The force balance of sea ice in a numerical model of the Arctic Ocean

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Abstract. The balance of forces in the sea ice model of Hibler [1979] is examined. The model predicts that internal stress gradients are an important force in much of the Arctic Ocean except in summer, when they are significant only off the northern coasts of Greenland and the Canadian Archipelago. A partition of the internal stress gradient between the pressure gradient and the viscous terms reveals that both are significant, although they operate on very different timescales. The acceleration term is generally negligible, while the sum of Coriolis plus sea surface tilt is small. Thus the seasonal average force balance in fall, winter, and spring is mostly between three terms of roughly equal magnitudes: air drag, water drag, and internal stress gradients. This is also true for the monthly average force balance. However, we find that there is a transition around the weekly timescale and that on a daily basis the force balance at a particular location and time is often between only two terms: either between air drag and water drag or between air drag and internal stress gradients. The model is in agreement with the observations of Thorndike and Colony [1982] in that the correlation between geostrophic wind forcing and the model’s ice velocity field is high. This result is discussed in the context of the force balance; we show that the presence of significant internal stress gradients does not preclude high wind–ice correlation. A breakdown of the internal stress gradient into component parts reveals that the shear viscous force is far from negligible, which casts strong doubt on the theoretical validity of the cavitating fluid approximation (in which this component is neglected). Finally, the role of ice pressure is examined by varying the parameter P*. We find a strong sensitivity in terms of the force balance, as well as ice thickness and velocity.

1. Introduction

What is the force balance of sea ice in the Arctic Ocean, and how does it vary with space and time? We refer to the following balance

\[ \rho h \frac{dU}{dt} = F_a + F_w + F_i + F_c + F_t \] (1)

where the acceleration term on the left-hand side is a function of the density \( \rho \), the average thickness \( h \), and the velocity \( U \). On the right-hand side are the vector forces (per unit area) due to air drag \( F_a \), water drag \( F_w \), internal stress gradient \( F_i \), Coriolis effect \( F_c \), and sea surface tilt \( F_t \).

This question has traditionally been addressed by examining data from two sources: manned ice camps and autonomous drifting buoys. Results from the Arctic Ice Dynamics Joint Experiment (AIDJEX) ice camps in the Beaufort Sea [e.g., Newton and Coachman, 1973; Hunkins, 1975; McPhee, 1980] indicate that in summer, ice moves mostly in response to air drag, water drag, Coriolis, and sea surface tilt, while in winter, internal stress gradients also play a significant role. The acceleration term is usually significant only for timescales smaller than a day. Note that internal stress gradients are not directly measured in these studies but are instead derived as a residual of the terms in (1), which are themselves calculated using forcing data (such as surface winds) together with analytical parameterizations.

Thorndike and Colony [1982] used a network of autonomous drifting buoy data to find the correlations between ice motion and the geostrophic wind. This was a statistical analysis of two well-observed variables and not an explicit force balance calculation. Thorndike and Colony [1982, Figure 4] show high correlations at points over 400 km away from coasts, even in winter. Lower correlations near shore and in the East Greenland Current were attributed to the effects of internal stress gradients and water drag, respectively. Their Figure 4 (bottom panel) also shows relatively lower correlations in the Eurasian Basin relative to the Canadian Basin. They conclude that for timescales longer than about a day the main force balance in the interior of the Arctic Ocean is between air drag, water drag, Coriolis, and sea surface tilt throughout the year.

If the AIDJEX ice camp results are taken to be representative of the interior (a borderline assumption given their mean offshore distance of about 400 km), then there is a conflict: internal stress gradients were large at the ice camp for all seasons except summer, while they were deemed small throughout the year in the buoy analysis of Thorndike and Colony [1982]. Of course, more field data would help to resolve this question. A first step toward this answer was taken by Hibler [1986], who used analytical and idealized numerical examples to show that internal stress gradients have the greatest effect on ice velocity when the air drag is weak (his Figure 6). He predicted that high correlation (between winds and ice motion) could exist even when internal stress gradients are significant. In a subsequent study [Hibler and Bryan, 1987] the air drag, modeled internal stress gradient, and observed buoy motion were compared at the mean position of eight drifting ice buoys in the Arctic Ocean. Significant internal stress gradients were pre-
dicted, generally directed opposite of the air drag vector. An interesting exception was a buoy in the Nansen Basin where the internal stress gradient was normal to the air drag, pointing towards the Eurasian continental slope. This is explained in section 3.2.

In this paper we extend the work of Hibler [1986] and Hibler and Bryan [1987] by investigating the temporal and spatial variations of the ice force balance as portrayed in a commonly used ice-ocean numerical model. It should be noted here that our goal in this study is not an exacting model-data comparison, although buoy drift statistics are cited and compared with model-simulated drifts. Here we concentrate on examining the model's version of reality and trying to understand the results based on intuition. There is renewed interest in the force balance, focusing mostly on the representation of internal stress gradients in sea ice models [Northwest Research Associates, Inc., 1995; M. D. Coon et al., An oriented thickness distribution for sea ice, submitted to the Journal of Geophysical Research, 1996]. Further, efforts are underway to compare and merge buoy and satellite ice motion vectors with model output [Maslaniak and Maybee, 1994; D. Thomas, The quality of sea ice velocity estimates, submitted to the Journal of Geophysical Research, 1997]. From both perspectives it seems worthwhile to dig deeply into a numerical model's representation of the forces that make sea ice move in the Arctic Ocean.

Among the questions we address are the following: What are the dominant terms in the ice force balance? How do these vary with space and time? Is the force balance a function of averaging period (e.g., daily versus seasonal)? Where and when does the internal stress gradient play a role and why? Finally, in order to resolve the previously mentioned conflict we ask, How can internal stress gradients be important if the ice velocity is fairly highly correlated with the geostrophic winds? In the following section the coupled sea ice-ocean model is briefly described. Results are then presented, including a correlation analysis as in work by Thordike and Colony [1982]. The seasonal and daily-force balances are discussed next, including a sensitivity study with varying ice pressure. The paper ends with our conclusions.

2. Model

We use the most common sea ice model in use today, that described by Hibler [1979]. This is a so-called "two-level" model, in which the distribution of ice thicknesses at each model gridpoint is simplified to an average thickness $h$ with areal concentration (or compactness) $A$. Air and water drags are parameterized with the usual quadratic bulk formulas, using the coefficients in work by Hibler [1979]. The other terms in (1) are also described by Hibler [1979].

(1) $F_i = F_{p}^{i} + F_{s}^{i} + F_{i}^{B} + F_{i}^{P}$

(Note that the terms in (1) have units of force per unit area, often referred to as "stress" by researchers in both the sea ice and ocean research communities, e.g., "air stress" or "water stress." Confusion may occur over, however, since $F_i = \nabla \sigma$ where $\sigma$ is known as the "stress tensor" and has units of force per unit length, not area. Here we refer to (1) as a "force" balance, with the implicit understanding that each of the terms in (1) has units of force per unit area. In keeping with common usage we refer to $\sigma$ as internal stress, and $F_i$ as the internal stress gradient. Also, we refer to $F_{a}$ as air drag, and $F_{w}$ as water drag.)

The viscous-plastic constitutive relation in work by Hibler [1979] allows a diagnostic partitioning of the internal stress gradient $F_i$ into three parts

\[ F_i = F_{P}^{i} + F_{s}^{i} + F_{b}^{i} \]  

where $F_{P}^{i}$ denotes the force from the pressure gradient, $F_{s}^{i}$ denotes the force from bulk viscosity, and $F_{b}^{i}$ denotes the force from shear viscosity. The term $F_{P}^{i}$ is given by

\[ F_{P}^{i} = -\frac{1}{2} \nabla P \]  

where the pressure $P$ (also known as "strength") is related to the mean thickness and concentration by

\[ P = \frac{P^{*}}{\alpha^2} h \exp[-C(1-A)] \]  

and $P^{*}$ and $C = 20$ are tuning constants. In work by Hibler [1979], $P^{*} = 5000 \text{ N m}^{-2}$, while in work by Hibler and Walsh [1987] the value was increased by a factor of 5.5 to $P^{*} = 27,500 \text{ N m}^{-2}$. All subsequent papers by Hibler and coworkers have used the latter value, which is tuned to give the best buoy/model ice velocity correlations when daily wind data are used. (Hibler [1979] used 8-day averaged geostrophic wind forcing.)

The inclusion of internal stress gradients in a sea ice model generally requires some parameterization for the ice pressure. In Overland and Pease [1988] and Hákkinen [1987, 1992] the pressure is similar to (4), except it is a function of $r$. In Rothrock [1975], Hibler [1980a], and Flato and Hibler [1995] it is a more common quadratic bulk formulas, using the coefficients in work by Hibler [1979].

The second part of the internal stress gradient comes from the bulk viscous force $F_i^{B}$,

\[ F_i^{B} = \nabla (\zeta \dot{\epsilon}_i) \]  

where $\zeta$ is the bulk viscosity and $\dot{\epsilon}_i = \nabla \times U$ is the divergence of the velocity field. The third part of the internal stress gradient is the shear viscous force $F_i^{S}$, given by

\[ F_{i}^{S} = \begin{bmatrix} F_{i}^{S}_{x} \\ F_{i}^{S}_{y} \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} (\eta \dot{\epsilon}_{i\alpha} \cos 2\phi) + \frac{\partial}{\partial y} (\eta \dot{\epsilon}_{i\beta} \sin 2\phi) \\ \frac{\partial}{\partial x} (\eta \dot{\epsilon}_{i\beta} \cos 2\phi) - \frac{\partial}{\partial y} (\eta \dot{\epsilon}_{i\alpha} \sin 2\phi) \end{bmatrix} \]  

where $\eta$ is the shear viscosity, $\dot{\epsilon}_{i\alpha} = \sqrt{a^2 + b^2}$ is the shear magnitude, $\phi = \frac{1}{2} \arctan(ab)$ is the principal direction of shearing, $a = \partial u / \partial y + \partial v / \partial x$, and $b = \partial u / \partial x - \partial v / \partial y$. The ice velocity $U$ has $x$ and $y$ components $(u,v)$. With reference to the strain rate tensor, $\dot{\epsilon}_i$ is the sum of the eigenvalues, $\dot{\epsilon}_i^M$ is the (positive) difference of the eigenvalues, and $\phi$ is the orientation of the eigenvector corresponding to the larger eigenvalue.

Following Hibler [1979], the bulk viscosity $\zeta$ and the shear viscosity $\eta$ are given by

\[ \zeta = \frac{P}{(2\Delta)} \]  

\[ \eta = \zeta / e^2 \]  

where $\Delta = \sqrt{\dot{\epsilon}_i^2 + \dot{\epsilon}_i^M / e^2}$ is the magnitude of the strain rate when $e = 1$. Here we follow Hibler [1979] and set $e = 2$, where $e$ is the ratio of the major to minor axis of the elliptical yield curve in principal stress space. Thus the viscosities $\zeta$ and $\eta$ are linear functions of the pressure $P$ and nonlinear functions of the strain rate. They are constrained to maximum values given by

\[ \zeta_{\text{max}} = (2.5 \times 10^8 \text{ s})P \]  

\[ \eta_{\text{max}} = \zeta_{\text{max}} / e^2 \]
in order to avoid a singularity for small strain rates (equations 7 and 8). Further details may be found in work by Hibler [1979].

Other decompositions of the internal stress gradient $F_i$ are of course possible. The above method has the following advantages. First, it clearly identifies the part that is independent of strain rate and dependent on the more slowly varying pressure gradient $F_i^P$. Second, it decomposes the remaining viscous terms into the part due to shear $F_i^S$ and bulk $F_i^B$ viscous forces. These vary with both strain rate and pressure gradient.

The ice model used here differs from that described by Hibler [1979] mainly in the parameterization of heat and salt fluxes, which are taken from Hibler [1980a]. Also, a snow layer is added, and sea ice salinity is assumed to be a constant 7 parts per thousand (ppt). These are described by Zhang et al. [1997]. Also, a new, faster numerical scheme for solving for ice velocity is used [Zhang and Hibler, 1997]. The ice model is coupled to a three-dimensional multilevel ocean model [Bryan, 1969; Cox, 1984]. In this paper the ocean model is important only in its contribution to the water drag and tilt terms in (1). The ocean model and its coupling to the sea ice model are described in detail by Zhang et al. [1996]. The only change relative to Zhang et al. [1997] is that we have opened two outflow channels in the Canadian Archipelago at Mc Clure and Nares Straits, with specified volume transports of 0.8 and 0.7 Sv, respectively. Damping to climatological values of tem-
temperature and salinity below the mixed layer with a 5 year timescale is used, which Zhang et al. [1997] suggest provides the most realistic surface properties.

The forcing is provided by fields of daily atmospheric surface pressure obtained by merging National Center for Atmospheric Research (NCAR) analysis with Arctic buoy data for the time period 1979–1985 (J. E. Walsh, private communication, 1991). These are converted into air drag using a bulk formula that includes fixed constants for the drag coefficient and turning angle [Hibler, 1979]. Surface air temperatures are interpolated from monthly average values, varying over the domain with interannual variability from Hansen and Lebedeff [1987] and are modified to account for ice feedback on air temperatures as in work by Hibler and Zhang [1993]. These are used to compute turbulent heat fluxes as well as downwelling longwave fluxes via the parameterizations in work by Parkinson and Washington [1979]. The coupled model is integrated to steady state over two cycles of the 7 years, 1979–1985, and then a third cycle is run (for which we show results). The grid resolution is 40 km, and the momentum time step is 1 day.

3. Results

In the following, we show results from a standard case in which the ice pressure parameter \( P^* = 27,500 \text{ N m}^{-2} \). Results from sensitivity studies in which \( P^* \) is varied are also shown.

Figures 1a and 1b show the mean winter ice velocity and ice thickness fields for our standard case. (The other panels show results from sensitivity studies which are discussed in section 3.4.) Vectors in all figures are plotted at 120 km resolution. There is a Beaufort Gyre and an outflow across the Eurasian Basin into and through the Fram Strait. Broadly speaking, the ice moves in response to the annual average high-pressure cell that sits over the Canadian Basin and the low-pressure cell centered off the northern Norwegian coast. The thickest sea ice is found north of Greenland and the Canadian Archipelago, caused in part by wind stress piling up the ice against this coast and in part by the cold air temperatures in this region. In the interests of space we have chosen not to show the fields from other seasons. However, these plots show remarkable variability in the pattern of ice motion, which indeed exists on monthly and even daily timescales. The reason is that the ice motion is highly correlated with the highly variable air drag, as discussed by Thorndike and Colony [1982] and demonstrated below for our model. The thickness pattern, however, undergoes more moderate changes and then only on monthly or seasonal timescales.

Figure 2a shows the ice compactness \( A \) in winter, when concentrations in the central Arctic Ocean are above 99%. In summer (Figure 2b), concentrations drop to 90% and below. Figure 3 shows the ice pressure \( P(x,y) \) in winter (July, August, September), fall (October, November, December), winter (January, February, March), and spring (April, May, June), where the seasonal averages are over the 7 years, 1979–1985. Figure 3e shows the annual average. The pressure during much of the ice growth season is simply a linear function of thickness. In the transition to the melt season and in summer, however, the drop in ice compactness plays an obvious role via (4). The domain average values are nearly constant for at least half the year but are reduced by a factor of 4 in summer.

The mean winter ocean surface geostrophic velocity (taken as the velocity at 37 m depth, the third level of the ocean model) is shown in Figure 4. The pattern resembles the ice motion but with obvious topographic steering and amplification along the shelf break and major ridge systems. In much of the Arctic, where these currents are small, the water drag \( F_w \) is mostly a quadratic function of ice velocity; that is, \( F_w \) is a passive drag on the underside of the ice pack. However, there are definitely regions where the ocean currents are comparable in magnitude to the seasonal average ice speed (north of Alaska and in the East Greenland Current) or even larger (along the inner edge of the Chukchi Plateau 500 km north of eastern Siberia).

3.1. Correlation Analysis

As discussed in section 1, Thorndike and Colony [1982] found that daily average sea ice velocity and geostrophic winds were highly correlated throughout the year but particularly in summer. Few, if any, published numerical model studies have performed a similar analysis using the geostrophic velocity forcing and the model's ice velocity output.

Figure 5 shows the model's geographic and seasonal variation of this squared correlation (in percent) and may be compared with Figure 4 and Table 3 in work by Thorndike and Colony [1982]. Generally speaking, the agreement is good. There is high correlation in the interior of the Arctic Ocean and much lower correlation near the coasts and in the East Greenland Current. These patterns were noted by Thorndike and Colony [1982]. The value in the Arctic Ocean interior is generally 80–90%, while the domain average values are lower by 10–20%. Correlations in the interior during winter are slightly higher than those obtained by Thorndike and
Figure 3. The ice pressure $P(x,y)$ in (a) winter (January, February, March), (b) spring (April, May, June), (c) summer (July, August, September), (d) fall (October, November, December), and (e) the annual average. Plots are averaged over the 7 years, 1979–1985. The contour interval is $10 \times 10^3 \, N \, m^{-1}$. The domain average value ($\times 10^3 \, N \, m^{-1}$) is shown in the lower right-hand corner of each panel.

Colonel [1982], which is probably a result of our use of a seasonally invariant air–ice drag coefficient (section 3.3). The highest correlation in our model is in the fall; the lowest is in the spring.

Figure 5 shows that the model reproduces the observed high correlation between geostrophic winds and ice motion. As we will demonstrate in the next section, it is possible for the model to provide solutions wherein this correlation is high even when the internal stress gradient term is a significant (or in fact dominant) part of the force balance.

3.2. Seasonal Force Balance

Figures 6–9 show the seasonal cycle of forces, and Figure 10 shows the annual average. The acceleration term on the left-hand side of (1) is several orders of magnitude smaller than the other terms and is thus not shown. Both the Coriolis and the sea surface tilt forces are generally small but nonzero. Because they are often oppositely directed, they sum to an even smaller vector in much of the domain, in keeping with AIDJEX observations from the Beaufort Sea [Newton and Coachman, 1973; Hunkins, 1975]. Panels (f), (g), and (h) in Figures 6–10 show the partition of internal stress gradient $F_i$ between the pressure gradient $F_{iP}$, the shear viscous force $F_{is}$, and the bulk viscous force $F_{iB}$ (2).

Summer. In summer (Figure 6), internal stress and internal stress gradients are small over most of the Arctic Ocean. This is because ice compactness is reduced in summer relative to winter (Figure 2), which strongly affects the pressure via the exponential in (4). When pressure is reduced, the viscous terms $F_{is}$ and $F_{iB}$ become nil since both shear and bulk viscosities vary linearly with $P$ (for a fixed value of strain rate magnitude). Figure 3 shows that pressure gradients are reduced as well. Thus, in the bulk of the Arctic Ocean the main force balance in summer is between air and water drags.
The only exception to this rule is the region several hundred kilometers within the coasts of Greenland and the Canadian Archipelago. Here ice remains compact (and thick) throughout the year. Thus there are strong pressure gradients (Figure 6f) which act to push ice offshore. The (mostly bulk) viscous forces almost completely oppose this, aided by the large viscosities that occur at high pressure and low strain rate (equations 7 and 8). The net effect (Figure 6c) is a very small but nonetheless offshore component. (We note that viscosities are actually quite high in this sea ice model, which creates markedly smooth ice motion fields on even a daily timescale (see Figure 12). As a point of comparison, the annual average, domain average lateral shear viscosity in the sea ice model ($1.5 \times 10^8$ m$^2$ s$^{-1}$) is about 4 orders of magnitude larger than in the ocean model ($8 \times 10^3$ m$^2$ s$^{-1}$).

**Fall.** In fall (Figure 7) the ice accelerates as the wind stress picks up, which in turn leads to a stronger water drag. The ice thickens, and concentration approaches 100%, creating higher ice pressures (Figure 3d). This allows the internal stress gradient to play a role in the force balance, although its magnitude is generally somewhat smaller than air or water drags at this time. The partition between pressure and viscous gradients in Figures 7f–7h is particularly illuminating. (Note that following (2), the vectors in Figures 7f–7h together sum to the vectors in Figure 7c.) The pressure gradient (Figure 7f) is of course strongest along the Greenland and Canadian coasts, where thickness gradients are largest. Elsewhere, the vectors continue to point away from this coast, across pressure (i.e., thickness) contours, all the way to the seasonal sea ice edges (e.g., north of Alaska). In the Eurasian Basin these vectors are directed across the ice outflow stream that exits through the Fram Strait (Figure 1a). This explains the force balance at the Nansen Basin buoy described by Hibler and Bryan [1987]; that is, it is the pressure gradient force that in this model acts to push the outflowing sea ice in the Transpolar Drift Stream "sideways" toward the Eurasian continent.

The viscous forces (Figures 7g and 7h) look very different. As in summer, they partially counteract the pressure gradient north of Greenland and Canada. But in addition, the shear viscosity acts to damp any jetlike features in the other terms of (1). One example of this is the high-velocity ocean jet along the east Siberian shelf break (Figure 4) which produces a complimentary "jet" in shear viscous stress gradient pointing in the opposite direction. Another example is the high shear in both ocean and air drags along the coasts of Greenland and the Canadian Archipelago. These jets are also opposed by shear viscous stress gradients pointing upstream.

In fact, the East Greenland Current is an interesting case study for examining the partition between pressure and viscous stress gradients. Close to the coast, internal stress gradients are as large or larger than the water drag term, while near the ice edge, water drag dominates. The explanation is as follows. A "no-slip" boundary condition at the coast creates high shear there, which generates high viscous stresses. Closer to the ice edge, shear is reduced, and the pressure gradient force dominates because of the rapid transition in pressure. A closer examination of this region (not shown) shows a rather smooth clockwise turning of total $F_i$ vectors from those pointing upstream along the coast to those pointing outward across the ice edge. As noted by Leppäranta and Hibler [1985], the viscous stress gradients largely prevent the pressure gradient term from either expanding the ice edge indefinitely (nonrotating case) or creating an along-edge jet (rotating case).

**Winter.** Figure 8 shows the force balance in winter. The water drag acts to oppose the ice motion (Figure 1a) in much of the Arctic, which in turn responds largely to the wind stress. The internal stress gradient looks similar to that in fall, although its magnitude is now comparable to water drag in much of the Arctic Ocean. In the Chukchi Sea the growth of sea ice has allowed sufficient ice pressure to oppose the very strong onshore air drag. In fact, the largest vectors in Figure 8c are found in the Chukchi Sea. Figures 8g and 8h show that significant viscous stress gradients also exist in this region.

**Spring.** Figure 9 shows the force balance in the spring. The internal stress gradient is now comparable and often larger in magnitude than the water drag. In this season the ice is thick and "locked up," so that wind forcing is counteracted to a large degree by the internal stress gradient.

**Annual Average.** Figure 10 shows the annual average force balance. The main balance in much of the Arctic Ocean is between air drag, water drag, and internal stress gradient. Coriolis and sea surface tilt are smaller, while their sum is smaller still. Viscous and pressure gradient terms both contribute significantly to the net internal stress gradient over much of the domain. In the interior of the Arctic Ocean the shear viscous term generally dominates the bulk viscous term.

**North Pole.** Figure 11 shows the seasonal force balance at the north pole. For simplicity, in all subsequent figures we have summed the Coriolis and sea surface tilt into a single vector denoted as $F_{ct}$. As noted previously, $F_i$ and $F_w$ are comparable in magnitude to the air drag during fall and winter. They are also both directed nearly 180° away from the ice velocity and from the air drag. This is one reason why the ice motion can be highly correlated with the geostrophic winds; both water drag and internal stress gradients often act as passive drags, slowing down the sea ice but not affecting its direction. As Figures 6–10 demonstrate, it is mostly the shear viscous term $F_{i\beta}$ that acts in this manner, while the pressure term $F_{iP}$ is a more time-independent force that can act at an angle to the wind-forced motion. (The seasonal average bulk viscous term $F_{i\beta}$ is generally negligible at the north pole and is significant only close to the North American coast.) Figure 11b shows that in the spring the internal stress gradient is larger in magnitude (at the pole) than the water drag, and the colinear relationship between $F_i$, $F_w$, and $F_a$ has begun to break down.
The balance looks remarkably similar to that observed by Hunkins [1975] in the Beaufort Sea. An exception is the negligible turning angle between modeled ice motion and air drag. This could be a result of the simplified parameterization of air drag used here (i.e., constant drag coefficient and turning angle). But it also may be a result of regional variations, since a comparison of Figure 1a and Figure 8a shows areas of large turning angle in the Beaufort Sea.

By summer (Figure 11c), internal stress gradients are negligible, and the balance is between air drag, water drag, and to a lesser degree, Coriolis plus tilt. This is the situation described by Thomdike and Colony [1982].

Comments on the Validity of the Cavitating Fluid Model. Figure 10a shows that the annual mean winds tend to push sea ice against Greenland and the Canadian Archipelago. The resulting high viscous stresses (Figures 10g and 10h) greatly slow down the ice, which together with net convergence and net atmospheric cooling conspire over time to produce very thick ice. The cavitating fluid approximation neglects shear viscous stresses $F_{vS}$. Flato and Hibler [1990, 1992] show that the biggest difference between mean thickness patterns produced by the cavitating fluid model and the viscous-plastic model is precisely in this region north of Greenland and the Canadian Archipelago. In the cavitating fluid model the lack of shear viscosity means that when the ice is pushed against the coast, it easily splits into two streams, one moving westward into the Beaufort Sea and one moving eastward into and through the Fram Strait. The velocities in this region are consistently higher in the cavitating fluid model. The result is little or no ice thickness buildup.

Thus a qualitative comparison of ice thickness and velocity patterns between the full viscous-plastic and the cavitating fluid mod-
Figure 6. The summer (July, August, September) sea ice force balance (1), where the individual terms are (a) air drag, (b) water drag, (c) internal stress gradient, (d) Coriolis, and (e) sea surface tilt. The inertial term in (1) is negligible. Units are newtons per square meter. Figures 6f–6h show the partition of internal stress gradient into the pressure gradient, shear viscous force, and bulk viscous force.

It seems that the cavitating fluid model therefore neglects a major part of the internal stress gradient. Consider the following:

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Figure 7. Same as Figure 6, but for fall (October, November, December).

(6) and (8) show that the shear viscous force is inversely proportional to the square of $e$, the aspect ratio of the elliptical yield curve in principal component space. (In fact, the cavitating fluid approximation is obtained by setting $e \to \infty$, which stretches the ellipse into a straight line.) This means that the magnitude of shear viscous forces is completely dependent on our choice of $e = 2$. A larger value for $e$ would lead (quadratically) to much reduced shear viscous forces in this model. So what is the "right" value for this parameter? The answer is difficult to determine from first principles. However, a study of satellite radar-derived ice motion fields [Stern et al., 1995] supported values of $e = 2$ or $e = 3$.

Thus it seems that within the context of the viscous-plastic rheology, shear viscous forces are indeed a significant component of the internal stress gradient and should not be neglected. The use of
Figure 8. Same as Figure 6, but for winter (January, February, March).

a cavitating fluid model cannot therefore be justified from a theoretical viewpoint, although its behavior might be suitable for climate-related simulations which have low temporal and/or spatial resolution [Flato and Hibler, 1990].

3.3. Daily Force Balance

A Case Study. Figure 12 shows the force balance on January 6, 1984. A strong offshore air drag jet (a signature of a frontal system that moved through this area over several days) is evident in the Beaufort Sea (Figure 12a). Broader regions of high air drag exist over the Lomonosov Ridge and in the Barents and Kara Seas, evidence of the mean jet between the Beaufort High and the Greenland-Iceland-Norwegian Sea Low. Even averaged over just 1 day the acceleration term in (1) is negligible, so the other forces in the equation must balance the air drag. Figure 12 shows that over much of the domain it is the internal stress gradient, and not the
water drag, that balances the air drag forcing. In particular, the viscous term (defined as the sum of shear and bulk viscous forces) plays the biggest role (Figure 12f) except within a few hundred kilometers of the coast, where the pressure gradient also contributes (Figure 12e). The partition of viscous forces between shear and bulk terms on this day (not shown) is spatially noisy but generally shows equal contributions from the two terms.

The only exception is in the Barents Sea, where the water drag is larger than the internal stress gradient. This is because ice pressure is small in the Barents Sea (Figure 3), so that viscous stresses are insufficient to balance the air drag.

Figure 12 presents a fairly common balance of forces. Thus, over short (daily) timescales in the interior of the Arctic Ocean the viscous stress generally balances localized air drag patterns. Over

Figure 9. Same as Figure 6, but for spring (April, May, June).
longer timescales, Figures 6–10 indicate that the viscous stresses mostly balance shears in the ocean currents (Figure 4) which themselves vary much more slowly than those in the winds.

The Arctic Average Force Balance. Figure 13 shows time series of the amplitude of the daily domain average force vectors, averaged into mean monthly values over the 7-year simulation. Also shown are the time series for two sensitivity experiments discussed in section 3.4. The annual average values of each component are also given in the figure.

Like previous figures, Figure 13 shows that the water drag and internal stress gradient both contribute to balancing the air drag, while the combined Coriolis plus tilt term is much smaller. Also, the internal stress gradient is most important in late winter and early spring, when it is larger than the water drag in amplitude.
Winter

In each term looks similar to the mean force balance shown in (1). The left-hand side is again shown in Figures 6-10. (Note that the results of Overland and Colony [1994] have shown that in the interior parts of the Arctic Ocean the seasonal force balance (Figure 6) looks similar to the daily force balance, i.e., free drift dominates.

Daily Versus Seasonal Force Balance. Broadly speaking, the daily average balance is often dominated by just two terms. These are either air drag and water drag or air drag and internal stress gradient. Figure 12 shows an example from winter when regional variations dictate which balance prevails. It is relatively less common to find an area where all three forces contribute equally. The same holds true for summer, of course, when the main balance in most of the Arctic is "free drift," i.e., a balance between air drag and water drag (and, to a lesser extent, Coriolis).

In contrast, the seasonal balance during winter (Figure 8) shows broad regions where all three forces (air drag, water drag, and internal stress gradient) contribute more or less equally. In summer the seasonal force balance (Figure 6) looks similar to the daily force balance, i.e., free drift dominates.

Plate 1 differs in the vector averaging periods from daily (Plate 1a) to seasonal (Plate 1d). The histogram has a resolution of 0.001 x 0.001. The amplitude is color coded, wherein red pixels represent counts greater than 0.04% of the total, and the other colors represent lesser amplitudes. The geometric "triangle inequality" dictates that if the force balance is composed of the terms on the right-hand side of (1), then all data in Plate 1 should fall within the open square defined by \(x + y > 1\), \(y - x < 1\), and \(x - y < 1\), where \(x = |F_a + F_w + F_i|\) and \(y = |F_i|/|F_a|\). Some data in Plate 1a lie just outside this region near (1,0) and represent instances when the acceleration term plays a small role.

Figure 14b shows a further breakdown of the energy dissipation by internal stresses into the three components identified in (2). The pressure gradient term is negligible, which makes sense considering that it is often normal to the ice motion (especially in the Eurasian Basin). The energy dissipation by shear viscous forces is about 60% of the total dissipation by internal stress gradient. As noted in section 3.2, this calls into question the validity of the cavitation fluid approximation, in which this term is neglected.

Energetics. The kinetic energy balance is derived by taking the dot product of the ice velocity with (1). The left-hand side is again negligible. Figure 14a shows the domain average balance for the standard case, with each term plotted as a time series. The terms are calculated as daily average values, but for ease of presentation they are shown binned into monthly means. The seasonal variation in each term looks similar to the mean force balance shown in Figure 13. The main difference is that the Coriolis plus tilt term contributes essentially nothing to the energy balance since this term is mostly normal to the motion. (The Coriolis term is of course exactly normal, while the tilt term is not necessarily so.) If we take the air drag as the main supply of kinetic energy (neglecting regions with high ocean currents), then the annual average energy balance (also shown in Figure 14) indicates that 75% of this energy is dissipated by the water drag and 25% by the internal stress gradients.

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Daily Versus Seasonal Force Balance. Broadly speaking, the daily average balance is often dominated by just two terms. These are either air drag and water drag or air drag and internal stress gradient. Figure 12 shows an example from winter when regional variations dictate which balance prevails. It is relatively less common to find an area where all three forces contribute equally. The same holds true for summer, of course, when the main balance in most of the Arctic is "free drift," i.e., a balance between air drag and water drag (and, to a lesser extent, Coriolis).

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Plate 1 shows how the force balance varies as a function of averaging period. Here we have plotted two-dimensional histograms of the magnitude of the internal stress gradient \(F_i\) versus the magnitude of the vector sum of water drag and Coriolis plus tilt \(F_w + F_{ct}\) both normalized by the air drag \(F_a\) for the entire 7-year experiment. Points that lie near (0,0) represent free drift (wherein internal stress gradients are unimportant) while points that lie near (0,1) represent locked-up ice (wherein water drag and Coriolis plus tilt are both small owing to small ice velocities). The panels in Plate 1 differ in the vector averaging periods from daily (Plate 1a) to seasonal (Plate 1d). The histogram has a resolution of 0.001 x 0.001. The amplitude is color coded, wherein red pixels represent counts greater than 0.04% of the total, and the other colors represent lesser amplitudes. The geometric "triangle inequality" dictates that if the force balance is composed of the terms on the right-hand side of (1), then all data in Plate 1 should fall within the open square defined by \(x + y > 1\), \(y - x < 1\), and \(x - y < 1\), where \(x = |F_a + F_w + F_{ct}|\) and \(y = |F_i|/|F_a|\). Some data in Plate 1a lie just outside this region near (1,0) and represent instances when the acceleration term plays a small role.

The figure shows that longer period averaging tends to "smear" the bimodal structure observed in the daily plot. There seems to be a transition near the weekly time scale: shorter period averaging shows that \(F_a\) is often balanced by either \(F_w + F_{ct}\) or by \(F_i\), while longer period averaging shows more of an equal balance. We suspect that the transition is tied to the timescale of synoptic atmospheric disturbances. What is very clear, however, is that the force balance is a strong function of the averaging period.

3.4. Varying the Ice Pressure Parameter \(P^*\)

What is the sensitivity of our model to variations in the ice pressure parameter \(P^*\)? Hibler [1979] found that the main effect of doubling \(P^*\) was a decrease in the spatial variation of mean ice thickness. Fleming [1989] confirmed this result (using a simplified ice rheology) and also found that the domain average ice thickness decreased with increasing \(P^*\). Direct observations of in situ stress

\[
\begin{align*}
\text{(a) Winter} & \quad \begin{cases} F_i \\ F_{ct} \\ U \end{cases} \\
\text{(b) Spring} & \quad \begin{cases} F_i \\ F_{ct} \\ U \end{cases} \\
\text{(c) Summer} & \quad \begin{cases} F_i, F_{ct} \\ U \end{cases} \\
\text{(d) Fall} & \quad \begin{cases} F_i \\ F_{ct} \\ U \end{cases}
\end{align*}
\]
Figure 12: The force balance, as in Figures 6-10, for January 6, 1984. The viscous forces have been combined into one vector denoted $F_{vis}$. Also shown is the ice velocity field on this day.
and strain with which we could determine the parameter $P^*$ are rare, although recent field campaigns may offer some hope [Coon et al., 1993; Northwest Research Associates, Inc., 1995]. Thus $P^*$ has traditionally been tuned in numerical models to provide reasonable fields of ice motion and thickness. In basin-wide simulations this yields typical ice pressures of order $10^4$–$10^5$ N m$^{-1}$ [Hibler, 1980b]. Further, Overland and Pease [1988] suggested that pressure depends on the numerical grid resolution, citing values from previous studies that ranged from $10^4$ N m$^{-1}$ (for grid resolution of the order of several kilometers) to $10^5$ N m$^{-1}$ (for 100 km resolution). Recently, Flato and Hibler [1995] have investigated the sensitivity of a multilevel ice thickness model to variations in pressure (also referred to as strength).

In this section we reexamine the sensitivity of the Hibler [1979] model to variations in $P^*$ in the context of the force balance. In our standard case, we set the ice pressure parameter $P^* = 2.75 \times 10^4$ N m$^{-2}$, which is the value that has generally been used in the two-level model since Hibler and Walsh [1982]. The original value in Hibler [1979] was smaller by a factor of 5.5; here we show results from two experiments, in which $P^*$ is decreased and increased by a factor of 5.0.

Figure 15 shows the wintertime internal stress gradient in the two sensitivity cases and may be compared with Figure 8c (the standard case). The corresponding ice motion and mean thickness fields are compared with the standard case in Figure 1. The magnitudes of the Arctic average force balance for these cases are compared to the standard case in Figure 13.

Decreasing $P^*$ by a factor of 5 has the effect of decreasing the internal stress gradient (both pressure gradient and viscous terms) throughout the year and especially in winter (Figure 15a). The

Figure 13. The seasonal variation in the domain average forces, i.e., in (a) air drag, (b) water drag, (c) internal stress gradient, and (d) Coriolis plus tilt. Shown are daily average values, binned into monthly intervals for ease of presentation, for the standard case (solid lines), for the case with reduced ice pressure coefficient (dotted lines), and for the case with increased ice pressure coefficient (dashed lines). The annual average forces are also shown in each panel.
result is faster mean ice motion and thicker ice piling up against the coast of North America (Figures 1c and 1d). Some of this thicker ice is swept into the accelerated Beaufort Gyre circulation and carried into the Canadian Basin, leading to increased mean ice thickness there as well. The domain average wintertime ice thickness is 35% higher than in the standard case. The force balance (Figure 13) is nearly in free drift, wherein the internal stress gradient is negligible. The annual average, domain average ice pressure $P$ in the standard case is about $3 \times 10^5$ N m$^{-1}$ (Figure 3c), while in this case with lower $P^*$ it is $8 \times 10^5$ N m$^{-1}$. Thus the reduced $P^*$ case produces ice pressures that are just below the lowest values cited by Hibler [1980b] and Overland and Pease [1988].

Increasing $P^*$ by a factor of 5 leads, not surprisingly, to much stronger internal stress gradients (Figure 15b). But since these are already a significant force in the overall balance in the standard case, increasing $P^*$ has a degenerate effect on the simulation, as shown in Figures 1e and 1f. The ice becomes locked up to an unrealistic degree, and motion essentially ceases. The ice thickness reaches a value determined mainly by equilibrium thermodynamics and as noted by Hibler [1979] and Fleming [1989] has less spatial variability than the standard case. However, the domain average wintertime ice thickness in this case is very close (6% higher) to that in the standard case. The force balance (Figure 13) is essentially between air drag and internal stress gradients, like a sheet of solid ice over which a wind is blowing. The annual average, domain average ice pressure $P$ in this case is $2 \times 10^5$ N m$^{-1}$, just above the highest values cited by Hibler [1980b] and Overland and Pease [1988].

The model is clearly sensitive to changes in $P^*$ over the range we have used; that is, the decreased $P^*$ case is nearly in free drift, while the increased $P^*$ case is essentially locked up. The model's sensitivity to this parameter is highly nonlinear, however. Figure 16 shows the annual mean, domain mean ice thickness as a function of $P^*$. Reducing $P^*$ below the standard value leads to a
Plate 1. Histograms showing how the force balance varies with averaging period. The amplitude of internal stress gradient $F_i$ is plotted against the amplitude of water drag and Coriolis plus tilt $F_w + F_{ct}$ both normalized by the amplitude of air drag $F_a$. The vector averaging interval is (a) daily, (b) weekly, (c) monthly, and (d) seasonal. Bins (0.001 x 0.001) with less than 0.01% of the total count are not shown, while those with 0.01–0.02% are blue, 0.02–0.03% are green, 0.03–0.04% are yellow, and greater than 0.04% are red. Points near (1,0) are in free drift, while those near (0,1) are moving very slowly. The daily force balance is often dominated by a balance between $F_i$ and $F_a$ or between $F_w + F_{ct}$ and $F_a$, while the balance over other averaging periods is more equitable between the various forces.

Figure 15. The internal stress gradient $F_i$ for the sensitivity studies in which the ice pressure coefficient $P*$ is, relative to the standard case, (a) reduced by a factor of 5 and (b) increased by a factor of 5.
rapid increase in ice mass; when $P^* = 1.375 \times 10^3$ N m$^{-2}$ (one twentieth of the standard value), thickness increases without limit north of Greenland, and the model run blows up. However, increasing $P^*$ above the standard value also increases ice mass. This is because when $P^*$ is high, the model approaches the thermodynamic limit (also shown in Figure 16), wherein ice cannot be transported from areas of growth (the Arctic Basin) to areas of melt (the Greenland and Chukchi Seas). The standard value of $P^*$ for daily wind-forced simulations was tuned to give the best comparison between buoy and model-simulated buoy drifts [Hibler and Walsh, 1982]. Our standard run gives an average ratio of the daily model-simulated buoy displacement to the actual buoy displacement of 1.25, while the case with $P^*$ reduced by a factor of 5 gives a ratio of 1.56. That is, in both simulations the model velocities are too high, but the standard case provides a better fit to the data.

It is sobering to note the large effect that varying a highly uncertain parameter such as $P^*$ can have on basic model outputs such as ice thickness and velocity. This point was also noted by Fleming and Semtner [1991]. They stress that the uncertainty in these kinematic parameters could very well dominate the uncertainties in thermodynamic parameters such as ice albedo.

4. Conclusions

4.1. Large Internal Stress Gradients Throughout the Arctic Ocean

In contrast to the inference of Thorndike and Colony [1982], we find that internal stress gradients are an important part of the force balance in much of the Arctic Ocean (Figures 6-13). Yet our model agrees with their observations that geostrophic winds and sea ice motion are highly correlated (Figure 5). These two statements are in fact not contradictory since internal stress gradients act largely as a passive drag, much like the water drag force. The wind–ice correlation would be even higher if it were not for the pressure gradient term, which is a function only of the slowly varying state variables ice thickness and concentration. The pressure gradient frequently acts at a large angle with respect to the motion or the winds, for example pushing the ice in the Transpolar Drift Stream toward the Eurasian coast (Figures 7-10).

4.2. Seasonal Variation of Ice Forces

The internal stress gradient is negligible in summer, comparable in magnitude to the water drag in fall and winter, and larger than water drag in spring (Figures 6-11 and Figure 13). This is true for the domain average daily force balance and for monthly and longer time averages at particular points in the interior of the Arctic Ocean. But it is not necessarily true for a given point on any given day (Figure 12).

The seasonal variation in air drag in our model (Figure 13a) is possibly overstated, given the observations of Overland and Colony [1994]. This of course would affect the entire force balance. We have used constant drag coefficients in the air-ice and ice-ocean quadratic drag laws. This is common practice but could be a large source of error in ice-ocean models. Improvement might come from imposing a fixed seasonal cycle in these drag coefficients or by adding boundary layer models above and below the ice.

Also, the force balance is a strong function of averaging period, as has been recognized since the AIDJEX project in the 1970s [e.g., Hunkins, 1975]. We find a transition near the weekly timescale (Plate 1), which we speculate is tied to the synoptic atmospheric variability.

4.3. Validity of the Cavitating Fluid Model

Shear viscous forces are an important part of the total internal stress gradient. This is evident in seasonal vector balances (Figures 7-10) and in the daily energy balance (Figure 14b). This casts strong doubt on the validity of the cavitating fluid model, in which these forces are neglected.

4.4. Sensitivity of Ice Thickness to the Parameter $P^*$

Mean ice thickness and velocity are strong functions of the uncertain parameter $P^*$ (Figure 1). The spatial patterns of mean ice thickness show more sensitivity to changes in $P^*$ than the domain average thickness (and thus the total ice mass). But the domain average thickness can also be quite sensitive (Figure 17).
4.5. Relationship Between Water Drag and Internal Stress Gradient

Water drag and the internal stress gradient often act in a similar manner. Consider a jet in the ice motion field. Water drag damps the velocity toward the surface currents, which are small in most of the domain (Figure 4). And since it is a quadratic damping, the fastest part of the jet is damped the most. The shear viscous part of the internal stress gradient acts to eliminate shear gradients, which in the end has the same effect as the water drag: the jet is eliminated. The main difference between water drag and the shear viscous force is that the former damps the ice motion to small values in most of the domain, while the latter tends to create a "smooth" velocity field, i.e. one with small shear gradients. This is in theory, in practice, we find that water drag is much larger than viscous forces in the domain average energetics (Figure 14), while the opposite is often true on smaller spatial scales (Figure 12).

4.6. Application to a Model With a Distribution of Ice Thicknesses

We have performed preliminary experiments using an ice model in which the ice thickness distribution is explicitly calculated [Flato and Hibler, 1995], coupled to our ocean model. The analysis indicates that many of our conclusions regarding the force balance are unchanged when using this newer model. The seasonal patterns of ice forces are quite similar to those shown in this paper. The exception is the region north of the Canadian Archipelago and Greenland, where the thickness distribution model predicts generally weaker internal stress gradients and thinner ice relative to the two-level model.

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