(C\) and \(R\) are definitely significant, that \(S\) is marginal, and that \(D\) might as well be set equal to the identity. The principal values of \(S\) are 1.5970 and 0.6228. Reducing each principal value by 1.0 implies a pure shear deformation of about 50%. The \(\pm 31\%\) confidence interval gives some idea how far to trust this shear estimate. This conclusion of course applies only to this particular set of data. The method is generally applicable and is a useful way to determine what parts of the motion are significant.

<table>
<thead>
<tr>
<th>TABLE 3. Variance for total deformation over 150 days. (Primes indicate that the array mean has been removed.)</th>
<th>Residual</th>
<th>Sum of squared residuals, in km²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>(x' - X')</td>
<td>26,025</td>
</tr>
<tr>
<td>Divergence and shear</td>
<td>(x' - DSX')</td>
<td>14,076</td>
</tr>
<tr>
<td>Divergence and rotation</td>
<td>(x' - DRX')</td>
<td>5,040</td>
</tr>
<tr>
<td>Shear and rotation</td>
<td>(x' - SRX')</td>
<td>712</td>
</tr>
<tr>
<td>Divergence, shear, and rotation</td>
<td>(x' - DSRX')</td>
<td>711</td>
</tr>
</tbody>
</table>

DISCUSSION

The data set considered here from spring and summer 1975 in the Beaufort Sea implies that the true deformation cannot be represented well as constant over lengths of 100 km. The scale of 100 km was originally selected for the measurement program based on an argument that it was large enough to contain many deformational features but small enough to resolve large-scale variations in the forcing field. By choosing a length which covered many floes, the argument ran, the velocity discontinuities between floes would tend to average out to the underlying linear velocity
field. The argument is incomplete because it ignores the magnitude of the discontinuities. For a field with large amplitude discontinuities, even if there are many of them and they do tend to average out, it will be difficult to estimate the deformation based on only a few measurements. Each measurement will be affected by large local "errors." These can be averaged out by taking more measurements, but not by taking a larger length scale.

Equation 17 can be used in advance to predict the accuracy of estimates from a given array of points, and it may be a useful tool for designing future experiments. Of course, our estimate of $O_w$ may not apply to different length scales.

Hibler et al. [1974] have discussed the variability of deformation estimates on a scale of about 10 km. Their term inhomogeneity variation refers to the uncertainty in deformation estimates due to the nonlinear variations of the motion over 10 km. There are important differences between their work and ours. They considered shorter length and time scales, and their data were obtained in March and April 1972. They used about a dozen measurement points. Some of their calculations may be distorted because nonindependent data appear to have been used. Nevertheless, their important conclusion that local variations significantly affect deformation estimates is well documented.

Following our definition, the nonlinear contribution to the velocity field, $\sigma_w^2$, is an increasing function of length scale. Whether $\sigma_w/L$ will increase or decrease with $L$ remains unanswered. The conjecture of Hibler et al. [1973], that $\sigma_w = L^{-1/2}$, should be regarded with caution.

The spatial velocity spectrum may appear somewhat as sketched in Figure 6. A length scale $L$ divides the spectrum into two parts, the areas of which are the variances of $\nu_L$ and of $w$. The results here imply that the area to the right of $f = 1/100$ km is roughly half the area to the left. In their Table 3 Hibler et al. [1974] indicate a value of $\sigma_w = 0.063$ cm sec$^{-1}$ determined over a 10 km scale. When compared with our value of $\sigma_w = 0.4$ cm sec$^{-1}$ for 100 km, Hibler's result implies that only a small percentage of the variance occurs at wavelengths of less than 10 km.

We have emphasized the difficulties in estimating the deformation of sea ice. These arise because of real nonlinear variations in the ice velocity field. The estimate of the variance of the nonlinear component given here can be used to design measurement programs aimed at estimating deformation. The general conclusion is that a dense array of measurements is needed to resolve the underlying deformation well. In addition to making it difficult to estimate the underlying deformation well, the sizable nonlinear variations imply that the underlying deformation does not fully characterize the deformation field. Isn't it time to re-examine our models of thickness distribution and stress-strain laws which are formulated in terms of the mean deformation?

**ACKNOWLEDGMENT**

We thank Rich Hall for providing the displacement measurements from Landsat images. This work was supported by the National Science Foundation Grant OPP71-09031, formerly GV 28807, to the University of Washington for the Arctic Sea Ice Study.
REFERENCES


FIGURES

Fig. 1. The variation of $u$, the $x$-component of displacement, with distance $x$ along a line. Period 21-23 March 1973. From ERTS imagery. Reproduced from Nye and Thomas [1974].
Fig. 2. Configuration of the measurement array at eight times separated by 20-day intervals during spring and summer 1975.

Fig. 3. Ratio of area at time $t$ to the area on 1 May 1975 for each inner triangle and for the least squares fit. Gaps occur in three of the curves because of missing data at one camp (BF).
Fig. 4. Daily estimates of $\sigma_u^2$. 
(+), $u$-component of velocity; 
(□), $v$-component of velocity.

Fig. 5. Histogram of spatial velocity derivatives for sea ice from 60 one-day periods in spring 1975.

Fig. 6. Power spectrum of spatially varying ice velocity. Curve is not based on data. Our only insight into the shape of the spectrum at this point is that the length scale of 100 km divides the variance (area under the spectrum) roughly in half.
Ice conditions and motion in the nearshore Beaufort Sea from 27 January to 3 February 1976 were strongly affected by ice stresses. We chose to simulate this response using the AIDJEX model. There is no motion during the first two days. When motions begin, they are westward. There is a time lag with ice in the eastern portion responding later. In the nearshore area a fast ice region exists that is separated from the moving pack by a discontinuity. These conditions are verified by NOAA satellite images and data from drifting buoys and AIDJEX manned camps. The model is shown to simulate these features accurately, including the velocity discontinuity. This test of the AIDJEX model shows that we understand how ice responds on the large scale to driving forces and are able to describe this relationship at times when the ice stress exerts a dominant influence on the response. This model allows us to use winds (including the large set of historical winds) to determine ice velocity (and trajectories) and to estimate the large-scale average forces that pack ice may exert.

INTRODUCTION

This report describes the ice conditions and dynamics in the Beaufort Sea from 27 January through 3 February 1976. In addition to describing observed response of the atmosphere, ice, and ocean, we present a simulation of the ice conditions using the AIDJEX ice model. The time period was chosen because it contains ice dynamics, conditions, and motion that are both interesting and well documented by high-quality data from the AIDJEX program.

The motion of the ice during this period is greatly influenced by the internal stress in the ice pack. A flaw lead is developed along the north coast of Alaska from Pt. Barrow to the Mackenzie Delta. Shoreward of the flaw lead the ice has little motion; however, seaward of the lead the ice shows
appreciable motion. However, even in the regions where there is appreciable motion the amount and direction of it is greatly influenced by the internal stress. A detailed description of these conditions is given in this report.

In Figure 1 we show the region of interest together with the position of data stations for the AIDJEX program (the data station numbers shown in Figure 1 are the same station numbers as those used by Thorndike and Cheung [1977] to report on sea ice motions observed during AIDJEX and as part of this work). Stations numbered 1, 3, and 2 were manned camps at which extensive measurements in the atmospheric and oceanic boundary layers were taken. These camps are also identified by radio call names of Caribou, Snow Bird, and Blue Fox, respectively. We have used directly the positions and barometric pressure measured at each camp. The data from the AIDJEX stations and NOAA satellite imagery are used in the next section of this report to describe the ice conditions during the time in question.

A simulation of the ice dynamics for the region shown in Figure 1 has been made using the AIDJEX ice model. In the simulation, part of the data from AIDJEX stations are used to drive the model and the remaining data are used to verify the quality of the simulation.

ICE CONDITIONS

Daily velocity (average velocity during a day) is shown for eight days for all stations in Figure 2. On 27 and 28 January there is essentially no motion of any station. As will be shown later, there is appreciable air stress applied to the ice during those days. On 29 January the westernmost stations begin to move, and over days 30 and 31 January there is a predominantly western motion of all stations except for those in the Alaskan nearshore area, where the stations have essentially no motion. During 1, 2, and 3 February the ice motion reverses. To obtain a more detailed view of how the ice motion develops during this time, velocity time histories are shown for several of the stations in Figures 3 and 4. Figure 3 indicates the north-south and east-west components of the velocity for stations 3 (Snow Bird) and 2 (Blue Fox). The major velocity component during the period is directed east-west. We also see that the east-west motion of station 3 begins a half day before station 2. This indicates that the
disturbance that causes the motion travels from west to east across the Beaufort Sea and produces motion in the ice which is predominantly in the east-west direction, an indication which is supported by motions of other stations, e.g., 66 and 17, shown in Figure 2. Figure 4 shows the velocity time history for stations 44, 1 (Caribou), and 22. From Figure 1 it can be seen that these stations are aligned more or less north-south. The largest velocity components at stations 44 and 1 are directed in the east-west direction. We also see that stations 1 and 44 begin their motion at essentially the same time, even though the peak occurs later. Again, the disturbance moves west to east. Station 22 shows no appreciable motion during the period. This indicates that there is a large velocity difference between stations 1 and 22, which is, of course, what was seen in Figure 2 with the average daily velocities.

The characterization of the motion that emerges from Figures 2, 3, and 4 is also indicated clearly in NOAA satellite imagery for the period shown in Figures 5 through 8. All figures show the locations of the AIDJEX stations, and Figure 5 shows the outline of the area of interest. The NOAA images show a development of a series of cracks in the ice running essentially north-south from the flaw lead to the northernmost boundary of the area of interest. The flaw lead is not apparent in the earlier images, but by 2 February it is fully developed. It is at the flaw lead that the velocity discontinuity arises which is apparent in Figures 2-4. It is the opening of cracks running north-south that produces the east-west motion of the stations shown in Figures 2-4. The progression of these cracks occurring in time sequence from west to east across the area is consistent with the velocity time histories shown in Figure 3.

In addition to examining the kinematics of sea ice, it is important to look at forces acting on the sea ice. We begin by discussing the atmospheric boundary layer model used in the AIDJEX model simulation. Specifically, we discuss the procedure for determining the drag coefficient and turning angle from observed conditions. A similar discussion follows for the oceanic boundary layer. Finally, we study the forces acting on the ice and the resulting motions of the manned camps Caribou and Blue Fox.

The barometric pressure field defines the atmospheric geostrophic flow $\mathbf{U}$. The planetary boundary layer relates the surface traction exerted by the atmosphere on the upper ice surface $\mathbf{T}_a$ to the geostrophic flow [Brown, 1976]. The
air stress is computed as a quadratic function of $||U||$ applied at an angle $\alpha$ counterclockwise from $\hat{y}$:

$$\vec{\tau}_a = \rho_a C_D \frac{1}{f_c} \vec{k} \times \nabla p$$

where

$$\rho_a = \text{air density},$$

$$f_c = 14.15 \times 10^{-5} \text{ sec}^{-1} \text{ is the Coriolis parameter at 76°N latitude},$$

$$C_D = \text{a dimensionless drag coefficient},$$

$$\vec{E}_a = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix},$$

$$\vec{k} = \text{unit vector upward and orthogonal to plane of motion},$$

$$\nabla p = \text{horizontal gradient of barometric pressure p.}$$

The surface air stress can also be related to the square of the mean wind speed measured at 10 m above the ice surface:

$$\tau_a = \rho_a C_{10} U_{10}^2$$

where $C_{10}$ is the 10 m drag coefficient. Combining (2) and (3) we can express $C_D$ as a function of $C_{10}$ and the ratio $U_{10}/G$:

$$C_D = C_{10} \left( \frac{U_{10}}{G} \right)^2$$

Some measurements of $C_{10}$ from a site near Big Bear in spring 1975 are reported by Leavitt et al. [1977]. The mean value of $C_{10}$ was $1.3 \times 10^{-3}$, but the measurements showed a variation with wind direction, from $1.0 \times 10^{-3}$ to $1.5 \times 10^{-3}$. These measurements were taken over smooth floes and do not include the effect of "form" drag due to pressure ridges and rubble fields. For typical ice conditions in the Beaufort Sea, Arya [1975] predicts that the drag due to ridges would be approximately equal to that over the smoother ice; for example, this would suggest $C_{10} \approx 2.6 \times 10^{-3}$. Carsey and Leavitt [1977] have calculated air stress by integrating wind profiles through the boundary layer. These wind
profiles were obtained by tracking the motion of balloons (pibals) as they ascended through the boundary layer. A preliminary estimate of $U_{10}$ from these data is $2.7 \times 10^{-3}$, which agrees with Arya. The confidence limit on this estimate is $0.7 \times 10^{-3}$.

Preliminary analysis of pibal data and recorded surface winds for this period suggest $(U_{10}/G)^2 = 0.3$ and a turning angle $\alpha = 28^\circ$. These values are used to compute the drag coefficient

$$C_D = 0.8 \times 10^{-3}$$

which is used to compute air stress from the geostrophic wind for the simulation. Further comparisons between surface and geostrophic winds suggest that the mean turning angle for this period is $35^\circ$ rather than $28^\circ$, but the standard deviation in this estimate is $15^\circ$. The value used is therefore within the range of uncertainty.

A comparison between stresses derived from geostrophic winds and surface winds at the manned camps is shown in Figures 9-11. The agreement is excellent except for 30 and 31 January, when the east-west component of air stress obtained from geostrophic velocity exceeds the value determined from surface winds. Barometric pressures at the manned camps have been found to be in error by 0.1 mb at this time. Corrected values reduce the air stress by about 20%. The corrected air stress is shown in Figures 9-11. The uncorrected value has been used in the simulation presented later in this report.

Figure 12 shows a balance of dynamic forces derived from smoothed records of measured quantities at station Caribou sampled at 1200 GMT on 30 January. The force balance is a sum of air stress $T_a$, water stress $T_w$, Coriolis force $f_c$, and a residual $\tilde{R}$ where

$$T_a + T_w + f_c + R = 0$$

The water stress is related to ice velocity by

$$T_w = \frac{\rho_c}{C_w} ||\tilde{v}|| \tilde{B} \cdot \tilde{v}$$

$$\tilde{B} = \begin{pmatrix} \cos(\pi+\beta) & -\sin(\pi+\beta) \\ \sin(\pi+\beta) & \cos(\pi+\beta) \end{pmatrix}$$
and Coriolis force is

\[ f_C = -m f_c \beta \times v \]

where \( \rho_w \) is the water density, \( m \) is the ice mass per unit area (300 gm cm\(^{-2} \)), \( f_c \) is the Coriolis parameter, and \( \beta \) is the angle of turning. The resultant vector \( \vec{R} \) is required to balance the equation and represents internal ice forces, sea tilt, and ice inertia. We have used summer conditions when the ice is not compact enough to support appreciable internal stress (i.e., \( \vec{R} \) is small) to evaluate the water stress constants. The best results were obtained with \( C_w = 0.0055 \) and \( \beta = 23^\circ \) using observed surface winds and a drag coefficient \( C_{10} = 0.0027 \). During 30 January it is clear that \( \vec{R} \) is an appreciable force acting on the ice. Therefore, ice stress is an important factor in any simulation of this period.

Figures 13, 14, and 15 show time series of forces and velocities as measured at the three manned camps. In the top segment of each plot are shown the air stress component as determined from the 10 m wind \( (T_a = \rho_a C_{10}U_{10}) \), along with the negative component of the resultant vector, \( \vec{R} \). From (6) and (7), it is clear that \( \vec{R} \) and \( T_a \) will be equal and opposite when there is no ice motion; thus the plotted curves coincide for the first few days. When the ice velocity increases, the water drag and Coriolis force become increasingly important. We further analyze the forces by considering how the ice would behave if it were too weak to support an internal stress gradient. Then \( \vec{R} = \vec{0} \) and (6) can be solved for the wind-driven velocity, \( v_{wd} \). Solutions for wind-driven drift are shown along with the measured velocities as the lower traces of each plot. Observed motions are constrained to an east-west direction by the ice stress. Wind-driven drift has a larger north-south component. This is also indicated by the sizable southward components of internal force on 30 January at Caribou (Figure 12) and Snow Bird even though there was practically no north-south component of surface wind.

An important question to ask is, how much does the internal ice stress affect the trajectory of a given point? To this end, Figure 16 indicates the observed trajectory of station 1 (Caribou) and the wind-driven trajectory. It can be seen clearly that the difference between these trajectories is very large. At the end of the eight-day period there is a difference in position of

42
approximately 25 km. The internal ice stress has retarded the motion of the ice by this amount.

MATHMATICAL MODEL

Conservation of momentum in this system accounts for air stress \( T_a \), water stress \( T_w \), divergence of ice stress \( \nabla \cdot \mathbf{v} \) (\( \mathbf{q} \) is the Cauchy stress in excess of isostatic equilibrium integrated through the thickness in this two-dimensional material model), Coriolis acceleration \(-m f_c \mathbf{k} \times \mathbf{v}\), and sea surface tilt \( (m f_c \mathbf{k} \times \mathbf{v}_g)\):

\[
m \mathbf{\ddot{v}} = T_a + T_w + \nabla \cdot \mathbf{q} - m f_c \mathbf{k} \times (\mathbf{v} - \mathbf{v}_g)
\]

where \( m = \text{mass per unit area}, \) and \( f_c = 14.15 \times 10^{-5} \) sec\(^{-1}\) is the Coriolis parameter at 76°N latitude. The notation ('') implies differentiation along the particle path, and \( \nabla \) is the spatial gradient operator.

The oceanic boundary layer is represented by a quadratic drag law similar to that used in the oceanic boundary layer as shown in equation (7). Water drag, however, is a function of the ice velocity relative to the geostrophic current \( \mathbf{v}_g \). The relationship is

\[
T_w = \rho_w C_w \| \mathbf{v} - \mathbf{v}_g \| \mathbf{E} (\mathbf{v} - \mathbf{v}_g)
\]

where all variables except \( \mathbf{v}_g \) have been defined previously. The geostrophic flow is assumed to be given by long-term mean observed values. In Figure 17 we present the values. Values at intermediate locations are computed by linear interpolation between values defined on the 75 km square grid.

The elastic-plastic constitutive law developed by the AIDJEX modeling group [Coon et al., 1974; Coon and Pritchard, 1975; Pritchard, 1975] relates stress to the deformations. We assume a stiff linear elastic response:

\[
\mathbf{q} = (M_1 - M_2) \frac{1}{2} \text{tr} \mathbf{\varepsilon} + 2M_2 \mathbf{\varepsilon}
\]

where \( \mathbf{\varepsilon} \) is the elastic strain. Moduli used in each simulation are presented in Table 1. The moduli are large enough that elastic strain cannot exceed 0.2%. The rate of change of elastic strain is determined from
TABLE 1
MODEL PARAMETERS FOR VARIOUS SIMULATIONS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol (Units)</th>
<th>Run 3B</th>
<th>Run 3C</th>
<th>Run 3D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strength</td>
<td>$p^*$ (dyn cm$^{-1}$)</td>
<td>$10^7$</td>
<td>$10^8$</td>
<td>$10^9$</td>
</tr>
<tr>
<td>Bulk Modulus</td>
<td>$M_1$ (dyn cm$^{-1}$)</td>
<td>$0.5 \times 10^{10}$</td>
<td>$0.5 \times 10^{11}$</td>
<td>$0.5 \times 10^{12}$</td>
</tr>
<tr>
<td>Shear Modulus</td>
<td>$M_2$ (dyn cm$^{-1}$)</td>
<td>$0.25 \times 10^{10}$</td>
<td>$0.25 \times 10^{10}$</td>
<td>$0.25 \times 10^{12}$</td>
</tr>
<tr>
<td>Time Step</td>
<td>$\Delta t$ (sec)</td>
<td>300</td>
<td>120</td>
<td>40</td>
</tr>
<tr>
<td>Mean Thickness</td>
<td>$m$ (gm cm$^{-2}$)</td>
<td>300</td>
<td>300</td>
<td>300</td>
</tr>
</tbody>
</table>

\[
\hat{e} - \hat{w} \varepsilon + \varepsilon \hat{w} = \mathcal{D} - \mathcal{D}_p
\]  
\[\phi = \sigma_{II} + \tan b \sigma_I (1 + \sigma_I/p^*)^{1/2}
\]

where stretching $\mathcal{D} = 1/2(L + L^T)$ and spin $\hat{w} = 1/2 (L - L^T)$ are obtained from the velocity gradient $L = \nabla \nu$. The plastic stretching $\mathcal{D}_p$ is defined by the normal flow rule

\[
\mathcal{D}_p = \lambda \frac{\partial \phi}{\partial \sigma}
\]

where $\lambda$ is a positive multiplier. Finally, the yield criterion

\[
\phi(\sigma, p^*) \leq 0
\]

completes the description. The yield surface has been assumed to have the shape of a "squished teardrop" as shown in Figure 18. The family of curves has been normalized by $p^*$. The surfaces are defined by

\[
\mathcal{D}_p = \lambda \frac{\partial \phi}{\partial \sigma}
\]

where $\lambda$ is a positive multiplier. Finally, the yield criterion

\[
\phi(\sigma, p^*) \leq 0
\]

completes the description. The yield surface has been assumed to have the shape of a "squished teardrop" as shown in Figure 18. The family of curves has been normalized by $p^*$. The surfaces are defined by

\[
\phi = \sigma_{II} + \tan b \sigma_I (1 + \sigma_I/p^*)^{1/2}
\]

where $b$ is the angle at which the curve approaches the origin ($\sigma = 0$). We have chosen $b = 30^\circ$ since this value has been found to be reasonable in previous simulations [Pritchard, Coon, and McPhee, 1977]. Yield strength $p^*$ determines the size of the surface given by equation (14). For the set of simulations we
have varied \( p^* \) as a parameter, setting it to a constant in each calculation. Thus, we have used a perfect plasticity model.

An important feature of the AIDJEX ice model is the ice thickness distribution. It is this variable that distinguishes ice conditions by describing the relative area covered by ice of each thickness. One of the properties of the ice model that depends on thickness distribution is the yield strength \( p^* \). At present, we believe that strengths found from thickness distributions are too low to allow realistic simulation of ice motion and deformation [Pritchard, 1977]. Therefore, we have bypassed this part of the model in favor of varying \( p^* \) as an arbitrary input parameter. The results of this work provide critical information on strength needed to simulate ice response and shall provide direction as we reformulate the redistribution function and energetics argument that enable us to determine strength from the thickness distribution.

QUASI-STEADY NUMERICAL INTEGRATION

In previous simulations [Coon et al., 1976; Pritchard, Coon, and McPhee, 1976], we have used air stress fields each six hours and boundary velocities each three hours with values determined at intermediate times by linear interpolation. Solutions were then obtained using a difference approximation known as the leapfrog scheme [Pritchard and Colony, 1976]. This scheme requires numerical time steps on the order of 2 minutes for cells that are 40 km wide using typical elastic parameter, say \( M_1 = \frac{1}{2} \times 10^{11} \text{ dyn cm}^{-1} \) and \( M_2 = \frac{1}{4} \times 10^{11} \text{ dyn cm}^{-1} \), and an area mass density of \( m = 300 \text{ gm cm}^{-2} \). The Courant condition is assumed to give \( c \triangle t/\triangle x \leq 1/2 \) where \( c = [(M_1 + M_2)/m]^{1/2} \). See Table 1 for values used in the simulation.

The fundamental concept of the AIDJEX model is that the physical processes of ridge building and lead formation are the mechanisms that provide deformation. The model further assumes that a large-scale spatial average (~100 km) is being described. We feel it is consistent with these ideas that temporal variations be resolved on scales of the order 1 day. To be more compatible with these concepts, we have modified the numerical scheme. We have averaged the air stress and the boundary velocity over each one-day interval and now seek to find the steady-state response of the model to the constant driving forces.
The ice acceleration may be rewritten
\[ \ddot{v} = \nu_t + \zeta v \]  
(15)
where \( \nu_t \) is the partial derivative of velocity \( v(x, t) \) with time and \( \zeta v \) represents advection. Since we seek steady solutions (by which we mean that velocity is constant, not zero), we see that an Eulerian formulation is simpler to visualize. In that case \( \nu_t = 0 \). The contribution of advection to the momentum balance is an apparent force
\[ f_a = m \zeta v \]  
(16)
The magnitude is on the order of
\[ \|f_a\| \leq m \|\zeta\| \cdot \|v\| \]  
(17)
where velocity gradient \( \|\zeta\| \approx 1 \times 10^{-5} \) sec\(^{-1} \) in the marginal ice zone and velocity \( \|v\| \approx 20 \) cm sec\(^{-1} \) in the pack ice so that as a worst case
\[ \|f_a\| \approx 0.06 \) dyn cm\(^{-2} \]  
(18)
which is an order of magnitude smaller than significant forces in equation (8). Therefore, we neglect advection in the simulations.

For completeness, we must similarly evaluate the advection of elastic strain in equation (11). However, we have no accurate estimate of the spatial gradient of elastic strain. Therefore, without proof we neglect this advection term, also, but note that the elastic response is as much a numerical artifact as a physical reality. Furthermore, elastic strains are constrained to be less than 0.2% by choosing moduli \( (M_1 \) and \( M_2) \) to be large. From these arguments, we feel that the assumption is valid.

Our results show that some variations still occur during the last cycle of iteration. At this time, we are assuming the solution to have reached steady state. Forces appear to vary less than 0.1 dyn cm\(^{-2} \) during the last hour of iteration (approximately 60 cycles), and this difference is acceptable. Since the quasi-steady concept is a new one, we have not felt justified in
developing a criterion to decide when the solution has converged because we have only begun to decide whether or not we should continue the quasi-steady solution method. Indications are that the method is an improvement over our previous scheme in which solutions vary continuously, and we shall begin to look into the convergence question in more detail.

PROBLEM CONFIGURATION

The locations of 17 data buoys and three manned camps have been shown in Figure 1. We have chosen to use the four northernmost and the two westernmost buoys (one buoy common to both) to provide boundary conditions for the simulation. The boundary is assumed fixed to shore along the North Slope from Pt. Barrow east to Banks Island. The motion of each of the other two sections of boundary is obtained using a spline interpolation polynomial with zero second derivatives at the end points. For example, a spline interpolation polynomial using four data buoys and a fixed point provides velocities at each grid point lying along the northern boundary of the grid. A generally rectangular grid is set up in the interior of the region. The interior points are chosen so that each additional buoy or camp lies either on a grid point or on a line to simplify interpolation of solutions for comparison. The 15 interior stations allow us to test the performance of the model in reproducing observed motions. We have regenerated a grid for each day of the calculation because of the motion that occurs. This detail is necessary so that the computed results can be interpolated properly for direct comparison with observations.

RESULTS

Wind-driven ice velocity is presented in Figure 19. This velocity is obtained as a balance of all forces considered in the complete AIDJEX model except for ice stress divergence, and it is calculated at each point independently of information at surrounding points by using the air stress fields shown in Figure 20.

On 27 and 28 January (a and b of Figure 20), in those areas in which air stress is small (for example, less than about 2 dyn cm\(^{-2}\) near the Alaska North
Slope), a low wind-driven velocity of about 5 cm sec\(^{-1}\) reflects mostly transport by the geostrophic ocean current; where wind speeds are larger (on the order of 25 cm sec\(^{-1}\)) the influence of a larger air stress dominates. These results may be compared with the observed ice drift (Figure 2), which is essentially zero during these two days.

On 29, 30, and 31 January (c, d, and e of Figure 20), the winds rise and the air stress increases to a range of values on the order of 2-5 dyn cm\(^{-2}\). The air stress is more nearly homogeneous on each day, with winds blowing toward the west. The wind-driven drift shows most of the domain moving to the northwest at about 20-30 cm sec\(^{-1}\). On 30 and 31 January, the wind pattern within about 100 km of the Alaskan North Slope shows a large gradient, with wind-driven drift results either zero or turned northward. The speeds are larger than those observed at these times; but more striking is the fact that the ice was observed to drift westward, but the simulated wind-driven drift is to the northwest.

During 1, 2, and 3 February (f, g, and h) the winds fall off and turn northward and finally to the northeast. At these times the wind-driven drift generally follows the direction of the observed motion, but speeds are typically twice as high as observed speeds. The most striking feature here, and one that is modeled poorly by wind-driven drift, is the nearshore behavior when ice is motionless in a band nearly 200 km wide along the North Slope and separated by the flaw lead that appears as a velocity discontinuity in the observed motions (Figure 2) and the satellite images (Figures 5-8). Instead, the wind-driven drift varies smoothly everywhere, with spatial gradients dictated by gradients in the air stress field.

The ice model is included in the simulation so that the effect of ice stress divergence may be considered. We have performed the simulation three times for the interval 27 January - 3 February. During each of these simulations the yield strength \(p^*\) was held at a different constant value throughout the domain. The values are shown in Table 1. The intermediate yield strength of \(p^* = 10^8\) dyn cm\(^{-1}\) was chosen to agree with the lower bound estimate of Pritchard [1977] during the interval 10-24 February 1977, just seven days after the simulation time chosen for the present study. Although a lower bound, the estimate is thought to be a reasonable estimate of the actual value. We have also used values an order of magnitude smaller and larger to
see how sensitive the resulting motions are to such variations. For convenience of presentation, we shall first discuss the velocity calculated using the intermediate yield strength (Run 3C). Then we shall show the effect on the velocity field due to changing yield strength. Finally, we shall return to the best estimate to look in more detail at the deformation and stress fields that are obtained.

The sequence of modeled ice velocities determined with yield strength $p^* = 10^8$ dyn cm$^{-1}$ is presented in Figure 21. The accuracy with which we have simulated velocity seems remarkable; it is comparable to observed velocity at each available station shown in Figure 2. During 27 and 28 January the model velocity is nearly zero throughout the domain, which agrees with observed motions. During 29 January as the winds rise we find motion to the northwest in the western half of the domain, which agrees with observed motions. Except for the two buoys at approximately 150 km northeast of Pt. Barrow, the velocity of each interior check buoy and manned camp is accurate to within a few centimeters per second.

The discrepancy between the two nearshore buoys and model results is caused at least in part by the velocity profile assumed as a boundary condition. We have interpolated the boundary velocities between the five buoys along the boundary to input the velocity of each grid point. Furthermore, all points along the fixed shore have zero velocity. Therefore, the boundary velocity smoothly approaches zero as we approach shore. This does not allow the large velocity gradients that are observed to appear near these regions.

During 30 January the ice speed increases to about 20 cm sec$^{-1}$ with the wind and the flow to the west. In the Alaska nearshore region steep gradients normal to the shoreline appear. The velocity out to about 80 km is small, comparing well with observed motions. To the east we find that the modeled velocity field continues to move with a lead opening along the shore at Banks Island. The observed velocity decays rapidly in the last 200 km of this region. It is possible that inaccuracies in the winds, incorrect yield surface shape, or the lack of tensile strength of the ice model could cause this error in the approximation.

During 31 January, winds are similar to 30 January, and with comparable velocities. By this day the region off Banks Island is observed to be moving
at the same velocity as the area to the west. The most striking feature of the velocity field during 31 January is the region of fast ice along the U.S.-Canada north slope that is separated from the moving pack ice by an abrupt change in velocity—a discontinuity. As seen in Figure 2e, the three buoys within this region are stationary and nearby ones are moving rapidly. We interpret this discontinuity as the flaw lead reaching from Pt. Barrow to Banks Island. We have simulated accurately both the existence and location of this discontinuity and the smooth velocity field in the pack ice.

During the last three days of the simulation (1, 2, and 3 February) the details of the velocity field become less interesting, but we find that the modeled velocity does come around and match the observed motions accurately. The comparison is accurate throughout the domain, with negligible motions in the Alaskan nearshore represented correctly.

It is important to point out that while it is true that the prescribed boundary velocity has a strong influence on the resulting velocity field, the accurate representation of the velocity field throughout the interior of the domain could be achieved only by a model that represents sea ice response correctly in a variety of deformation states. We shall demonstrate this point by simulating the response using the same plasticity model but with different values of yield strength.

In Figure 22 we present the set of eight velocity fields that result when a yield strength \( p^* = 10^7 \) dyn cm\(^{-1} \) is used (Run 3B). A comparison with wind-driven drift velocities (Figure 19) shows that velocities modeled by the low-strength yield surfaces are similar. In the interior the velocities compare within a few centimeters per second in magnitude and are oriented in approximately the same direction but turned consistently 10-20 degrees to the left by ice stress divergence. Therefore, as with wind-driven drift, the weak ice model does not provide an accurate simulation of observed buoy and manned camp motions. For example, during the first two days when winds are too low to move the ice we find that the weak ice model allows motion. Furthermore, at the boundaries where the velocity is specified by the buoy motions, we find the weak ice model allows a discontinuity to develop in the velocity field. The jump in velocity persists at almost all boundary locations for the entire eight-day period. Finally, the behavior of the weak ice model in the nearshore does not simulate the discontinuous behavior.
that is observed to exist. The modeled velocity field instead varies smoothly to the zero boundary value specified at the shore. This is not a reasonable representation of the fast ice zone seen in satellite images, in the buoy motions, and in the modeled results with yield strength $p^* = 10^8$ dyn cm$^{-1}$ (Run 3C).

To learn how sensitive modeled velocities are to changes in yield strength we felt it to be worthwhile also to perform a simulation with $p^* = 10^9$ dyn cm$^{-1}$ (Run 3D), an order of magnitude larger than the estimated value. However, we felt it unnecessary to simulate all eight days. During the first two days (27 and 28 January) when winds were low and boundary velocities motionless, the model predicted no motion with a yield strength of $p^* = 10^8$ dyn cm$^{-1}$. It is therefore not possible to change this velocity field by increasing yield strength.

Since sea ice conditions were similar during the next three days (29, 30 and 31 January), we felt it necessary to simulate only one day. We have chosen 30 January as representative. The modeled velocity field during 30 January is presented in Figure 23 for the high yield strength $p^* = 10^9$ dyn cm$^{-1}$. Although the nearshore region has zero velocity it is seen that the width is far larger than the fast ice region defined by buoy motions (Figure 2). Approximately half of the entire domain is predicted to be at rest. As expected, the strength is so high that no discontinuous behavior is exhibited at boundaries, but modeled velocities in the interior do not compare closely with observed velocities. Results similar to those for the days 27-31 January are expected if the last three days are simulated (1-3 February).

In summary, we find that the perfect plasticity model may be used to simulate the observed velocity field quite accurately when the yield strength is estimated correctly. In particular, fast ice regions in the nearshore are accurately delineated and pack ice motion is accurately represented. The interface between these regions is narrow and is approximated by a discontinuity in the theoretical model (and by a rapid variation across 2-3 cells in the difference approximation). Furthermore, we have learned how sensitive modeled velocity fields are to variations in yield strength. Variations of an order of magnitude provide modeled velocities that are similar to wind-driven if too weak, and strongly dominated by boundary conditions if too strong.
Our attention turns back to the intermediate strength simulation (Run 3C). We now present a more detailed view of the results since the velocity field has been shown to be accurate.

The deformation is shown in Figure 24 for each of the eight days simulated using the strength estimated at $p^* = 10^8$ dyn cm$^{-1}$ (Run 3C). We have presented the stretching $\mathcal{Q}$ which is the symmetric part of the velocity gradient $\mathcal{Q} = \nabla \mathcal{V}$. This is the variable considered as strain rate in small deformation theories and is the variable most descriptive of velocity differences throughout the domain. Within each cell of the numerical grid we have displayed the principal values of stretching oriented in the correct directions. Opening and closing in each direction are differentiated by dashed and solid lines, respectively. A line of 1 cm length on the figure represents a stretching value of $8 \times 10^{-7}$ sec (approximately 8% per day).

During the first two days (27 and 28 January) deformations are negligible, as were the motions. During 29 January deformations begin to occur around the nearshore with opening at Banks Island and with both shearing and opening north of Alaska. We note that principal values of equal magnitude and opposite sign represent pure shearing--that is, shearing accompanied by no dilatation (area changes). During 30 and 31 January a similar pattern of deformation occurs, but principal values are larger. Maximum shearing ($D_{II} = D_1 - D_2$, the difference between principal values) in the two cells approximately 100 km north of the U.S.-Canada land mass is about $16 \times 10^{-7}$ sec (-16% per day). This larger deformation is calculated in a narrow band about two computational cells wide (~80 km) and represents the velocity discontinuity that we discussed earlier (Figure 2).

Since the numerical technique does not predict discontinuities explicitly, we must interpret these features by studying both the velocity and the deformation fields. It is seen that deformation in the center of the domain is an order of magnitude smaller than in the nearshore region. It should be pointed out that we had expected a region of uniaxial opening to occur along a line running generally northward from the shear zone. This is seen in the satellite images (Figures 5-8) in the form of leads running north to south. This feature of the observed conditions is not represented in the simulation. We shall return to the discrepancy later.

During the last three days (1-3 February) the deformations do not show
a simple and significant pattern until 3 February, when the northwest corner is seen to undergo shearing of about 6% per day, whereas the entire part of the domain near shore is not deforming.

The stress states simulated during the eight-day period are presented in Figure 25. At each node point we have determined the stress state as the average value found in the surrounding nodes. The principal stress values are shown proportional to line length in the directions in which they occur. A line length of 1 cm represents a value of stress equal to the yield strength of $10^8$ dyn cm$^{-1}$. While we cannot test the stress state by direct comparison with observations, we can learn at least to some extent whether the stress state is physically reasonable. For example, we see that during 28 January - 1 February, when ice is blown away from Banks Island, the stress is small in that region. This result is desirable since we expect little ice stress to arise in regions that are undergoing opening. Similarly, where the ice is being blown into a region the stress is seen to be larger (e.g., the western boundary on 30 January).

However, we are not satisfied that principal values of stress in the center of the domain during 30 January are on the order of $5 \times 10^7$ dyn cm$^{-1}$. It is in this region that we have seen the leads opening in a generally north-south direction. We find it difficult to understand how stress may be transmitted across these leads in an east-west direction. We believe the shape of the yield surface must be modified to correct the stress state in this region. We have preliminary results of such a simulation using the triangular yield curve shown in Figure 26. Using the triangular yield curve allows the stress state to be uniaxial where plastic flow occurs. Preliminary results indicate that the principal values are aligned with the leads and a zero stress occurs normal to the leads. We further believe that the uniaxial opening deformation that is not observed in the present simulation (with a squished teardrop) will probably occur when the triangle yield surface is used. In summary, we find the stress fields to be reasonable except for one detail and we understand how that problem may be eliminated.

The force balance at each location provides important insights relating response to the driving forces. In Figure 27 we present the sequence of plots showing the forces acting at the node nearest the location of manned camp Caribou. In Figure 28 we present similar results for the node nearest the
location of manned camp Blue Fox. The differences between the two plots depict spatial variations that occur between points about 150 km apart. During each day the results are similar but can vary by about 25 percent. In each plot of force balance we present the calculated ice drift as a dashed vector. The air stress $\mathbf{\tau}_a$, water stress $\mathbf{\tau}_w$, ice stress divergence $\mathbf{f}_\sigma$, and Coriolis force $\mathbf{f}_c$ are also shown. In addition, we show a vector $\mathbf{E}$ that is required to sum forces to zero. It is composed of sea surface tilt, of inertia which is a measure of the lack of convergence to steady-state conditions, and of plotting errors on the order of 0.1 dyn cm$^{-1}$. During 30 and 31 January when winds and ice motion are highest, the force balance plots are especially useful. Since these two days are similar we concentrate on results of only one day, 30 January.

We confine our further attention to Figure 27 and consider the results at Caribou because these may be checked directly with Figure 12. It should be noted that during 30 January (see Figure 2d) the observed motion of Caribou and Snow Bird appear to be about 20-30 degrees counterclockwise from other nearby points. This appears from satellite images to occur because the camps are on a large single floe that is surrounded by several leads and is rotating. We should not be confused by this anomalous motion.

The comparison between modeled and observed forces is reasonably accurate. Many of the differences can be explained. First, we have already shown (Figure 9) that the geostrophic air stress input to the model is about 20% too large. This is shown also in the force balance. It is a cause of a large difference between $\mathbf{f}_\sigma$ and $\mathbf{R}$ (we note that stress divergence should be the largest contributor to $\mathbf{R}$) because a reduction of $\mathbf{\tau}_a$ by 20% would cause that vector difference to be subtracted from $\mathbf{f}_\sigma$. Furthermore, the anomalous observed velocity of Caribou means that the modeled velocity is a better representation of large-scale motion than the motion measured at the camp. This observed velocity in turn is used to compute $\mathbf{f}_c$ and $\mathbf{\tau}_w$ in Figure 12. Rotating each clockwise would better align the "observed" forces with computed results. The water stress computed from observations neglects geostrophic ocean currents which are about 5 cm sec$^{-1}$ in the direction of motion. This accounts for the fact that the water stress in Figure 12 is about twice as large as modeled. The consequence of this neglect of $\mathbf{v}_g$ is to reduce $\mathbf{\tau}_w$. Thus, halving $\mathbf{\tau}_w$ automatically adds the vector difference to $\mathbf{R}$ because $\mathbf{R}$
force balance reduces the discrepancy to about 0.5 dyn cm⁻¹. In that case \( f_0 \) and \( P \) are aligned to within about 20°. This comparison is thought to be excellent considering the uncertainty in each of the many components.

**SUMMARY**

The AIDJEX model has been shown to provide a physically realistic simulation of the dynamic response of sea ice to winds during the winter when ice stress is significant. Furthermore, the motion is seen to compare extremely well with observed motions of buoys and manned camps. In the nearshore regions the plasticity model represents fast ice areas. These areas are separated from the moving pack ice by rapid variations or discontinuities. The location of the flaw lead agrees with satellite images. A close look at deformations and stress shows that we may improve some details of the response by changing the yield surface shape, and we expect to pursue that work soon.

In addition to gaining a greater scientific understanding by studying the results of this simulation, we are a large step closer to answering a question of more immediate concern. That is, how can we relate the ice drift to given wind conditions? We have shown that the AIDJEX model, including air, ice, and ocean components, allows the large-scale ice drift to be determined. Thus, it is now possible to use the large set of historical wind data to drive a model and determine what ice drift occurs in a wide variety of conditions. Since ice drift data are so sparse, this provides a dramatic increase in our knowledge of ice trajectories.

Ice trajectories are not the only variables that become better known as a result of this modeling effort. The stress state in the ice cover is also important and our model allows stress to be studied. The stress in the AIDJEX model is interpreted as the large-scale average of the forces that are transmitted between floes on the small scale. Although much work remains to relate the large-scale stress to forces that may be exerted on an individual ship or marine structure, we are sure that the large-scale stress is an indicator of the relative size of those forces.
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Fig. 1. Numerical grid with manned camp and buoy locations. Station numbers are consistent with Thorndike and Cheung (1977). Manned camps are identified with their initials and the following station numbers: 1, Caribou; 3, Snow Bird; 2, Blue Fox.
Fig. 2. Daily averages of data buoy and manned camp velocities. Scale vector is 25 cm sec$^{-1}$. The model boundary is roughly the U.S. and Canadian coastline.
Fig. 2 (cont.). Daily averages of data buoy and manned camp velocities. Scale vector is 25 cm sec$^{-1}$. The model boundary is roughly the U.S. and Canadian coastline.
Fig. 3. Time history of observed velocity from data buoys at stations 3 (Snow Bird) and 2 (Blue Fox). Components at each location are east and north.
Fig. 4. Time history of observed velocity from data buoys at stations 1 (Caribou), 22, and 44. Components at each location are east and north.
Fig. 5 (on facing page). Reproduction of NOAA-4, IR-VHRR images (orbit number 5487, frames I1F0001 and I2F2238) covering the simulation region on 27 January 1976 at approximately 2100 GMT. The boundary of the numerical grid (Fig. 1) is shown. Triangles and circles indicate locations of manned camps and data buoys, respectively.

Fig. 6 (on facing page). Reproduction of NOAA-4, IR-VHRR images (orbit number 5524, frames I1F0001 and I2F2238) covering the simulation region on 30 January 1976 at approximately 2100 GMT. Triangles and circles indicate locations of manned camps and data buoys, respectively.
Fig. 7 (on facing page). Reproduction of NOAA-4, IR-VHRR images (orbit number 5549, frames I1F0001 and I2F2238) covering the simulation region on 1 February 1976 at approximately 2100 GMT. Triangles and circles indicate locations of manned camps and data buoys, respectively.

Fig. 8 (on facing page). Reproduction of NOAA-4, IR-VHRR images (orbit number 5562, frames I1F0001 and I2F2238) covering the simulation region on 2 February 1976 at approximately 2100 GMT. Triangles and circles indicate locations of manned camps and data buoys, respectively.
Fig. 9. Air stress time history at station 1 (Caribou). Components are shown in east and north directions. The dotted lines (...) represent the best estimate from 10 m winds; dashed lines (---) represent values from geostrophic winds, linearly interpolated between data points each 6 hours; the bold line (—) represents modified values after barometric pressures were corrected.
Fig. 10. Air stress time history at station 3 (Snow Bird). Components are shown in east and north directions. The dots (…) represent the best estimate from 10 m winds; dashed lines (--•--) represent values from geostrophic winds, linearly interpolated between data points each 6 hours; the bold line (-----) represents modified values after barometric pressures were corrected.
Fig. 11. Air stress time history at station 2 (Blue Fox). Components are shown in east and north directions. The dots (...) represent the best estimate from 10 m winds; dashed lines (---) represent values from geostrophic winds, linearly interpolated between data points each 6 hours; the bold line (----) represents modified values after barometric pressures were corrected.
Fig. 12. Force balance at station 1 (Caribou) at 1200 GMT on 30 January 1976.
Fig. 13. Time history of driving forces and resulting velocity at station 1 (Caribou). Components are given in geographic coordinates. Subscripts E and N indicate positive values of components to the east and north. Air stress $\tau_a$ and the residual $\mathcal{R}$ (composed largely of ice stress divergence) are shown, as are the observed and wind-driven velocities.
Fig. 14. Time history of driving forces and resulting velocity at station 3 (Snow Bird). Components are given in geographic coordinates. Subscripts E and N indicate positive values of the components to the east and north. Air stress $\tau_a$ and the residual $\vec{R}$ (composed largely of ice stress divergence) are shown, as are the observed and wind-driven velocities.
Fig. 15. Time history of driving forces and resulting velocity at station 2 (Blue Fox). Components are given in geographic coordinates. Subscripts E and N indicate positive values of the components to the east and north. Air stress $\tau_a$ and the residual $\tau$ (composed largely of ice stress divergence) are shown, as are the observed and wind-driven velocities.
Fig. 16. Trajectory of station 1 (Caribou). Observed drift track shown as (---), wind-driven track as (---). Circles indicate locations on indicated dates at 0000 GMT.
Fig. 17. Geostrophic ocean current from long-term dynamic topography. Scale arrow is 10 cm sec\(^{-1}\). Dots represent zero current.
Fig. 18. Squished teardrop yield curve.
Fig. 19. Wind-driven (free-drift) ice velocity. Scale vector is 25 cm sec\(^{-1}\).
Fig. 19 (cont.). Wind-driven (free-drift) ice velocity. Scale vector is 25 cm sec$^{-1}$.
Fig. 20. Daily average of air stress field. Scale vector is 4 dyn cm$^{-1}$. 
Fig. 20 (cont.). Daily average of air stress field. Scale vector is 4 dyn cm$^{-1}$. 

- e)  31 January
- f)  1 February
- g)  2 February
- h)  3 February
Fig. 21. Modeled ice velocity field with yield strength $p^* = 10^8$ dyn cm$^{-1}$. Scale vector is 25 cm sec$^{-1}$. 
Fig. 21 (cont.). Modeled ice velocity field with yield strength $p^* = 10^8$ dyn cm$^{-1}$. Scale vector is 25 cm sec$^{-1}$. 
Fig. 22. Modeled ice velocity field with yield strength $p^* = 10^7$ dyn cm$^{-1}$. Scale vector is 25 cm sec$^{-1}$. 
Fig. 22 (cont.). Modeled ice velocity field with yield strength $p^* = 10^7$ dyn cm$^{-1}$. Scale vector is 25 cm sec$^{-1}$. 
Fig. 23. Modeled ice velocity field with yield strength $p^* = 10^9$ dyn cm$^{-1}$. Scale vector is 25 cm sec$^{-1}$. 
Fig. 24. Stretching tensor field (daily strain) with principal values proportional to line length in directions shown. Dashed lines indicate opening, solid lines closing. Scale vector is $8 \times 10^{-7}$ sec$^{-1}$ (approximately 8% per day).
Fig. 24 (cont.). Stretching tensor field (daily strain) with principal values proportional to line length in directions shown. Dashed lines indicate opening, solid lines closing. Scale vector is $8 \times 10^{-7}$ sec$^{-1}$ (approximately 8% per day).
Fig. 24 (cont.). Stretching tensor field (daily strain) with principal values proportional to line length in directions shown. Dashed lines indicate opening, solid lines closing. Scale vector is $8 \times 10^{-7}$ sec$^{-1}$ (approximately 8% per day).
Fig. 24 (cont.). Stretching tensor field (daily strain) with principal values proportional to line length in directions shown. Dashed lines indicate opening, solid lines closing. Scale vector is $8 \times 10^{-7} \text{ sec}^{-1}$ (approximately 8% per day).
Fig. 25. Stress tensor field with principal values (all compressive) proportional to line lengths in directions shown. Scale vector is $10^8$ dyn cm$^{-1}$. 
Fig. 25 (cont.). Stress tensor field with principal values (all compressive) proportional to line lengths in directions shown. Scale vector is $10^6$ dyn cm$^{-1}$.
Fig. 26. Triangle yield curve.
Fig. 27. Force balance at node nearest Caribou for each day. Scale vectors show magnitude of forces (per unit area) and velocity (dashed). Air stress $\tau_a$, water stress $\tau_w$, ice stress divergence $f_\sigma$, Coriolis force $f_C$, and ice velocity $\chi$ are each shown. If any vector is missing, it is zero on that day. Cartesian axes are aligned with all other axes in this report.
Fig. 28. Force balance at node nearest Blue Fox for each day. Scale vectors show magnitude of forces (per unit area) and velocity (dashed). Air stress $\tau_a$, water stress $\tau_w$, ice stress divergence $f_g$, Coriolis force $f_c$, and ice velocity $v$ are each shown. If any vector is missing, it is zero in this report.
AN EXAMINATION OF THE VISCOUS WIND-DRIVEN CIRCULATION OF THE ARCTIC ICE COVER OVER A TWO YEAR PERIOD

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ABSTRACT

To demonstrate that many alleged shortcomings of viscous models are more apparent than intrinsic, a detailed re-examination of the viscous approach is made by comparing predicted with observed ice drift in the Arctic basin over a two-year period employing a viscous constitutive law having both bulk and shear viscosities. Using available atmospheric and oceanic input data in 8-day averaged form and seasonally varying viscosity parameters without spatial variations, numerical drift calculations for the Arctic Basin are carried out at 4-day intervals over a two-year period employing periodic boundary conditions. Drift predictions are compared with the observed drift of three contemporaneous drifting stations with reasonable agreement. The largest errors are found to occur in late summer, and may be due to nonsteady current effects. Numerical finite-difference boundary value calculations are also made using seasonal averaged input data. These boundary value calculations show that reduction of the shear viscosity (while still maintaining a large bulk viscosity) reduces the excessive stiffening often found in viscous models while still maintaining substantial changes in drift direction due to boundaries. Sensitivity studies show steady current effects to be small for drift rates over tens of days but not negligible for cumulative drift over years.

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INTRODUCTION

Modeling of sea ice drift and deformation has recently received substantial attention in the scientific community. From work to date, it has become apparent that to model pack ice motions adequately requires some constitutive law describing the nature of the interaction of the pack ice with itself. For this purpose early models of sea ice often utilized viscous formulations similar to those used in atmospheric modeling [for example, Laikhtman, 1961, pp. 156-163]. In probably the most highly developed models of this genre [Doronin, 1970] the ice has been considered to be a Newtonian viscous fluid with a shear viscosity which may vary in space and time. However, such models have been found to have several drawbacks: (1) even though the viscosity is variable there can be no resistance to compression since no bulk viscosity is included; (2) viscosities large enough to give significant internal ice stress tend to produce excessive stiffening [Rothrock, 1975a]; and (3) on a local scale ice does not appear to behave in a viscous manner.

Because of these shortcomings, plastic laws, which seem to better represent the physics of sea ice on the geophysical scale, have become quite popular recently [Coon et al., 1974]. However, reflection on the shortcomings of viscous laws suggests that the problems may be due less to the "viscous continuum" concept than to the particular form of the viscous law assumed and to the inadequacy of input data. Moreover, it is possible using statistical arguments [Hibler, 1977] to justify a viscous law for the time-averaged response of pack ice even though the local behavior is not viscous. Such argument suggest that the viscous fluid approach is not necessarily inconsistent with plastic models, but rather a manifestation of considering different time and space scales.

Consequently, it was felt to be worthwhile to make a careful re-examination of the viscous approach using available data for tests before dismissing the approach as unworkable. For this purpose a numerical study is carried out here with the pack ice considered to be a viscous medium characterized by both bulk and shear viscosities and pressure term. Following Ling and Campbell [1976], we refer to such a continuum model as a mesoscale fluid. The addition of bulk viscosity, as shown by some of the results in this paper, is a particularly important facet of the mesoscale fluid, and in many cases it substantially changes
results obtained from the earlier Newtonian "viscous" approach. In this study numerical model calculations for the drift and deformation of the arctic ice cover over approximately a two-year period (May 1962 - April 1964) are carried out using available daily atmospheric pressure data for the Northern hemisphere compiled by NCAR and mean oceanographic current data reported by Coachman [Coachman and Aagaard, 1974]. For validation, comparisons between the predicted and the observed drift and relative drift of one Russian and two U.S. drifting stations are made. In these calculations the viscosity and pressure parameters are allowed to vary in time but not in space, periodic boundary conditions are used, and both predictions and observations are made on 8-day averaged intervals. Considerable effort is devoted to examining the empirical seasonally varying viscosity giving the best fit to observations. For more detailed boundary value studies we also carry out finite-difference boundary-value seasonal drift calculations for a grid over the whole Arctic Basin using seasonal winds and various combinations of constant viscosities and boundary conditions.

It should be emphasized that this work is essentially a "mechanistic" model study in that it is primarily intended to illustrate physical principles and determine the sensitivity to and significance of the forces causing ice drift. Certainly in a more complete "simulation" model additional features must be included; most notably, spatial variations in viscosity and pressure must be allowed by coupling these parameters to an ice thickness distribution model including thermodynamics as is done in the Coon et al. [1974] plastic model. Such developments are in progress, and preliminary results will be published in the ICSI/AIDJEX Symposium proceedings.

PREVIOUS WORK

To place this investigation in perspective, it is useful to give a brief chronological review of the highlights of earlier work. The earliest approaches to predicting ice drift, some of which are still in use, were empirical models, the most popular being Zubov's rule [Zubov, 1943], which simply states that the ice motion is a linear function of the local winds. Various ad hoc corrections to such rules have been added. In one, the ice-to-wind speed ratio is decreased as the wind increases, a modification which,
although generally giving a correct seasonal effect because of stronger winds in winter, appears to have little physical basis. Another has been to take the ice motion to be a linear function of the local ocean currents, an approach used by Bryan et al. [1975] to couple world ocean and atmospheric simulation models.

The early efforts to take into account the ice interaction with itself have tended to use Newtonian fluid concepts taken from meteorology, and in fact the viscous model suggested by Laikhtman formed the last chapter of his monograph on a boundary layer theory [Laikhtman, 1961]. Subsequent parameter studies by Yegorov [1970, 1971] contained very few changes in the basic model although a series solution was used to extract numerical values. In the Newtonian viscous law a stress/strain rate relationship of the form

$$\sigma_{ij} = 2\eta \dot{\varepsilon}_{ij} - n\delta_{kk} \delta_{ij}$$

is assumed, where $\sigma_{ij}$ and $\dot{\varepsilon}_{ij}$ are the stress and strain rate tensors respectively and $\eta$ is the shear viscosity. Probably the most complete development of such a simple Newtonian viscous approach has been by Doronin [1970], in which the viscosity is taken to be a function of the compactness of the ice which is calculated by means of a continuity equation, an inclusion which to some extent makes the model complete and capable of simulation. A particular application of Newtonian viscous ice models of considerable interest has been the calculation of the steady circulation of ice in the Arctic Basin [Campbell, 1965] using an idealized mean annual geostrophic wind field as estimated by Felzenbaum [1965].

Around 1970 with the formation of the Arctic Ice Dynamics Joint Experiment Program (AIDJEX), attention turned to a more careful examination of appropriate constitutive laws. Glen [1970] noted that a logical extension of the Newtonian viscous law could be made by including a bulk viscosity $\zeta$, yielding a stress/strain rate relation

$$\sigma_{ij} = 2\eta \dot{\varepsilon}_{ij} + (\eta - \zeta)\dot{\varepsilon}_{kk} \delta_{ij}$$

This law, with the addition of a pressure term, becomes the mesoscale fluid. Using an idealized steady wind field Campbell and Rasmussen [1972] examined
the Glen law as well as other variations although no application to actual data and observations was made. Subsequently, comparisons of predicted and observed deformation rate results over one month were made by Hibler [1974] using a drift theory in the infinite boundary limit employing the Glen law, with reasonable agreement.

Moving away from the viscous approach Rothrock [1975b] re-examined the steady circulation in the Arctic Basin using, like Campbell, the Felzenbaum geostrophic wind data. Rothrock assumed the ice is incompressible, and argued that his results were an improvement over Campbell's. However, as we shall see later in this paper many of the improvements attributed to incompressibility can be accounted for by using appropriate bulk viscosities. At about the same time, the concept of a plastic sea ice constitutive law was proposed by Coon [1972] and eventually merged into a complete model [Coon et al., 1974] by coupling the plastic strength of the ice to an ice thickness distribution model [Thorndike et al., 1975] through energetic considerations [Rothrock, 1975c].

As is apparent from this brief review, the present study lies somewhere between the plastic approach and Newtonian viscous methods. Also, in terms of the drift predictions made here there are certain similarities to the mean annual circulation calculations, since calculations over the whole Arctic Basin are considered. However, there is substantially more detail in this study since, instead of using idealized mean annual data and doing a single calculation, we make use of available real geostrophic wind data varying in time over a two-year period and allow the viscosity also to vary in time.

**DRIFT MODEL**

For drift and deformation prediction we utilized a diagnostic (acceleration terms are neglected) linear drift model similar to that used by Hilber [1974]. In such a model the wind and water stress contributions are taken to be linear functions of the wind, current, and ice velocities. For more general calculations it would probably be appropriate to use nonlinear versions of these external stresses (a modification quite easy to make for the wind...
stress component), but for our purposes here the linear relationship was felt to be adequate. Numerical solutions of the drift equation were carried out by discrete transform methods for the infinite boundary case and finite difference methods for the boundary value case. The numerical infinite boundary solutions have the advantage of being very rapid, thus allowing a large number of simulations. Also, comparison of infinite boundary and finite boundary solutions yields insight into the effects of boundaries on the ice drift.

**Governing Equations**

The basic governing equation for this model, applicable to steady equilibrium drift, is given by

\[-m f w^2 + T_w + T_a + F + G + T = 0\]  

(1)

where \( \mathbf{v} \) is the ice velocity, \( f \) the Coriolis parameter, \( \mathbf{k} \) a unit vector normal to the \( x,y \) plane, \( m \) the ice mass per unit area, \( F \) the force due to variation in internal ice stress, \( T_w \) and \( T_a \) the water and air stresses on the ice, and \( G \) and \( T \) the effect of long-term geostrophic currents and ocean tilt on the ice motion. The components of current and air stresses are given by a modified Ekman layer theory [see, for example, Rothrock, 1975b]:

\[ \tau_{ax} = B(\cos \phi U_g - \sin \phi V_g) \]

\[ \tau_{ay} = B(\sin \phi U_g + \cos \phi V_g) \]

\[ G_x + \tau_{wx} = D(\cos \theta (u - U_w) + \sin \theta (v - V_w)) \]

\[ G_y + \tau_{wy} = -D(\sin \theta (u - U_w) + \cos \theta (v - V_w)) \]

(2)

where \( \phi \) and \( \theta \) are Ekman angles in the air and water respectively, \( u \) and \( v \) are the \( x \) and \( y \) components of the ice velocity, \( U_g \) and \( V_g \) are the \( x \) and \( y \) components of the geostrophic wind, \( U_w \) and \( V_w \) are the \( x \) and \( y \) components of the geostrophic ocean flow beneath the Ekman layer, and \( B \) and \( D \) are constant wind and water stress parameters. In (2), the \( G \) components are those components linear in \( U_w \) and \( V_w \). In the case of \( G \) being neglected the sea ice is effectively considered to be moving across a stagnant ocean. The geostrophic current
flow is computed by $U_w = g f^{-1} k \times \nabla H$, where $g$ is the gravitational acceleration. Coachman's long-term values have been used here for the sea surface height $H$ [Coachman and Aagaard, 1974]. The tilt component is shown by $T = -mg\nabla H$, and the geostrophic wind is related to the surface atmospheric pressure $P$ by $U_g = (\rho_a f)^{-1} k \times \nabla P$ with $\rho_a$ the density of air.

The force due to internal ice stress, $F$, is obtained from the mesoscale fluid constitutive law which is the same as that proposed by Glen [1970] with the addition of a pressure term:

$$\sigma_{ij} = 2\eta \varepsilon_{ij} + (\zeta - \eta) \varepsilon_{kk} \delta_{ij} + P \delta_{ij}$$

where $\eta$ and $\zeta$ are the shear and bulk viscosities characterizing the ice cover on the geophysical scale and $P$ is the pressure. In general, $F$ is given by

$$F_I = \frac{\partial}{\partial x_j} \sigma_{ij}$$

In all the calculations considered in this paper $\eta$, $\zeta$, and $P$ do not vary in space so that $F_I$ takes on the simple form

$$F_I = \eta \nabla^2 u_i + \zeta \nabla (\nabla u_i)$$

where $u_i = u$ and $u_2 = v$. The relationship of this constitutive law to plastic law is discussed briefly in the next section.

**Viscous Constitutive Law as a Stochastic Average of Plasticity**

Although it seems physically reasonable that on the local scale sea ice should exhibit some sort of brittle, plastic behavior, it does not necessarily follow that the spatial-and/or time-averaged behavior will be the same. Certainly, in normal fluids, for example, the local molecular behavior may consist of elastic collisions, whereas aggregate behavior may be more viscous in nature due to averaging over large numbers of random collisions.

For sea ice the concept of a viscous constitutive law arising from a stochastic averaging of local plastic behavior may be illustrated by conceptually considering a one-dimensional rigid plastic material which has zero
stress under tension and a fixed compressive stress, say $P_k$, under compression (see Figure 1). If over some given time interval $T$ the strain rate fluctuates randomly about a mean value, then the mean stress-strain rate relationship will not lie on the plastic curve, but rather on some relatively smooth curve as illustrated by the light solid line in Figure 1. Basically, the idea is that if the average strain rate is positive and small there will be times during the interval $T$ where the instantaneous strain will be negative, so that on the average the mean stress will be less. Such arguments can also be made in terms of spatial averages where we might think of a large region of sea ice which on the average is diverging but which at individual locations is converging.

More formal application of these arguments [Hibler, 1977] in two dimensions indicates that similar effects may occur even without the sign of the strain rate components changing, due to independent variations of one component from another. Considering particular plastic laws, analytical
relationships between the average stress and strain can be obtained. However, the essential idea is simply that random motions will tend to smear out the sharp plastic transition as shown in Figure 1 so that it may be approximated by a linear law (see the dashed curve in the figure) which in two dimensions takes on the form of the mesoscale fluid constitutive law.

**Numerical Solution: Periodic Boundary Case**

In component form the basic governing equation becomes two coupled second-order partial differential equations:

\[
\lambda \nu + D \nu \sin \theta - D \nu \cos \theta + F_x = \tau^e_x
\]

\[
-\lambda \nu - D \nu \sin \theta - D \nu \cos \theta + F_y = \tau^e_y
\]

where

\[
F_x = (\eta + \zeta) \frac{\partial^2 u}{\partial x^2} + \eta \frac{\partial^2 u}{\partial y^2} + \frac{\zeta \partial^2 v}{\partial x \partial y}
\]

\[
F_y = (\eta + \zeta) \frac{\partial^2 v}{\partial y^2} + \eta \frac{\partial^2 v}{\partial x^2} + \frac{\zeta \partial^2 u}{\partial x \partial y}
\]

where \( \lambda = mf \), \( \tau^e_x \) and \( \tau^e_y \) represent the external wind and water stress components linear in \( U_g + U_w \), and \( u \) and \( v \) are the \( x \) and \( y \) components of the ice velocity. We have dropped the pressure term since we will consider only cases where it is assumed to be constant in space. To solve for the infinite boundary case, we analytically Fourier-transform the equations and solve for the transformed values of \( u \) and \( v \). This yields equations of the form (using tildes to denote wavenumber space variables)

\[
\tilde{u}(k_x, k_y) = \sum_{j=1}^{2} \tilde{g}_{xj} \tilde{\tau}^e_j (k_x, k_y)
\]

\[
\tilde{v}(k_x, k_y) = \sum_{j=1}^{2} \tilde{g}_{yj} \tilde{\tau}^e_j (k_x, k_y)
\]

103
where \( T_1 \) and \( T_2 \) are the \( x \) and \( y \) components of the transformed external stresses. The actual expressions for \( G_{xj} \) and \( G_{yj} \) are complicated algebraically but straightforward to derive, so we omit these results here.

To approximate the infinite transform we utilize discrete two-dimensional Fourier transforms on a \( 16 \times 16 \) grid with a grid spacing of 250 km (to be described later). Consequently, we are technically doing a periodic boundary solution, although by using an appropriately large grid a good approximation to an infinite boundary solution is obtained. (Alternatively we may consider the forcing field, and the ice drift to be infinite but with a periodicity of \( 16 \times 250 \) km = 4000 km.) Using two-dimensional fast Fourier-transforms, we invert the \( \tilde{G}_{ij}(k_x,k_y) \) back to real space grid values which are then convolved periodically with the \( T_j(x,y) \) at the 256 grid points. This was found to be faster than transforming all the variables if the variables if the drift at only a few grid points is desired. To predict the ice motion at a location between grid points, we linearly interpolate using the drift values at the three nearest grid points.

The principal justification for the infinite boundary (or in practice, the periodic boundary) solution is that the infinite boundary response function \( G_{ij}(x) \) falls off to small values for large \( x \) with the fall-off being the slowest for higher viscosities. For example, the response function for drift rate magnitude due to \( x \) stress components for viscosity parameters of \( \eta = \zeta = 4 \times 10 \) kg sec\(^{-1}\), was found to fall off to less than 15% of the maximum at a distance of 875 km.

**Numerical Solution: Finite Boundary Case**

For a conventional boundary value solution we made use of finite difference techniques. Referring back to (4), \( F_x \) and \( F_y \) were approximated by second-order finite differences. Assuming a regular grid of mesh size \( h \):

\[
(F_x)_{i,j} = \frac{1}{h^2} \left\{ (\eta + \zeta)[u_{i+1,j} + u_{i-1,j}] + \eta[u_{i,j+1} + u_{i,j-1}] \\
+ \frac{\zeta}{4} [v_{i+1,j} + v_{i-1,j} - v_{i-1,j+1} - v_{i-1,j-1}] - 2(2\eta + \zeta)u_{i,j} \right\}
\]  

(6)
where the subscripts $i,j$ denote the $i,j$th grid point. A rectangular $14 \times 10$ grid with 250 km mesh (a subgrid of the $16 \times 16$ grid used in the infinite boundary solution) was used to approximate the Arctic Basin. Dirichlet (no slip) boundary conditions were used at all boundary points except two grid points located between Spitsbergen and Greenland, where Neumann boundary conditions are used in order to allow the ice to flow out. The actual form of the Neumann condition is obtained by considering a layer of zero viscosity (but pressure equal to the interior) outside the boundaries at this location and then approximating the derivatives of ice stress by finite differences which allow the viscosity to vary as well as the ice velocity.

The finite difference equations using these mixed boundary conditions were solved by Gauss Seidel relaxation techniques [Ames, 1969, p. 104].

INPUT DATA

Gridding of Atmospheric and Oceanic Input Fields

The basic input fields for predicting the sea ice drift and deformation are the atmospheric pressure and the sea surface dynamic height. For computational purposes we reduced all input field to a $16 \times 16$ 250 km mesh square grid. This spacing was determined as reasonable considering the accuracy of the input pressure data, areal coverage, and computer time.

The grid, shown in Figure 2, is oriented with the $x$-axis parallel to the $150^\circ$ W longitude and the $y$-axis parallel to the $60^\circ$ W longitude. In this configuration the geographic north pole conveniently has the coordinates (11,6). This orientation also has the $y$-axis roughly parallel to the north coast of Alaska. Also shown in this figure is the $14 \times 10$ subgrid used for boundary value calculations.
Fig. 2. Square mesh grid used for numerical calculations.

The sea level barometric pressure data were obtained from the National Center for Atmospheric Research. The data were recorded at 12-hour intervals and data points were located every five degrees of latitude and longitude. For our purposes the pressure data were averaged over 8-day intervals and a grid of the averaged data prepared at 4-day intervals. Original data to NCAR were furnished by the Navy's operational analysis program (Jenne, personal communication).
An interpolation algorithm developed by Rankin [1975] was used to interpolate the data from geographic coordinates to the \( x,y \) grid format. The process consists of three basic steps. First, input data are assigned to the closest intersection of grid lines using a weighted average of all input values lying within a square centered on each grid point, the length of a side being equal to the grid interval. This procedure results in a partially filled grid which makes the remaining grid calculations much faster. The second step is to fill in the borders and any other large gaps (more than grid intervals) of the partially filled grid. This is accomplished with an interpolation technique from Shepard [1968] which uses a variation of the method of weighted averages over a circular disc centering on the intersection. Finally, remaining unfilled grid intersections are completed using the cubic spline technique [Davis and Kontis, 1970]. The cubic spline procedure yields a curve that passes through each given data value exactly and has continuous first and second derivatives. The spline is first run in the \( y \) direction, filling in the missing values at intersections, then run in the \( x \) direction, and the two values determined for each intersection are averaged. The splining procedure, while yielding the most accurate interpolated values, requires the two previous steps to prevent uncontrolled interpolation likely to occur over large data gaps.

The gridded data were visually compared with data that had been gridded by hand on several occasions before deciding to use the procedure. As an additional check on the automatic gridding procedure as well as on the overall accuracy of the barometric pressure data in the Arctic, data observations of station pressure from Arlis II were obtained from the National Climatic Center. Figure 3 shows the station observations compared with a pressure value interpolated from the \( 16 \times 16 \) grid at the station location. Although there is generally good agreement between the two curves, the NCAR data do not always reproduce the rapid variations in the pressure, a fact commensurate with the probability that the estimated pressure field is somewhat smoother both in space and time than the actual field.

For the oceanic dynamic height data we used the average values obtained by Coachman, through interpolation of numerous measurements. Since in this case only one fixed set of data is used it was convenient to prepare a data
grid using a visual overlay of Coachman's dynamic height contour maps [Coachman and Aagaard, 1974] and interpolating by hand.

Calculation of Geostrophic Winds and Currents

Calculation of the geostrophic winds, currents, and tilt requires differentiation of the pressure and dynamic height fields. For this purpose we utilized discrete transform methods which allowed us to simultaneously smooth the fields while obtaining the required stress fields. To carry out this operation the appropriate data grid was first transformed into discrete wavenumber space using two-dimensional fast Fourier transforms. The smoothing and differentiation were then carried out by multiplication in wavenumber space, after which the grid was transformed back to real space. Again, as mentioned before, since we are using discrete transforms this operation can
be viewed as a periodic filtering procedure.

For smoothing we used a two-dimensional cosine bell:

\[
F(k) = 0.5 \left[ 1 + \cos \left( \frac{\sqrt{k_x^2 + k_y^2}}{2f_1} \right) \right]
\]

(7)

for

\[
\sqrt{k_x^2 + k_y^2} \leq 2\pi f_1
\]

and

\[
F(k) = 0 \text{ for } \sqrt{k_x^2 + k_y^2} > 2\pi f_1
\]

where \(f_1\), the cut-off frequency, was chosen to be the Nyquist frequency of \((1/500) \text{ km}^{-1}\). This type of smoothing is analogous to the Hanning smoothing in one dimension.

**Observed Drift Data**

For comparison with model predictions we made use of position measurements of three contemporaneous drifting stations—T-3, Arlis II, and NP-10—over a two-year period from May 1962 to April 1964. The original and final positions of the stations are shown in a later figure (Figure 11). These data were averaged over 8-day intervals to suppress rather large measurement errors, and then resampled every 4 days. (This is the same type of smoothing as was applied to the gridded geostrophic wind time series.) These drift data have also been used by Thorndike et al. [1975] in a test of an ice thickness model of sea ice, and the raw position data were obtained from the AIDJEX Office at the University of Washington. These drift records were used to test model predictions by comparing individual station drift rates and the deformation of the triangle formed by the three stations.
INFINITE BOUNDARY RESULTS

For the bulk of the comparison between the predicted and observed drift characteristics of the three drifting stations the infinite boundary solution of the drift model was utilized. Since the drifting stations are generally more than 500 km from shore, this neglect of boundaries is a reasonable starting point especially within the framework of a spatially constant viscosity calculation where the appropriate boundary conditions that should be used are not clearly defined in any case.

Determination of Seasonal Viscosity Variation

In order to estimate seasonal variations in the viscosity parameters in the model an empirical fit between the observed and predicted drift of the three ice stations over a two-year period was carried out. For this purpose predicted \( x \) and \( y \) drift rates for each station were generated every four days over a period of 760 days using the eight values of viscosity listed in Table 1 which were taken to vary in a logarithmic manner. For this fit and for subsequent comparisons the bulk and shear viscosities were taken to be equal.

TABLE 1
NUMERICAL PARAMETERS USED IN DRIFT PREDICTION

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B )</td>
<td>0.0146 kg sec(^{-1}) m(^2)</td>
<td>( \eta, \zeta )</td>
<td>( 0.45 \times 10^{10} ) kg sec(^{-1})</td>
</tr>
<tr>
<td>( D )</td>
<td>0.59 kg sec(^{-1}) m(^2)</td>
<td></td>
<td>( 0.94 \times 10^{10} ) kg sec(^{-1})</td>
</tr>
<tr>
<td>( f )</td>
<td>( 1.46 \times 10^{-4} ) sec(^{-1})</td>
<td></td>
<td>( 2.0 \times 10^{10} ) kg sec(^{-1})</td>
</tr>
<tr>
<td>( g )</td>
<td>9.832 m sec(^{-2})</td>
<td></td>
<td>( 4.2 \times 10^{10} ) kg sec(^{-1})</td>
</tr>
<tr>
<td>( \phi )</td>
<td>( \theta = 30 )</td>
<td></td>
<td>( 9.0 \times 10^{10} ) kg sec(^{-1})</td>
</tr>
<tr>
<td>( m )</td>
<td>( 3.0 \times 10^{3} ) kg m(^{-2})</td>
<td></td>
<td>( 19.0 \times 10^{10} ) kg sec(^{-1})</td>
</tr>
<tr>
<td>( \rho_a )</td>
<td>1.3 kg m(^{-3})</td>
<td></td>
<td>( 40.0 \times 10^{10} ) kg sec(^{-1})</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( 85.0 \times 10^{10} ) kg sec(^{-1})</td>
</tr>
</tbody>
</table>

In general, to predict a given drift rate for a given drifting station, the velocity vector is centered on the station location at the beginning of a 4-
day interval by interpolating drift rates from the three nearest grid points. Consequently, the predicted drift rates from currents and tilt, although taken from fixed dynamic height contours, will vary in time as the position of the stations changes. However, as this variation was found to be small (as shown later) compared with day-to-day variations in geostrophic winds, the geostrophic current and tilt components were neglected in the calculation of the short-term drift rates used to estimate the best fit viscosity.

The standard values of the numerical parameters used in the calculations are shown in Table 1. These values are similar to those used by Rothrock [1975b]. Taking 8-day averages of the observed drift rates (to compare with the average predicted drift rates), the sum of squared differences between the three stations average predicted and observed drift rates (x and y components) was calculated for each viscosity. The best fit viscosity was obtained by minimizing the sum of square over running 60 intervals.

The viscosity results are shown in Figure 4, together with a curve (dashed line) proportional to the 60-day averaged growth rate of 50 cm ice applicable for this observational data as taken from Thorndike et al. [1975]. We have also shown in Figure 4 the effect on the best fit viscosity of increasing and decreasing the wind stress coefficient by 50% and increasing the water stress coefficient. The most salient characteristic of this figure is the seasonal variation of the best fit viscosity (standard case) with the smoothed growth rate showing a good correlation. The best correlation (0.80 significant at the 0.01% level) occurred with zero lag time, that is, when the viscosity does not lead or lag behind the growth rate. The regression equation for the best fit viscosity is given by

\[
\ln (\eta) = \ln (\zeta) = 0.92 \langle G \rangle + 24.73
\]  

where \(\langle G \rangle\) is the 60-day averaged growth rate of 50 cm thick ice in units of cm day\(^{-1}\). Figure 4 also shows the viscosity generated by the regression equation. The best fit viscosities for different wind and water stress parameters are similar in form to the standard case, with the curves being shifted up for higher wind stress (or lower water stress) and down for lower wind stress.
Fig. 4. (a) Viscosity values (solid line) giving the best fit between predicted and observed drift rates for standard values of the wind and water stress parameters $B$ and $D$. The dashed line is a regression fit of viscosity upon the 60-day averaged growth rate of ice 50 cm thick (see eq. 5).

(b) Effect on the best fit viscosity of increasing (curve 1) and decreasing (curve 3) the wind stress parameter and increasing the water stress coefficient (curve 2). (c) Mean square error between predicted drift and observed drift rate using the best fit viscosity.

In Figure 4-c is shown the error in the velocity for the best fit viscosity, which tends to be of the order of 1 or 2 cm per second (0.5-1.0 nautical miles for day). Interestingly enough the error tends to get very large around September, an effect which coincides with a slight oscillation in the best fit viscosity to high and then back to low values, as can be seen in Figure 4-a. Since September more or less marks the end of the summer melt season, these effects may indicate that certain aspects of the model assumptions are breaking down then, one possibility being that the ocean current structure has become modified due to large amounts of open water.
The dominant effect which seems to account for much of the seasonal viscosity variation exhibited in Figure 4 is a decrease in drift rate amplitude as the viscosity is increased. This is illustrated in Figure 5, where we show the observed and predicted 3-station average velocity for a low viscosity of $1.0 \times 10^{10}$ kg sec$^{-1}$, a high viscosity of $4.0 \times 10^{11}$ kg sec$^{-1}$, and a seasonal viscosity varying according to equation (3). Physically the increase in viscosity increases the capability of the ice pack to transmit stress so that the drift tends to be driven by the average wind over a large region rather than the local wind. A second effect, which will be illustrated later, is a general counterclockwise rotation of the predicted drift direction of up to 20° as the viscosity is increased.

Drift Rate and Deformation Rate Results Using a Seasonal Viscosity

Having obtained an estimated seasonal viscosity as a linear function of growth rate, drift and deformation predictions over the two-year period to compare with the observed data may now be carried out. To make this calculation complete, geostrophic currents and tilt were also included in the model predictions. To show that the effect of currents and tilt on predicted drift rates is indeed as small as previously asserted we show, in Figure 6, 25-day smoothed predicted $x$ and $y$ drift rates of ice island Arlis II with and without current and tilt. (The smoothing of these records was done with unity gain low-pass filter [Hibler, 1972], which has a frequency response similar to a 25-day running mean without the spurious side lobes.) As can be seen, the effect of currents and tilt tends to be more of a constant additive effect and thus can effectively be neglected in drift prediction for short time intervals although (as we shall see later) these effects are non-negligible in cumulative drift.

In Figures 7 and 8 the predicted and observed drift rate time series for all three stations using 25-day and 8-day smoothing are shown. Regarding the 25-day smoothed results in Figure 6, the agreement for all stations is reasonably good, with NP-10 being the best and T-3 probably the poorest. There does appear to be a slight reduction in amplitude of the oscillation in the predicted curves relative to the observations. Going to the 8-day smoothing
Fig. 5. Average drift rate of the three drifting stations. The top curves show 8-day averaged observed and predicted drift rates (based on a seasonally varying viscosity). The bottom curves show predicted drift rates using constant viscosities of $1.0 \times 10^{10}$ kg sec$^{-1}$ (low viscosity) and $4.0 \times 10^{11}$ kg sec$^{-1}$ (high viscosity).
Fig. 6. Effect of currents and tilt on the 25-day smoothed predicted drift rate of Arlis II.

in Figure 8 the agreement is still reasonable, but the detailed rapid variations are less well predicted. Although this could be an "artifact" of the model, it is quite probable that we are beginning to approach the limitation of the NCAR input data which really represent spatially smoothed interpolations from only a few points in the Arctic Basin. Looking back at Figure 5, for example, which represents the 3-station average drift rate (using 8-day smoothing), there is better agreement between the predicted and observed fine structure in time. These results suggest that the atmospheric pressure field does not faithfully represent the rapid spatial variations in pressure. Also,
in the case of T-3, it is in a region of high ocean currents, and it is possible that deviations in the predictions are due to inadequate current input.

These conclusions are also commensurate with the deformation rate comparisons shown in Figures 9 and 10. In Figure 9 we have compared the predicted and observed differential drift between stations by examining distance change per unit time. By not dividing by the distance between stations to obtain strain rates we have minimized the biasing of the prediction by observations, although some bias is, of course, always present since the observed positions of the stations are used to locate the predicted velocity vector. The agreement here is again reasonable and provides a more sensitive test of spatial resolution in the model and the input data. The problem of the smaller fluctuation amplitude of the predictions has become more pronounced here and would be even more marked if we were to show the 8-day smoothed results. It is also worth
Fig. 8. Predicted and observed drift rates of the three drifting stations with 8-day smoothing.
Fig. 9. Predicted and observed distance changes between stations per unit time.

noting that Arlis II and NP-10 (see Figure 11) are quite close together so that poor spatial resolution of the atmospheric pressure field data becomes a critical factor.

Comparison of the predicted and observed divergence rates, the 45-day smoothed version of which is shown in Figure 10 with reasonable agreement, also amplifies the problem of fluctuations. More specifically, in the comparison of the divergence rate on a shorter time scale, the high viscosity values tend to overdamp the predicted divergence rate oscillation. Again, since oscillations in the predicted divergence rate are dependent on rapid variations
in the pressure field, part of this problem may be due to poor input data. Also, the use of a nonlinear wind stress model would amplify the spatial variations in pressure and might improve the predictions.

On the positive side, the good agreement that is observed between the observed and predicted differential drift rates, especially if we consider longer time averages, is encouraging considering the simplicity of this model and the quality of the input data. Basically, the results are reasonably good, indicating that the inclusion of a seasonally varying viscosity in a mesoscale fluid drift model does seem to explain the salient characteristics of the drift rate. Moreover, the fact that this general agreement can be obtained even if gradient current effects are neglected is relevant to drift forecasting problems where adequate current data are often scarce.
Cumulative Drift Comparisons

In the examination of drift rates one is primarily concerned with the variation and fluctuation in time of the drift of the ice pack. The cumulative drift, on the other hand, is an integrated measure of the drift records and is more closely akin to circulation studies of the Arctic ice cover. Not surprisingly, in such an integrated measure of the ice drift the gradient currents begin to play an important role, as do boundary conditions. Consequently, the comparison of cumulative drift supplies a rather independent test of the model results, since the available variation in the changeable viscosity parameters was used up in fitting the short-term drift rates which were relatively insensitive to current effects.

The effect of the various current and wind components on the cumulative drift is shown in Figures 11 and 12. In Figure 11, the predicted $x$ and $y$ drift rates for each station with and without gradient currents and tilt were integrated over a 720-day period and compared with the final position. In Figure 12, the predicted drift rate of the center of the triangle of the three stations was integrated over 60-day intervals and compared with the observed rate. We also show in part b of this figure the low viscosity prediction in order to give some idea of the free drift result. The slight difference in dates between these figures is due to a different starting time chosen for Figure 11.

Overall, it can be seen that the gradient currents account for about 30% of the drift, with the direction being generally toward the Greenland-Spitsbergen passage, while tilt accounts for perhaps 15% of the cumulative effect and has an average direction essentially parallel to the Canadian Archipelago. It should also be noted that currents play a substantially greater role in summer, an effect which can be seen by careful examination of Figure 12. As we shall see in the next section on boundary value calculations, this may be due largely to the generally lower wind speeds in summer. Although there are some disagreements between the simulated individual stations' net drift over the two-year period and that observed, the agreement is generally quite good considering the cumulative error that can creep into such integrated effects.
Fig. 11. Predicted and observed net drift of the three drifting stations with and without current and tilt effects.

Fig. 12. Predicted and observed three-station average drift integrated over 60-day intervals.
To examine the individual station drift in more detail we show in Figure 13 predicted and observed 60-day cumulative drift tracks for each station, together with the low viscosity prediction for comparison. Examination of this figure together with the spatial average cumulative drift in Figure 12 indicates that some of the largest disagreements are found during the end of the summer season, a fact commensurate with the large error for the best viscosity found earlier (see Figure 4-c) during the same time. Drawing heavily on Figure 12, showing the current effects, it might be deduced that much of this deviation is due to incorrect currents, a fact supported by more detailed current measurements by Coachman and Aagaard [1974] which indicate increased current speeds in late summer and fall. One possible explanation for such an effect might be the increased stress transferral into the ocean due to greater percentage of open water.

Another overall effect that shows up in Figure 13 is the general counterclockwise rotation from $10^\circ$ to $20^\circ$ of the drift vector as the viscosity is increased. Also, with regard to overall differences between predicted and observed drift, the NP-10 predicted drift is generally too far clockwise whereas the Arlis II and T-3 results are too far counterclockwise. As we discuss next, boundary effects may very well have an effect on such rotation of directions.

Finally, to lead into the next section on seasonal boundary value calculations and to illustrate the summer/winter contrasts, we have made seasonal comparisons between the predicted and observed drift of the three-station triangle. This comparison, shown in Figure 14, again shows that much of the cumulative drift error occurs in summer (when the variable viscosity generally is coincident with the low viscosity case), whereas in the winter season the high viscosity prediction is in reasonable agreement with the observations.

**SEASONAL BOUNDARY VALUE CALCULATIONS**

In the preceding section we concentrated on a comparison of the temporal variations of the predicted and observed drift of three particular drifting stations. To get a more synoptic view of the Arctic it is useful to carry out seasonal drift predictions over the whole Arctic Basin using a finite
Fig. 13. Predicted and observed cumulative 60-day drift for each drifting station.
Fig. 14. Seasonal predicted and observed drift of the three drifting stations.
boundary calculation. Such a calculation yields insight into the effect of boundary conditions on the drift. Moreover, the nearshore ice predictions tend to be quite sensitive to the ratio of the bulk and shear viscosity parameters.

In such a boundary value solution it should be emphasized that because of the simplicity of the model used here (most notably no spatial variation of the dynamical parameters) the boundary value solutions should not be construed to yield accurate structure very near the shore; instead, they serve as a tool for estimating boundary stress transferral.

**Seasonal Circulation of the Arctic Ice Cover**

To estimate the seasonal circulation of the Arctic ice cover we averaged the wind field over the same intervals as were used in the seasonal drifting station comparisons in Figure 14, and carried out boundary value calculations for the predicted drift rates with no-slip boundary conditions except at two boundary points between Spitsbergen and Greenland where Neumann boundary conditions were used. A viscosity of $4 \times 10^{10}$ kg sec$^{-1}$ was used for the summer case and a value of $3 \times 10^{11}$ k sec$^{-1}$ for winter. The drift results and forcing geostrophic wind fields are shown in Figure 15. Probably the most dominant characteristic of this figure is the increase in ice-to-wind velocity in summer as the viscosity increases, an effect which is in general agreement with observations [Skiles, 1968]. This effect is manifested by the average magnitude of ice velocities being about the same in summer and winter even though the average wind magnitudes decrease substantially in summer. Physically, this phenomenon follows from the increase of viscosity in the model in winter, which increases the capability of the ice pack to transmit stress so that the drift tends to be driven by the average wind over a large region rather than the local wind.

With regard to the general characteristics of the ice motion, there are two factors worth noting. First, both summer seasons have an easterly wind component off the North Slope of Alaska which, coupled with a turning angle to the right of the ice, would tend to push the ice offshore and create favorable shipping conditions. In a historical study of the nearshore summer ice
Fig. 15(A). Seasonal geostrophic wind. A velocity vector one grid space long represents $3.0 \text{ m sec}^{-1}$. No-slip boundary conditions were used except between Spitsbergen and Greenland, where a zero viscosity layer was introduced to allow outflow. See Fig. 14 labels for exact dates.

conditions off Alaska, Barnett [1976], in fact, found these particular summers to be largely icefree and favorable for shipping. Second, there is a substantial breakup in the classical Arctic high pressure pattern as we go from winter to summer. Moreover, there is a year-to-year variation in the winter pattern, with the first winter having a slightly more standard Arctic high pattern along the lines of the mean annual Felzenbaum data used by Campbell [1965] and
Fig. 15(B). Seasonal predicted drift rates. A velocity vector one grid space long represents 0.025 m sec$^{-1}$. No-slip boundary conditions were used except between Spitsbergen and Greenland, where a zero viscosity layer was introduced to allow outflow. See Fig. 14 labels for exact dates.

Rothrock [1975b] for their circulation studies. The seasonal and year-to-year variations suggest that to make a comparison with observations that is at all meaningful, it is necessary to make use of actual wind data for a given season rather than idealized mean annual input.
Effects of Variations in Boundary Conditions and Viscosity Parameters

With regard to boundary effects, examination of the winter plots shows a marked stiffening of the ice cover off the Alaska coast and near the Chukchi Sea compared with the wind characteristics. That this is primarily a boundary effect can be illustrated by comparing the winter no-slip calculation with a "natural" boundary condition which allows the boundaries to move. For this latter boundary condition we use the standard infinite boundary transform solution, but set the geostrophic input wind field to zero everywhere outside the 14 × 10 grid. This allows the boundaries to move in the natural direction they are being pushed by the neighboring ice and local winds. The results for winter 2 are shown in Figure 16. Although there is some difference in turning angles in the center of the basin between these two calculations, the dominant difference is the excessive stiffening, in the no-slip case, of the ice motions in the region near the Chukchi Sea and to a lesser extent in the Beaufort Sea.

However, it turns out that much of this excessive stiffening effect is due to too high a shear viscosity. Lowering the shear viscosity as shown in the winter 2 drift calculation in Figure 16 to one-third the bulk viscosity generally loosens up this region and allows greater motion. To illustrate the effect of shear and bulk viscosity in the extreme we also show in Figure

Fig. 16. Winter 2 predicted drift rates for (left) the standard no-slip boundary conditions and (right) a "natural" boundary condition obtained by solving over a larger grid with zero geostrophic wind outside the boundaries but with no restrictions on the boundary motion. A velocity vector one grid space long represents 0.025 m sec⁻¹.
the same calculation for zero shear and bulk viscosities separately. With only a bulk viscosity the pack loosens up considerably while still maintaining a large turning drift angle especially near the Canadian Archipelago. The pure bulk viscosity case is in many respects similar to Rothrock's incompressible case in that it has large turning angles away from shore, but has velocity magnitudes that are rather high. The pure shear viscosity case, on the other hand, effectively reproduces Campbell's Newtonian viscous model with its excessive stiffening problem at high viscosities.

Fig. 17. Winter 2 seasonal drift calculations for different shear-to-bulk viscosity ratios. No-slip boundary conditions were used except between Spitsbergen and Greenland, where a zero viscosity layer was introduced to allow outflow. (a) Standard case, $\eta=3.10^{11}$ kg sec$^{-1}$; (b) deduced shear viscosity, $\zeta=3.10^{11}$ kg sec$^{-1}$, $\eta=10^{11}$ kg sec$^{-1}$; (c) zero shear viscosity, $\zeta=3.10^{11}$ kg sec$^{-1}$, $\eta=0$; (d) zero bulk viscosity, $\zeta=0$, $\eta=3.10^{11}$ kg sec$^{-1}$. A velocity vector one grid space long represents 0.025 m sec$^{-1}$.
CONCLUSIONS

Although the model calculations presented here are limited in scope, the comparisons and sensitivity studies do yield several points relevant to drift modeling.

Overall, the reasonable agreement between the direction and magnitude of the predicted and observed drift of ice stations far from shore indicates that many of the inadequacies in previous "viscous" Arctic circulation calculations may be remedied without drastic changes in the constitutive law by (a) allowing the dynamical parameters to vary seasonally, (b) using observed rather than idealized pressure data for input to model calculations, and (c) including a bulk viscosity as well as a shear viscosity in the sea ice constitutive law.

With regard to boundary effects, the comparison of different boundary conditions does yield some changes in direction of the predicted drift rates in the central Arctic far from shore. However, the dominant effect of boundaries is to produce an excessive stiffening near the Alaskan coast, an effect which turns out to be quite sensitive to the shear viscosity. Reduction of the shear viscosity, while maintaining the bulk viscosity, reduces such stiffening while still maintaining substantial changes in the drift direction due to boundaries. The results underscore the importance of the addition of the bulk viscosity parameter to the ice constitutive law.

The effects of steady currents on the ice drift rates were found to be generally negligible for drift rates averaged over intervals less than a month, a fact of considerable aid in sea ice forecasting. However, for long-term cumulative drift the geostrophic current effects cannot be neglected.

Seasonally, the effect of currents is much more pronounced in summer and in late summer. Cumulation drift comparisons as well as the best fit viscosity residual errors suggest that the current structure may be substantially modified in September, probably due to looser ice conditions over the summer. It would seem fair to infer that variations in wind to water stress transmittal due to changing ice conditions has an important effect on the current structure of the Arctic Ocean.

Overall, it appears that the most critical inadequacy of the model calculations made here is the neglect of spatial variations in the dynamical
parameters which should be coupled in a consistent way with ice thickness characteristics. This neglect becomes especially troublesome near boundaries. Such variations are now being modeled, and preliminary results will be published in the proceedings of the ICSI/AIDJEX Symposium.

ACKNOWLEDGMENT

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CHARACTERISTICS OF ARCTIC STRATUS CLOUDS OVER THE BEAUFORT SEA DURING AIDJEX

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ABSTRACT

The stratus cloud cover over Barrow, AIDJEX camp Big Bear, and the Beaufort Sea from May to September 1975 was studied by means of surface and satellite observations. Contrary to previous reports of consistently high long-term summer mean cloudiness, the values for 1975 showed a period of considerably low cloudiness. Comparison with the wind field suggests that a southerly component in the 700 mb winds corresponds to an increase in cloudiness and a northerly component corresponds to a decrease. The unusual northerly 700 mb flow during this year may explain both the low summer cloudiness and the severe ice conditions in the Beaufort Sea.

The temperature and emissivity profiles measured from an instrumented aircraft in May 1976 indicate that within a few days of formation of stratus cloud overcast the emissivity at the top part of a cloud becomes low and the temperature inversion drops down to the middle part of the cloud. Continuous solar heating and the effect of long-wave scattering may be responsible for these changes.

INTRODUCTION

During the summer months low stratus clouds cover the Arctic Basin almost continuously. This persistent cloudiness appears to be a steady climatological feature and hence has important implications for the radiation budget of the Arctic. Previous investigations of the cloud cover by Huschke [1969], Vowinckel and Orvig [1970], and Henderson [1967] show that the amount of cloudiness and the frequency of occurrence are essentially a step function of time and that the increase of cloudiness during the summer is due to
the presence of low level stratus. The monthly variation of spatially averaged cloud cover is given by Huschke [1969] and is reproduced in Figure 1. It is evident from this figure that except for the Canadian Arctic the average cloud cover from May to September is nearly 65% and never less than 60% for any period. The layering feature of these clouds is reported by Jayaweera and Ohtake [1970], who also indicate that these clouds are essentially super-cooled and have few ice crystals.

In their description of the mechanism for the formation and layering of arctic stratus, Herman and Goody [1976] suggest that condensation is induced in initially unsaturated continental polar air as it flows over the pack ice by diffusive cooling to the colder ice surface and long-wave emission to space. Intense mixing within the cloud is generated by a strong cooling due to radiative emission at the cloud top. The layered structure is attributed to a greenhouse mechanism whereby solar radiation penetrates to the interior of the cloud and causes evaporation there. At the same time the top remains cold due to long-wave emission to space and the base remains cold since the surface temperature is fixed near 0°C. Herman and Goody's model predicts that condensation occurs within a day and the clouds persist for several days due to the absence of effective dissipative mechanisms.
In this paper, we use surface observations and satellite images to examine the amounts of cloud cover over the Beaufort Sea and then compare these amounts with the wind field to infer the large-scale air flow in the presence of a cloud cover. Because stratus clouds have a low liquid water content and their emissivity is therefore considered to be less than 1, it is difficult to estimate long-wave radiative fluxes from cloud top temperatures.

Vertical profiles of emissivity and air temperature of stratus clouds were measured during a cloud episode over the Beaufort Sea near Barrow, Alaska, in May 1976. The measurements are also described in this report.

CLOUD COVER AMOUNTS

Mapping

The study of cloud coverage was limited to the region shown in Figure 2, an area that includes the Beaufort Sea, the northern Alaska coast, and the AIDJEX area. The cloud cover amount (defined as the fraction of the area covered by clouds other than high cirrus) was obtained from NOAA-3 and NOAA-4 satellite images made by VHRR (Very High Resolution Radiometer) in the visible band (0.6–0.7 μm) and the thermal infrared band (10.5–12.5 μm) with a resolution of 1 km. Many techniques are available for mapping clouds from satellite images at lower latitudes, but the snow or ice cover and the temperature inversions at the lowest layers in the clouds make these techniques unusable in Arctic studies. Therefore, it is not possible to develop a standard identification applicable in all cases to the mapping of arctic stratus clouds.

The method adopted here to map arctic stratus clouds utilizes both visible and infrared images, in particular the shadows and highlights in the visible and abrupt temperature changes in the infrared. Solar elevation in the Arctic is low and sunlight is available continuously for most of the summer. The stratus clouds in the visible show highlights on the side facing the sun and dark shadows on the other side. Thus clouds often appear as an indentation in the image (see Figure 3a), although the contrast in the albedo is small. From the position of the sun it is possible to distinguish the
Fig. 2. Area for which the cloud cover amounts were obtained.

the clouds from the underlying sea surface. This method is satisfactory unless the entire area of the Arctic Ocean in the image is cloud covered so that the cloud layer can be mistaken for the ice surface. Under these circumstances the exact location of the coastline can be used to differentiate clouds from the ice surface.

The infrared images often show sharp contrasts in temperature between cloud tops and ice surface. These differences may be enhanced by choosing a gray scale to cover a small appropriate temperature scale (as in Figure 3b) so that isotherms can be drawn. Sharp changes in temperature are assumed to be a boundary between two cloud layers or between a cloud layer and the ice surface. Using these characteristics we were able to map the cloud cover of
Fig. 3a. NOAA-4 satellite image in the visible band showing arctic stratus clouds as indentation.
Fig. 3b. Enhanced infrared image for the same area as Fig. 3a, showing radiative temperature isotherms. (The distance between the two vertical lines is 2000 km.)
arctic stratus using all the satellite images from 1975 for the area shown in Figure 2.

Data for computing mean daily and weekly values of cloud cover were obtained as surface observations made by the National Weather Service at Barrow for the skies over Barrow and as hourly meteorological observations at Big Bear for the AIDJEX area during April-August 1975.

Results

The weekly mean cloud cover amount from May until September 1975 is shown in Figure 4 for the Beaufort Sea, Barrow, and Big Bear. Although the mean for the entire period is still rather high (approximately 6/10) and is not much different from the previous estimates of Huschke [1969] and Vowinckel and Orvig [1970] (see Figure 1), our results show considerable fluctuation in the mean weekly cloud amount for 1975, especially over the Beaufort Sea and Big Bear. However, our results were confined to one year, and the cloud cover in 1975 may have been anomalous. Indeed, the weather in summer 1975 did depart considerably from the mean long-term weather, giving rise to anomalous ice conditions [Wendler and Jayaweera, 1976].

Mean Weekly Status Cloud Cover for Summer 1975

Fig. 4. Weekly mean cloud cover amounts over the Beaufort Sea, AIDJEX camp Big Bear, and Barrow for 1975.
Another inference from Figure 4 is that the cloud cover at Barrow is considerably different from that over the AIDJEX camp. These results show that cloud conditions can vary considerably from place to place, especially near the coast where the presence of land may have local effects. Therefore, one must be cautious when making inferences about the cloud cover over the Arctic Ocean that are based on data from stations on land or near the coast.

Herman and Goody [1976] suggest that stratus clouds are initiated by condensation of continental polar air as it moves over the cold ice pack. To test this suggestion we compared the daily surface, 850 mb, and 700 mb north-south wind component at Barrow with the cloud cover amount. The wind components were derived from the winds reported daily at the National Weather Service at Barrow, which measures surface winds hourly and upper level winds twice a day with a radiosonde. The comparison, in Figure 5, reveals a good

![Daily cloud cover amount at Barrow (1975)](image1)

![Daily surface wind at Barrow (1975)](image2)

![700mb daily wind at Barrow](image3)

Fig. 5. Comparison between cloud cover amount at Barrow with the north-south wind component at surface and 700 mb level.
relation between the cloudiness and the southerly wind component at the 700 mb level. The surface winds show no correlation at all, and a correlation with winds at 850 mb is inconclusive. This comparison supports Herman and Goody's contention to the extent that cold arctic ice causes stratus clouds by modifying synoptic-scale continental air above the boundary layer. Low-level flow may give rise to arctic fog, but stratus clouds are not necessarily formed by the lifting of the fog.

The absence of 700 mb winds at Big Bear made it impossible to compare the wind directions and cloud cover over the camp. However, the National Weather Service synoptic maps at 700 mb may be used for determining the wind for comparison with the cloud cover over the Beaufort Sea. Assuming geostrophic flow, the mean 700 mb winds were computed for two-week periods from April until August of 1975 for this region. The mean winds show a northerly component for all periods except from mid-June to mid-July, when the winds were northeasterly over the Beaufort Sea and correspond to a minimum in the cloud cover amount. This rather qualitative analysis tends to confirm that the small amount of cloudiness coincides with the changes in wind direction from southerly to northerly. To confirm a close correlation between the wind and cloud cover would require a detailed analysis of the 700 mb wind field.

COMPARISON BETWEEN ICE AND CLOUD COVER

The summer of 1975 was noted for its unusual ice conditions in the Beaufort Sea. The ice movement was late and incomplete. Analyzing the meteorological situation pertaining to the movement of ice, Wendler and Jayaweera [1975] found that a persistent low pressure system over the high Canadian Arctic added a northwesterly wind component to the average wind; the resulting wind kept the ice near the shore. Using the deviations of the height of the 700 mb level for summer 1975 from the long-term mean, they showed that the wind vector had a southerly component only in June. For the remainder of the summer there was a persistent northerly component. Comparison of Figures 1 and 4 show that, except in early June, the cloud cover for this year is consistently less than that inferred from past climatological data. Therefore, less than usual offshore ice movement seems to be related
to the less than usual cloud cover over the Beaufort Sea. Both these factors display a strong dependence on the 700 mb synoptic-scale wind field over the area.

VERTICAL TEMPERATURE AND CLOUD EMISSIVITIES

The meteorological properties of stratus clouds and the environment—in particular, humidity, air temperature, and the upward spectral thermal radiative temperature in the atmospheric window region—were measured in May 1976 from a Cessna 180 aircraft. These observations were designed to study the variations of cloud properties during a single stratus cloud episode over Barrow. A stratus cloud episode is defined as a period during which the sky is overcast without a break. Twice-daily (morning and afternoon) measurements of temperature, relative humidity, and the spectral radiative temperature were obtained for six days during a cloud episode which started with an overcast sky on 20 May and lasted until 26 May. The spectral radiometer used during these experiments was a Barnes PRT-5 radiometer. A thermistor was used for air temperature measurements, and the humidity was measured with a Vaisala-hygrometer.

The Barnes PRT-5 radiometer was mounted inside the cabin over a porthole looking down so that it measured upward spectral radiance. If $R_1$ is the upward long-wave flux at the bottom of the cloud layer, $R_2$ the upward long-wave flux at the top of the layer, and $\sigma T^4$ the black body radiation at the average temperature of the cloud layer, then the emissivity

$$E = \frac{R_1 - R_2}{R_1 - \sigma T^4}$$

This expression can be used to calculate the effective flux emissivity of any stratus cloud or a layer thereof.

A disadvantage of using upward radiance to determine emissivity is the influence of the underlying surface. As pointed out by Griggs [1968], the radiance from the top of the layer becomes insensitive to emissivity when the cloud temperature is close to that of the ice surface below. Therefore, for very low thin clouds, such as fog, the value for emissivity obtained by this method may be in error.
The vertical temperature and radiant flux measurements using the Cessna 180 were made in the vicinity of Barrow. The first flight was made on the first overcast day after a period of partial cloudiness. Air or cloud temperature, spectral radiative temperature, and air humidity were measured at various altitudes below, within, and above the clouds along horizontal flight legs, each about three minutes long. The first leg was flown about 100 m above the ground to record the surface properties. Then to avoid aircraft icing, measurements were made about 50 m above the cloud top along a leg directly above the first. The rest of the sampling was done within the cloud in legs vertically separated from each other by about 100 m until the cloud base was reached.

The thermistor for measuring outside temperature was mounted on the underside of the right wing of the aircraft and shielded with reflective material to avoid icing and solar heating. The sensor was calibrated in an ice bath before and after every experiment run. The output was recorded on a multivolt-meter single pen recorder.

The performance and accuracy of the Barnes PRT-5 instrument were checked by flying about 30 m above a lead. During the entire experiment, the indicated radiative temperature of the PRT-5 over a lead was -1.8°C. Very little or no turbulence occurred during any run so that the effect of slight pitching and rolling of the aircraft on the look-direction of the PRT-5 were considered to be negligible.

The profiles of outside (air) temperature and spectral radiative (IR) temperature for three experimental runs are shown in Figures 6, 7, and 8. The cloud conditions described below correspond to three distinct situations of the stratus cloud during this particular episode.

(1) 21 May 1976, 1402-1439 local time (Figure 6)

The second day of the stratus cloud episode. The profiles for the first and second days are similar; both are typical of stratus cloud conditions found elsewhere. The cloud was thick enough to be black, and the radiative temperature at cloud top was nearly equal to the cloud top temperature. Furthermore, radiative cooling at the top caused a sharp temperature inversion at the cloud top which was well defined.
Fig. 6. Air, IR, and dew point temperature profiles below, within, and above the stratus clouds, 21 May 1976, 1402-1439 local time. Emissivity ($\varepsilon$) values for various cloud layers are indicated.

(2) 22 May 1976, 0920-1009 local time (Figure 7)

The profiles within the cloud showed marked changes on this day. The cloud top was tenuous so that it was difficult to measure its exact altitude. In Figure 7, the maximum observed height is indicated. Variation by as much as 500 m was observed during five up-and-down passes made through the cloud top to determine its height. The temperature structure was unlike that of the previous day, with the inversion occurring less than half-way up from the cloud base. Even for the rest of the lower section, the temperature was very peculiar: the top three quarters of the cloud had no effect on the radiant
flux emitted by the cloud. This peculiar profile is quite contrary to accepted profiles of stratus clouds. The temperature inversion near the middle of the cloud and a steady temperature increase all the way to the top may indicate the effect of solar heating. But the peculiarly steady IR temperature may indicate the importance of scattering [see for example Platt, 1972]. Further measurement may be required to confirm this effect prior to calculating the transmission coefficient of scattering.

Fig. 7. Air, IR, and dew point temperature profiles below, within, and above the stratus clouds, 22 May 1976, 0920-1009 local time. Emissivity (E) values for various cloud layers are indicated.
This is the first instance where the situation described above changed: the cloud developed an interstice which divided it into two layers. It is interesting that the interstice occurred at about the same height at which the temperature inversion occurred on 22 May (Fig. 7). The lower cloud has lowered its base, but inversion conditions and radiance effects are very similar to that shown in Figure 7. The top layer has become thicker and the increase in cloud temperature is reflected in the radiance. Here again the sharp temperature increase at the cloud top is not observed. The cloud is thickest in the lowest part of the bottom layer. The high emissivity values indicate a high liquid water content for the lower parts of the cloud [Paltridge, 1974].

Later on, in the afternoon of this day, an altostratus layer at a much higher latitude was observed in the north. Passes under this cloud showed that it had a considerable effect on the profiles of air and IR temperature of the upper layer, but no effect on the lower layer. The effect of this higher cloud was to increase temperatures and radiance of the top layer.

Just before the cloud episode ended on 26 May, a flight through the cloud showed that the clear interstice had disappeared, giving the impression of one solid layer almost 1600 m thick. However, the cloud was very tenuous in the top half, with clear regions at irregular intervals. The temperature profile showed an initial small decrease in the temperature in the lower half of the cloud and an inversion with a slight increase in temperature in the tenuous cloud region.

DISCUSSION

Except for one case of a stratocumulus reported by Platt [1972], the temperature and spectral radiance profiles described in this report are unique in character. Arctic stratus clouds develop by radiative cooling at the bottom, not at the top. The continuous sunlight during the summer shows its effect by warming the top part of the cloud so that considerable increase in temperature occurs with the inversion near the middle of the cloud. Inter-
Fig. 8. Air, IR, and dew point temperature profiles below, within, and above the stratus clouds, 24 May 1974, 1030-1110 local time. Emissivity ($\varepsilon$) values for various cloud layers are indicated.

...stices form where the heating due to solar radiation exceeds the long-wave cooling, or near the inversions. As suggested by Herman and Goody [1976], solar radiation can penetrate farther into the cloud, while long-wave cooling effects are confined to the top and bottom. Continuous solar heating attenuates the top layers of the cloud, so that
they have negligible effect on the long-wave radiance flux. The small amount of radiation emitted by the cloud particles may be scattered away so that, even with a considerable increase in the cloud temperature, the long-wave flux remains essentially a constant through the top part of the cloud.

The results described here are preliminary and need verification. The profiles described show the considerable effect that stratus clouds have on the radiation field and hence on the heat budget of the Arctic. The increase and decrease in cloudiness seem to be related strongly to the wind field, an observation consistent with that of Reed and Kunkel [1960]. Southerly flow advecting warm air shows a high correlation with the formation of stratus clouds. The primary source for dissipation of these summer stratus clouds is the advection of cold dry air from the north.

ACKNOWLEDGMENT

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TILT-METER MEASUREMENTS DURING COLLISION OF TWO FLOES,
1972 AIDJEX PILOT STUDY

by

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ABSTRACT

An event recorded by a biaxial tilt-meter during a storm at the camp of the 1972 AIDJEX pilot study located on the Beaufort Sea pack ice is interpreted as an acceleration caused by the collision of a faster drifting floe with the station floe. It is estimated that the acceleration lasted between 20 and 40 seconds and resulted in a velocity increase of between 84 and 120 m hr\(^{-1}\). The energy imparted per square kilometer to the station floe during collision is estimated to be between \(5.7 \times 10^6\) and \(8.1 \times 10^6\) joules.

The station was equipped with an acoustic bottom reference (ABR) system with a sampling rate of one minute. However, the positional scatter is too large and the sample rate too low to be able to identify the collision event reliably from the ABR data alone.

On 4 April 1972, the AIDJEX camp, located on the pack ice of the Beaufort Sea, experienced a number of shocks that felt like earthquakes and shook the buildings. One of these shocks, occurring at 1021 GMT, was recorded on a biaxial tilt-meter used to measure ocean and ice tilts.

The tilt-meter consists of two hydrostatic levels, each 120 m long, oriented perpendicularly to each other. The displacement of the fluid in the pots at opposite ends of each level is sensed with floats and displacement transducers and recorded on strip chart. The dynamic response of the fluid is such that the system is critically damped. The theory of hydrostatic levels and a description of the apparatus are given in greater detail by Weber [1974] and Weber and Erdelyi [1975].
The sudden apparent tilt changes recorded by the tilt-meter during the event (Figure 1) are likely due to a rapid horizontal acceleration or deceleration of the station floe. The deflections are positive and correspond to apparent downward tilts of 29.1 μrad to the east and 27.3 μrad to the south.

Fig. 1. Components of ice tilt measured with an orthogonal pair of hydrostatic levels at the AIDJEX camp on 4 April 1972. Positive tilt (downwards on the charts) is caused either by the downward tilt of the floe to the east and south or by horizontal accelerations to the west and north. The event of 1021 GMT is interpreted as having been caused by the collision of a faster drifting floe with the AIDJEX station floe.

The records therefore indicate an apparent tilt of 39.9 μrad caused by either an acceleration of a floe drifting northwest (317°) or by a deceleration of a floe drifting southeast (137°). From NavSat positions recorded on 4 April at 0000 GMT (lat. 75°01.5'N, long. 148°24.1'W) and at 1300 GMT (lat. 75°02.1'N, long. 148°32.6'W) it was determined that the station was drifting in the direction of 285° clockwise from north at an average speed of 325 m hr⁻¹. At the time of the event the wind was blowing to the west-southwest (242°) at 5.2 m sec⁻¹. The parameters are illustrated in Figure 2. Because the drift
Fig. 2. Average drift direction between 0000 and 1300 GMT and wind direction at collision time. Direction of acceleration (or deceleration) of station floe determined from the tilt-meter.

direction is to the northwest and in the same general direction (west-northwest) as the apparent tilt axis (northwest), it is thought that the shock was caused by a sudden acceleration, preceded by collision of a faster drifting floe with the station floe. The records show that the displacement of the floats in the pots due to the collision lasted less than two minutes, after which the floats resumed their former positions.

The magnitude of the acceleration can be estimated by considering the differential equation of motion of the vertical displacement, \( h(t) \), of the fluid column in the pots of the hydrostatic level:

\[
\ddot{h}(t) + 2\omega \dot{h}(t) + \omega^2 h(t) = \frac{A_1}{A_2} \alpha \tag{1}
\]

[Weber, 1974, eq. 9] where \( A_1 \) and \( A_2 \) represent, respectively, the cross-sectional areas of the tube (\( 1.27 \times 10^{-4} \text{ m}^2 \)) and the pots (\( 16.9 \times 10^{-4} \text{ m}^2 \)), and \( \alpha \) represents a constant horizontal acceleration in the direction of the level axis. Assuming that the acceleration lasts for \( \tau \) seconds and has the
shape of a box car pulse of magnitude $a_0$, then for

\[
\begin{align*}
  t < 0 & , \quad a = 0 ; \\
  0 \leq t \leq \tau & , \quad a = a_0 ; \\
  t > \tau & , \quad a = 0 .
\end{align*}
\]

By substituting the apparent tilt $\alpha(t)$ for $h(t)/s$, where $s$ equals the level length (120 m), and by using Laplace transforms, the following solution for the equation of motion is obtained:

\[
\alpha(t) = \frac{a_0}{2g} \left[ 1 - e^{-\omega t} (1 + \omega t) \right], \quad 0 < t < \tau
\]

\[
\alpha(t) = \frac{a_0}{2g} \left( e^{-\omega (t-\tau)} [1 + \omega (t-\tau)] - e^{\omega t} (1 + \omega t) \right), \quad t > \tau
\]

(2)

Here $\omega^2$ equals $2gA_1/sA_2$, where $g$ represents the gravitational acceleration (9.82 m sec$^{-2}$). A plot of equation (2) with $\tau$ as parameter is shown in Figure 3.

![Figure 3](image)

**Fig. 3.** Apparent tilt with time, $\alpha(t)$, of a tilt-meter caused by a constant acceleration, $a_0$, of duration $\tau$ in the direction of the instrument axis.
From the records (Fig. 1) it can be seen that for the E-W component the pulse width is 120 seconds at the most. For the N-S component the pulse is easier to interpret and its width lies between 72 and 96 seconds. Comparison with Figure 3 shows that this corresponds to a pulse with an acceleration of duration $\tau$ between 20 and 40 seconds. For $\tau = 20$ sec the maximum tilt change of 39.9 $\mu$rad corresponds to 68% of the value $a_0/2g$. Therefore,

$$39.9 \times 10^{-6} = 0.68 \frac{a_0}{2g}$$

or

$$a_0 = 11.5 \times 10^{-4} \text{ m sec}^{-1}$$

The velocity increase, $\Delta V$, over the time period $\tau = 20$ sec is

$$\Delta V = a_0 \tau = 2.30 \times 10^{-2} \text{ m sec}^{-1} = 83 \text{ m hr}^{-1}$$

For $\tau = 40$ sec the maximum tilt change is 94% of $a_0/2g$. The corresponding figures are $a_0 = 8.33 \times 10^{-4}$ m sec$^{-2}$ for the acceleration and $\Delta V = 3.34 \times 10^{-2}$ m sec$^{-1}$, or 120 m hr$^{-1}$ for the velocity increase.

The energy imparted to the AIDJEX floe during the collision is equal to the difference between the energy lost by the faster drifting floe and the energy dissipated by deformation (break-up, rafting, and formation of pressure ridges). We can estimate the energy imparted per square kilometer of floe area to the AIDJEX floe. If $m$ is the ice mass, $V_0$ is the average drift velocity, and $\Delta V$ is the velocity increase, the energy, $\Delta E$, imparted per square kilometer is

$$\Delta E = mV_0\Delta V$$

For an average floe thickness of 3 m and an ice density of 900 kg m$^{-3}$, the mass $m$ is $27 \times 10^8$ kg. For an acceleration lasting 20 seconds the imparted energy is $5.6 \times 10^6$ joules, or 1.6 kWh, and for a 40-second acceleration the corresponding figure is $8.1 \times 10^6$ joules, or 2.3 kWh. In other words, the energy imparted per square kilometer of floe area during the collision is about equal to the energy used to supply the AIDJEX camp with electric power for five minutes.
The station was equipped with an acoustic bottom reference (ABR) system that sampled the positions once every minute [Thorndike, 1973]. The ABR position fixes for the period from 0930 to 1040 GMT surrounding the event are illustrated in Figure 4. The scatter of the position points is caused partly by the geometrical configuration of the transducer-transponder array and partly by variations in the sound velocity of the water. In reality the drift path ought to be an almost straight line. The average velocity over the 50-minute period is 286 m hr⁻¹ in almost exactly the same direction (286°) as the average velocity over the 13-hour period determined from the NavSat fixes (285°).

![Graph showing station position fixes and average drift direction]

**Fig. 4.** Station position fixes at 1-minute intervals between 0950 and 1040 GMT determined from an acoustic bottom reference system, and average drift direction over the same time period.

We now wish to see whether the velocity change caused by the collision process can be detected in the ABR data. To minimize the effect of the scatter the position fixes are first projected onto the average drift direction and the drift speed variations calculated at 1-minute intervals. The angle of projection is then chosen for minimum velocity scatter (in this case 49°, Figure 4). The drift speed, as a running average over 4-minute periods, is plotted in Figure 5, lower graph. The scatter is still large and of the
same magnitude as the velocity change we are trying to detect. These values, averaged a second time over 7-minute intervals (upper graph), show that the average speed increases, reaching a maximum at collision time, and then decreases slowly over the next 20 minutes.

The result is not entirely convincing. The velocity increase is probably, but not necessarily, due to the collision. The scatter of the position points is too large and the collision event is too short to be detected reliably. This example demonstrates that it is impossible from the ABR data alone to detect, much less measure, a velocity change caused by a collision event unless the sampling rate is much increased and the geometrical configuration of the transducer-transponder array is more favorable.

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SYNTHETIC APERTURE RADAR IMAGERY
OF THE AIDJEX TRIANGLE

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ABSTRACT

Imaging radar mosaics of the AIDJEX triangle are presented together with a brief discussion of the radar sensor used in the data collection.

INTRODUCTION

During the 1975-76 AIDJEX field program, airborne radar imagery of the AIDJEX triangle was obtained during 15 flights in April, August, and October 1975 and April 1976 as part of the AIDJEX remote sensing activities [Weeks et al., 1974; Campbell et al., 1976]. This report presents the preliminary mosaics and discusses the radar instrument used.

RADAR SENSOR AND IMAGING GEOMETRY

The sensor used was the Jet Propulsion Laboratory (JPL) L-band dual-polarization synthetic-aperture imaging radar. During the April 1976 flights, an X-band channel was added to the system. The radar, as well as other remote sensors, was mounted on the NASA Ames CV-990 aircraft.

The basic parameters of the radar sensor are summarized in Table 1. This sensor is a research instrument and, as such, has been subjected to numerous
TABLE 1
JPL IMAGING RADAR PARAMETERS

<table>
<thead>
<tr>
<th></th>
<th>L-Band</th>
<th>X-Band</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmitted peak power</td>
<td>7 kW</td>
<td>50 kW</td>
</tr>
<tr>
<td>Frequency (wavelength)</td>
<td>1215 MHz (25 cm)</td>
<td>9600 MHz (3 cm)</td>
</tr>
<tr>
<td>PRF</td>
<td>1 kHz at platform velocity ($v = 250 \text{ m sec}^{-1}$)</td>
<td>Same as L-band</td>
</tr>
<tr>
<td>Pulsewidth</td>
<td>1.25 \text{ (\mu)sec}</td>
<td>Same as L-band</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>10 MHz</td>
<td>Same as L-band</td>
</tr>
<tr>
<td>Antenna, phased array</td>
<td>75 cm $\times$ 25 cm for each polarization</td>
<td>81 cm $\times$ 3 cm for each polarization</td>
</tr>
<tr>
<td>Polarization</td>
<td>HH, HV, and VV</td>
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</tr>
<tr>
<td>Beam</td>
<td>Range beamwidth centered at azimuth beamwidth 45° of vertical azimuth beamwidth 18°</td>
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</tr>
<tr>
<td>Recorder</td>
<td>Goodyear 102</td>
<td>Same</td>
</tr>
</tbody>
</table>

As a result, the quality of the data is much better for the last three series of flights than for the first one, in April 1975. A Sensitivity Time Control (STC) system has been added which increases the receiver gain as a function of time to compensate for the decrease in brightness across the image. Most of the data were taken with HH polarization (i.e., transmitted and received waves are horizontally polarized). Some data were taken in the HV and VV polarization modes.

The radar produces a high resolution (25 $\times$ 25 m), two-dimensional representation of the Earth’s surface. The image brightness is directly proportional to the radar backscatter cross section of the surface, which in turn is a function of the properties (roughness, inhomogeneities, and dielectric constant) of the surface and the region immediately below it. Given sufficient power, the radar resolution is independent of the altitude of the sensor platform. Thus, the type of image presented here is somewhat similar...
to what is expected from the imaging radar sensor; the reader is referred to the literature [Goodman, 1968; Rihaczek, 1969; Harger, 1970].

The JPL SLAR system is different from the usual synthetic-aperture imaging radars which operate at grazing angles, in that it is specifically designed to simulate a spaceborne radar. Thus, we obtain data from the nadir location (directly along the flight line beneath the aircraft) and out to an angle of 55° off nadir. As seen in Figure 1, this configuration leads to a geometric deformation (compression) in the near range. Although the compression can be geometrically corrected through digital processing [Thompson et al., 1972], the images presented in this report have not been corrected. Figure 2 gives an example of such geometric correction.

Another observable effect in the radar imagery is a change in the average brightness across the image. This effect is due to a decrease in the average backscatter cross section and the increase in the distance from the radar antenna to the Earth's surface (i.e., slant range) as the incidence angle increases. Thus, on the average, the imagery appears brighter in the near range (especially at the nadir), and this obviously has a direct influence on the interpretation. The controllable calibrated gain system (STC) which increases the receiver gain compensates for this decrease in brightness. However, although this gain compensation is integrated into the system, it is still difficult to make absolute comparisons of image brightness from ground areas at different ranges of the radar imager. A comparison of the April 1975 and October 1975 mosaics clearly shows the results of the addition of STC to the radar system.

Because a synthetic-aperture radar uses coherent electromagnetic waves to synthesize the long baseline aperture, there is a speckled appearance in the radar imagery similar to that observed when a rough surface is illuminated with coherent laser light. This speckling must also be considered when conducting detailed interpretations.

Although the altitude has no major effect on the resolution of the radar imagery (because it is synthetic-aperture rather than real aperture or "brute force" radar), the width of the imaged swath on the Earth's surface is directly related to the altitude of the aircraft. The antenna has a "field of view" which extends from nadir through 90° to the right of the aircraft. However,
the area imaged is limited by the maximum width of the film in the optical recorder. For the JPL system this is limited to a 14 km swath for a flight altitude of 10,000 m.

Radar Mosaics

Figure 3 indicates the general drift of the AIDJEX triangle during the study period. A total of 18 mosaics of imaging radar data are presented in Figures 4-21 as a catalog of the available data (Table 2). During the flights over the triangle, it was often possible to fly a full and complete radar mosaic of the area. In other instances, the constraints of other aspects of the mission allowed only a partial mosaic. This is because the SAR collects data only to the right of the aircraft flight track; two flights in opposite directions, or more closely spaced flights in one direction, would be required for complete SAR mosaics. For both complete and incomplete mosaics, we have placed the imagery on a grid to indicate their locations within the AIDJEX triangle.

### Table 2
SAR Mosaics of the AIDJEX Triangle Included in This Report

<table>
<thead>
<tr>
<th>Figure No.</th>
<th>Phase</th>
<th>Date</th>
<th>Band/Polarization</th>
</tr>
</thead>
<tbody>
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<td>4</td>
<td>I</td>
<td>13 Apr 1975</td>
<td>L(HH)</td>
</tr>
<tr>
<td>5</td>
<td>I</td>
<td>21 Apr 1975</td>
<td>L(HH)</td>
</tr>
<tr>
<td>6</td>
<td>I</td>
<td>22 Apr 1975</td>
<td>L(HH)</td>
</tr>
<tr>
<td>7</td>
<td>I</td>
<td>24 Apr 1975</td>
<td>L(HH)</td>
</tr>
<tr>
<td>8</td>
<td>I</td>
<td>26 Apr 1975</td>
<td>L(HH)</td>
</tr>
<tr>
<td>9,10</td>
<td>II</td>
<td>18 Aug 1975</td>
<td>L(HH), L(HV)</td>
</tr>
<tr>
<td>11</td>
<td>II</td>
<td>22 Aug 1975</td>
<td>L(HH)</td>
</tr>
<tr>
<td>12</td>
<td>II</td>
<td>27 Aug 1975</td>
<td>L(HH)</td>
</tr>
<tr>
<td>13</td>
<td>III</td>
<td>10 Oct 1975</td>
<td>L(HH)</td>
</tr>
<tr>
<td>14</td>
<td>III</td>
<td>15 Oct 1975</td>
<td>L(HH)</td>
</tr>
<tr>
<td>15</td>
<td>III</td>
<td>26 Oct 1975</td>
<td>L(HH)</td>
</tr>
<tr>
<td>16,17</td>
<td>IV</td>
<td>12 Apr 1976</td>
<td>L(HH), X(HH)</td>
</tr>
<tr>
<td>18,19</td>
<td>IV</td>
<td>16 Apr 1976</td>
<td>L(HH), X(HH)</td>
</tr>
<tr>
<td>20,21</td>
<td>IV</td>
<td>21 Apr 1976</td>
<td>L(HH), X(HH)</td>
</tr>
</tbody>
</table>
Each mosaic has been annotated as being "uncorrected." This term refers to three items:

1. No type of averaging has been applied to the signal for the changes in radar sensitivity with increased range. This is especially prominent in the phase I mosaics (April 1975), which were collected before the STC was added to the system.

2. No geometric corrections have been performed on the mosaics to remove distortions resulting from the slant range format of the SAR configurations (Figure 1).

3. The mosaics have not been made with "photogrammetric" accuracy and, therefore, the positions of the various data strips on the underlying grid are offered primarily for general locations and not for detailed studies.

ACKNOWLEDGMENT

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REFERENCES


FIGURE LEGENDS

Fig. 1. Geometry of synthetic aperture radar imagery from JPL radar system.

Fig. 2. Geometric correction of JPL L(HH) SAR imagery. (2a) original imaged data, (2b) geometrically corrected SAR data. Note major geometric changes in near range (left, stretching) and far range (right, compression).

Fig. 3. Locations of AIDJEX triangle during collection of SAR data.

Fig. 4. Uncorrected L(HH) mosaic, 13 April 1975.

Fig. 5. Uncorrected L(HH) mosaic, 21 April 1975.

Fig. 6. Uncorrected L(HH) mosaic, 22 April 1975.

Fig. 7. Uncorrected L(HH) mosaic, 24 April 1975. Note presence of only parallel data sets, without overlap, making it impossible to tie the data together.

Fig. 8. Uncorrected L(HH) mosaic, 26 April 1975. (See comment for Fig. 7.)

Fig. 9. Uncorrected L(HH) mosaic, 18 August 1975. Note difference in tone between northernmost data run and rest of mosaic; it is a function of the system settings and can be corrected by image processing.

Fig. 10. Uncorrected L(HV) mosaic, 18 August 1975. (Compare with Fig. 9.)

Fig. 11. Uncorrected L(HH) mosaic, 22 August 1975. (See comment for Fig. 7.)

Fig. 12. Uncorrected L(HH) mosaic, 27 August 1975. (See comment for Fig. 7.) Banding in central strip of mosaic is caused by recording film's being light-struck at end of roll.

Fig. 13. Uncorrected L(HH) mosaic, 10 October 1975. (See comment for Fig. 9.)

Fig. 14. Uncorrected L(HH) mosaic, 15 October 1975. (See comment for Fig. 12.)

Fig. 15. Uncorrected L(HH) mosaic, 26 October 1975. Note lack of image overlap, a result of slight navigational error in south-central portion of area.

Fig. 16. Uncorrected L(HH) mosaic, 12 April 1975. (See comment for Fig. 7.)

Fig. 17. Uncorrected X(HH) mosaic, 12 April 1976. Compare with Fig. 16.

Fig. 18. Uncorrected L(HH) mosaic, 16 April 1976. Note large amount of overlap and small areas of missing data in southwest portion of study area.

Fig. 19. Uncorrected L(HH) mosaic, 16 April 1976. Compare with Fig. 18.

Fig. 20. Uncorrected L(HH) mosaic, 21 April 1976. Note breaks in south-central data strip caused by slight change in azimuth scale when compared with adjacent strip.

Fig. 21. Uncorrected X(HH) mosaic, 21 April 1976. Compare with Fig. 20.
ARCTIC EXPERIMENT PROGRAM—1

AIDJEX TEST SITE SYNTHETIC APERTURE RADAR MOSAIC
L-BAND -HH POLARIZATION (UNCORRECTED)

Figure 5  21 APRIL 1975
Figure 6
22 APRIL 1975
Figure 8

26 APRIL 1975
ARCTIC EXPERIMENT PROGRAM - II

AIDJEX TEST SITE SYNTHETIC APERTURE RADAR MOSAIC
L-BAND - HH POLARIZATION (UNCORRECTED)

Figure 9

18 AUGUST 1975
Figure 16 12 APRIL 1976

ARCTIC EXPERIMENT PROGRAM-IV

AIDJEX TEST SITE SYNTHETIC APERTURE RADAR MOSAIC
L-BAND -HH POLARIZATION (UNCORRECTED)
AIDJEX TEST SITE SYNTHETIC APERTURE RADAR MOSAIC
L-BAND -HH POLARIZATION (UNCORRECTED)

Figure 18  16 APRIL 1976
ARCTIC EXPERIMENT PROGRAM - IV

AIDJEX TEST SITE SYNTHETIC APERTURE RADAR MOSAIC
X - BAND - HH POLARIZATION (UNCORRECTED)

Figure 19
16 APRIL 1976
ARCTIC EXPERIMENT PROGRAM - IV

AIDJEX TEST SITE SYNTHETIC APERTURE RADAR MOSAIC
X-BAND - HH POLARIZATION (UNCORRECTED)

21 APRIL 1976

Figure 21